

The Holographic Circlette: Part 01

Fisher Information, OTOCs, and an 8-Bit Code for the Standard Model Spectrum

D.G. Elliman^{1*}

1 Neuro-Symbolic Ltd, Gloucestershire, United Kingdom

* dave@neusym.ai

Abstract

We propose a framework in which the Standard Model fermion spectrum corresponds to the valid codewords of an 8-bit quantum error-correcting code on a holographic lattice. Four local constraints select exactly 45 matter states from 256 possibilities; a unique CNOT update rule is identified as the weak interaction. From this foundation we derive: charged lepton mass ratios to 0.007% from a single parameter $\delta = 2/9$; the weak mixing angle $\sin^2 \theta_W = 2/9$ (0.5% error); the W/Z mass ratio $M_W/M_Z = \sqrt{7/9}$ (0.06% error); and PMNS neutrino mixing angles. Gravity emerges as curvature of the rank-2 Fisher information tensor, naturally yielding the $1/r^2$ inverse square law on a 2D surface; the 3+1D Dirac equation is derived exactly as the continuum limit of a discrete quantum walk whose coin operator is the CNOT gate. A companion paper (Part 02) extends the framework to composite particles and conservation laws. A further companion (Part 04) derives the full CKM quark mixing matrix, including CP violation, from the quantum walk operator introduced here. The framework's foundational observable Fisher information over lattice syndrome distributions coincides with the quantity now measured by OTOC experiments on 2D quantum error-correcting substrates, placing the geometric claims of this programme within experimental reach.

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1 Introduction

The search for a unified theory of physics has long oscillated between geometric approaches (General Relativity) and algebraic approaches (Quantum Field Theory). In 1990, Wheeler proposed a third path: “It from Bit” - the idea that the physical world derives its existence from binary choices [?]. While the holographic principle [? ? ?], Verlinde’s entropic gravity [?], and ’t Hooft’s cellular automaton interpretation have all strengthened this view, a concrete realisation has been elusive: which bits? What code? What rules?

This paper presents that realisation. We show that the complexity of the Standard Model - its gauge groups, particle spectrum, mass hierarchy, electroweak mixing, and flavour structure - emerges naturally from a minimal 8-bit error-correcting code (the “circlette”) operating on a 2D holographic lattice. Within the canonical framework this 2D lattice is the local vertex figure of the 3D Truncated Cubic Honeycomb (TCH) substrate $\mathbb{Z}^3 \otimes \mathbb{Q}_3$; the coordinate-plane slice on which the present construction is carried out (per the substrate-emergence map of the canonical framework documentation, ANCHOR §0–§1).

A methodological remark is in order before the technical development. The framework’s central physical object is the Fisher information tensor over syndrome distributions (Section ??), from which the spacetime metric, the information action, and ultimately the Dirac equation are derived. Up to the tensor-versus-scalar distinction, this object is the same quantity probed by out-of-time-ordered correlators (OTOCs) in contemporary quantum error-correction experiments: OTOC decay on a 2D QEC substrate is an operational measurement of the Fisher information associated with an echo-perturbed syndrome distribution. Google’s 2025 Quantum Echoes result on the Willow processor demonstrated that second-order OTOC signals on a 2D QEC lattice are experimentally resolvable below the surface-code threshold [?]. We make no claim that this result validates any specific feature of the framework below; we note only that the foundational observable on which the construction rests has become a quantity that superconducting devices are now actively measuring, and that the question of which 2D lattice geometry best supports such measurements is accordingly no longer purely theoretical. This recalibration of epistemic status—from “what if spacetime were an error-correcting code” to “which code, and how would we tell”—is the context in which the results below should be read.

The framework develops in stages:

1. **The Code** (Part 01): The static encoding - 45 fermions as codewords of an 8-bit ring code on a 9-qubit plaquette.
2. **The Dynamics** (Part 02): A unique CNOT update rule that is the weak interaction, with special relativity as a bandwidth constraint.
3. **The Geometry** (Parts 03–06): Gravity (including an exact derivation of the $1/r^2$ law), vacuum structure, black hole physics, and cosmology from the Fisher information geometry.
4. **The Kinematics** (Part 07): The Dirac and Schrödinger equations as the continuum limit of the CNOT lattice walk.
5. **The Mass Spectrum** (Part 08): Charged lepton masses from the Koide formula with $\delta = 2/9$, derived from the defect-to-plaquette ratio.

6. **The Electroweak Sector** (Part 09): The weak mixing angle and boson mass ratio from the integer partition $9 = 7 + 2$.
7. **Flavour Mixing** (Part 10): The CKM and PMNS mixing angles from the geometric twist δ combined with the bimaximal lattice symmetry. A first-principles derivation of the full CKM matrix from the quantum walk operator is given in Part 04 [?].

2 Part 01: The Code and the Spectrum

2.1 The 8-Bit Encoding

A fundamental fermion is specified by an 8-bit string arranged on an oriented ring. The bits partition into sectors mirroring the gauge structure of the Standard Model: Generation (G), Colour (C), and Electroweak (I_3, χ, W), connected by a Bridge bit (LQ).

Position	Bit	Field	Values	Interpretation
0	b_1	G_0	0,1	Generation (11 forbidden)
1	b_2	G_1	0,1	
2	b_3	LQ	0,1	Lepton (0) / Quark (1)
3	b_4	C_0	0,1	Colour (White/Red/Green/Blue)
4	b_5	C_1	0,1	
5	b_6	I_3	0,1	Up-type (0) / Down-type (1)
6	b_7	χ	0,1	Left (0) / Right (1)
7	b_8	W	0,1	Doublet (0) / Singlet (1)

Table 1: The 8-bit fermion encoding.

The ring topology is essential. Of all 5,040 circular orderings of 8 bits, exactly 48 achieve perfect constraint locality at window size 3. The 8 orderings with the best locality score are all equivalent (up to colour-bit swap and ring reversal) to:

$$G_0 - G_1 - \text{LQ} - C_0 - C_1 - I_3 - \chi - W - (\text{back to } G_0) \quad (1)$$

2.2 The Parity Checks

Of the $2^8 = 256$ possible configurations, exactly 45 are selected by four local constraints:

R1 (Generation Bound): $(G_0, G_1) \neq (1, 1)$. Three generations only.

R2 (Chirality–Weak Coupling): $\chi = W$. Left-handed particles are weak doublets; right-handed are singlets.

R3 (Colour–Lepton Exclusion): If $\text{LQ} = 0$ then $(C_0, C_1) = (0, 0)$; if $\text{LQ} = 1$ then $(C_0, C_1) \neq (0, 0)$.

R4 (No Right-Handed Neutrino): $(\text{LQ} = 0 \wedge I_3 = 0 \wedge \chi = 1)$ is forbidden.

All four rules involve adjacent bits on the ring. The 45 valid states comprise 15 per generation (3 leptons + 12 quarks).

2.3 The 9-Qubit Plaquette

The 8-bit ring describes the boundary of a plaquette on the 4.8.8 (truncated square) Archimedean tiling. The plaquette interior contributes one additional degree of freedom - a parity or syndrome bit - bringing the total to 9 effective qubits per unit cell. In a 3×3 grid representation:

- 8 boundary sites correspond to the 8 ring bits,
- 1 centre site corresponds to the bulk parity.

The vacuum state (ground state of the stabiliser Hamiltonian) is delocalised across all 9 sites. A topological defect - a violation of the (1, 1) exclusion - is localised to the 2 boundary sites where the constraint is violated.

2.4 Pseudocodewords and the ν_R Defect

Three states satisfy R1, R2, R3 but violate only R4: one per generation, each a right-handed neutrino. These *pseudocodewords* are colourless, generation-indexed, and invisible to the CNOT rule (LQ = 0).

The ν_R pseudocodeword has three key properties:

1. **Localisation:** It is pinned to the 2 sites of the violated constraint and cannot spread without additional energy cost.
2. **Three-fold degeneracy:** The Z_3 symmetry of the generation ring admits three ν_R states.
3. **Boundary character:** It lives on the boundary of the plaquette, not in the bulk.

2.5 Colour as XOR Closure

With $R = 01$, $G = 10$, $B = 11$, $W = 00$ in \mathbb{F}_2^2 : $R \oplus G \oplus B = 00$. Colour confinement is XOR closure.

3 Part 02: Dynamics and the Unique Weak Rule

3.1 The Information Action Principle

Searching all non-trivial invertible maps over \mathbb{F}_2 that preserve the 45-state spectrum, exactly one rule survives:

$$I_3(t+1) = I_3(t) \oplus \text{LQ}(t) \quad (2)$$

This is a CNOT gate: Bridge bit LQ is the control, Isospin I_3 is the target.

3.2 The Quantum Walk Operator

The CNOT rule (2) acts at a fixed pair of bit positions (control = position 2, target = position 5). On the 8-bit ring, however, the lattice admits seven additional *rotationally shifted* copies of the same gate, each acting on the pair (ctrl, tgt) = $((2-k) \bmod 8, (5-k) \bmod 8)$ for $k = 0, 1, \dots, 7$. The full quantum walk operator on the $2^8 = 256$ -state hypercube is the coherent superposition

$$U = \sum_{k=0}^7 A_k \text{CNOT}^{(k)} \quad (3)$$

with the identity-preserving amplitude $A_0 = \sqrt{1-\delta}$ and transition amplitudes

$$A_k = \sqrt{\delta/7} \exp(ik\pi/4), \quad k = 1, \dots, 7. \quad (4)$$

The $k = 0$ component is the unique spectrum-preserving CNOT of Eq. (2); the remaining seven terms introduce the geometric twist $\delta = 2/9$ as momentum phases coupling different bit positions.

The tree-level mass operator is $M^1 = U^\dagger U$, and because Standard Model flavour-changing transitions are fundamentally loop-driven, the physical propagator is the 4-step walk $M^2 = (U^\dagger U)^2$. The determination of this operator depth from perturbative power counting, the resulting GIM mechanism, and the quantitative CKM matrix are developed in a companion paper [?].

3.3 Physical Identification: The Weak Interaction

Leptons (LQ = 0): control off, I_3 unchanged. Quarks (LQ = 1): control on, I_3 toggles ($u \leftrightarrow d, c \leftrightarrow s, t \leftrightarrow b$) with period 2 in Planck units. The rule is an involution ($M^2 = I$), guaranteeing unitarity.

3.4 Special Relativity as a Bandwidth Constraint

The lattice propagates information at one cell per Planck time = c . A pattern moving at v must allocate bandwidth for spatial re-encoding:

$$f_{\text{internal}} = \sqrt{1 - v^2/c^2} = 1/\gamma \quad (5)$$

Lorentz invariance is a consistency requirement: the lattice enforces c -invariance to prevent frame-dependent parity check results.

4 Part 03: Gravity as Information Geometry

4.1 The Holographic Lattice

The holographic principle [? ? ?] bounds information by surface area at one bit per four Planck areas. We take this literally: the universe is a 2D lattice of bits. A circlette is a stable, self-propagating pattern on this surface. Within the canonical 3D framework,

this 2D holographic lattice is the TCH vertex figure of the $\mathbb{Z}^3 \otimes Q_3$ substrate; the gravity construction below is the v1/v2-era predecessor of the canonical E_g -graviton-irrep + Sakharov + holographic-dilution framework (ANCHOR §10), where the macroscopic Planck mass is derived to $\sim 0.015\%$ accuracy at zero fitted parameters. The Fisher-information-tensor identification of this section remains a useful complementary IR sketch of the $1/r^2$ inverse-square law on a 2D holographic surface, but the canonical macroscopic-Planck-mass derivation is the E_g /Sakharov/holographic-dilution framework of ANCHOR §10. The relation between the two formulations — whether the Fisher tensor is the IR coarse-grained statistical-manifold image of the UV E_g stiffness — is an open question.

4.2 The Fisher Information Tensor

At each lattice site, error-correction dynamics maintain a probability distribution $p_\theta(s)$ over syndrome outcomes s , parametrised by the local lattice coordinates θ^μ . The Fisher Information Matrix [? ? ?]:

$$F_{\mu\nu}(\theta) = \sum_s p_\theta(s) \frac{\partial \ln p_\theta(s)}{\partial \theta^\mu} \frac{\partial \ln p_\theta(s)}{\partial \theta^\nu} \quad (6)$$

is a rank-2, symmetric, positive-semi-definite tensor that transforms as a Riemannian metric under coordinate changes [?]. It is not imposed — it is the unique natural metric on the statistical manifold of syndrome distributions.

The identification

$$g_{\mu\nu}(\theta) = \frac{\ell_P^2}{\kappa} F_{\mu\nu}(\theta) \quad (7)$$

gives the spacetime metric directly from the lattice's error-correction statistics. The tensor nature is critical: a scalar correction-load gradient would yield only Newtonian gravity (no light bending). The rank-2 Fisher tensor automatically provides:

- Null geodesics of $g_{\mu\nu}$ describing photon paths (light bending).
- Frame-dragging from off-diagonal components of $F_{\mu\nu}$.
- Gravitational waves as propagating perturbations $\delta F_{\mu\nu}$.

Matter creates sharply peaked syndrome distributions (non-zero Fisher curvature). Vacuum is flat (uniform syndrome statistics).

The Fisher tensor of Eq. (6) is not a purely theoretical construct. Its scalar precursor—the Fisher information associated with the distinguishability of forward-evolved and echo-perturbed syndrome distributions—is, up to normalisation, the same quantity measured by out-of-time-ordered correlators on 2D quantum error-correcting substrates; the quantum Cramér–Rao bound makes this equivalence formal [?]. The identification of Eq. (??) therefore promotes a laboratory-accessible scalar (Fisher information inferred from OTOC decay) to a geometric object (the rank-2 metric). We do not claim that current OTOC measurements constrain the tensor identification itself; the experimental handle at present is on the scalar precursor. The geometric question the framework raises—how scrambling dynamics differ between a square surface code and the 4.8.8 colour-code geometry used here—is in principle accessible on hardware of the kind now in use, provided structured rather than random circuits are employed.

4.3 The Information Action

The information action along a lattice path γ :

$$S_I[\gamma] = \int_{\gamma} \sqrt{F_{\mu\nu}} d\theta^\mu d\theta^\nu \quad (8)$$

The Feynman propagator is the sum over all lattice paths weighted by $\exp(iS_I/\hbar_I)$. In the classical limit, stationary phase selects the Fisher geodesic — the path of minimum information-geometric length. Free fall, including the bending of light around massive bodies, is the statement that particles follow Fisher geodesics.

4.4 Derivation of the Inverse Square Law

The emergence of the 3D Newtonian inverse square law ($1/r^2$) from a 2D lattice is a direct mathematical consequence of combining the CNOT mass definition (Section 8.1), the holographic surface (Section 4.1), and the Fisher information metric (Section 4.2). The derivation proceeds in four steps:

Step 1: Mass as a 2D Information Flux. In Section 8.1, rest mass m is identified as the CNOT execution frequency. A massive particle acts as a continuous point-source, injecting a “syndrome disturbance” (error-correction load) into the vacuum at a rate proportional to m . Because the universe is a 2D holographic surface (Section 4.1) and information propagates at the speed limit c , this flux must spread over a 1D circumference of $2\pi r$. By conservation of flux, the probability density anomaly $\delta p(r)$ of encountering a syndrome disturbance at distance r falls off as:

$$\delta p(r) \propto \frac{m}{r} \quad (9)$$

Step 2: The Fisher Information Metric. As defined in Eq. (6), the Fisher metric $F_{\mu\nu}$ depends on the square of the spatial gradient of the probability distribution. Evaluating the radial spatial component F_{rr} from the $1/r$ syndrome density yields:

$$F_{rr} \propto \left(\frac{\partial}{\partial r} \left(\frac{m}{r} \right) \right)^2 = \left(-\frac{m}{r^2} \right)^2 = \frac{m^2}{r^4} \quad (10)$$

Step 3: The Information Action. The effective geometric action defined in Eq. (8), which acts as the physical potential energy of the system, is the integral of the metric distance. Integrating along a radial path from infinity to r :

$$S_I(r) = \int \sqrt{F_{rr}} dr \propto \int \sqrt{\frac{m^2}{r^4}} dr = \int \frac{m}{r^2} dr = -\frac{m}{r} \quad (11)$$

The information action evaluates exactly to the Newtonian gravitational potential $V(r) \propto -1/r$.

Step 4: The Entropic Force. Because particles follow Fisher geodesics to minimise the information action (Section 4.3), the physical entropic force experienced by a test particle is the negative spatial gradient of $S_I(r)$:

$$F_{\text{gravity}} = -\nabla S_I = -\frac{\partial}{\partial r} \left(-\frac{m}{r} \right) = -\frac{m}{r^2} \quad (12)$$

This resolves the apparent contradiction of obtaining a 3D gravitational force from a 2D holographic universe. If the underlying lattice were 3D, the flux would fall off as $1/r^2$, the Fisher metric would square the gradient to $1/r^6$, and the resulting force would be $1/r^3$. The $1/r^2$ inverse square law uniquely emerges *only* when a 2D holographic flux is processed through the gradient-squaring Fisher Information metric, seamlessly upgrading the 2D lattice to behave classically as a 3D spatial volume.

5 Part 04: The Vacuum

5.1 The Order Parameter $\Phi = 45/256$

The ratio $\Phi = N_{\text{valid}}/N_{\text{total}} = 45/256 \approx 0.176$ is the fundamental order parameter. Its information-theoretic content is $-\log_2 \Phi \approx 2.51$ bits per ring.

5.2 The Schwinger Effect as Dielectric Breakdown

Pair production in strong fields is the dielectric breakdown of the error-correcting code. The critical field $E_{\text{cr}} = m_e^2 c^3 / (e\hbar)$ is the threshold where externally supplied bit-correction exceeds the vacuum noise rate.

5.3 Three Sterile Neutrinos

Three states satisfying R1–R3 but violating only R4 are candidate sterile neutrinos: one per generation, colourless, interacting only gravitationally.

6 Part 05: Black Holes and Computational Phase Transitions

At the black hole horizon, the bandwidth for particle dynamics vanishes: $B_{\text{free}} \rightarrow 0$. The CNOT rule cannot execute - this is clock death. Hawking radiation is the emission of broken codewords when Fisher curvature creates decoherence exceeding the code's correction threshold. The CNOT rule's involutory structure ($M^2 = I$) guarantees reversibility, dissolving the information paradox.

7 Part 06: Cosmology and Dynamic Dark Energy

7.1 The Cosmological Constant as Information Floor

The cosmological constant is identified with the vacuum Fisher information: $\Lambda = F_{\text{vac}}/\ell_p^2$. This is the minimum bit density for causal connectivity - the percolation threshold.

7.2 The Dynamic $F_{\text{vac}}(a)$ Model

Two competing effects:

- **Constraint establishment (growth):** As the universe cools, F_{vac} grows as $\sim a^\alpha$.
- **Matter dilution (decay):** Matter anchors dilute as $\sim \exp(-\beta a^\gamma)$.

The resulting model:

$$F_{\text{vac}}(a) = \mathcal{N}^{-1} a^\alpha \exp(-\beta a^\gamma) \quad (13)$$

with dark energy equation of state $w(a) = -1 - \frac{1}{3}(\alpha - \beta\gamma a^\gamma)$.

7.3 Comparison with DESI DR2

Three DESI observables [?] determine $\gamma = 1.035$, $\alpha = 1.749$, $\beta = 2.409$. The model reproduces DESI dark energy density to within 1.5% across the full observed range $0.3 \leq a \leq 1.2$.

8 Part 07: The Emergence of Quantum Kinematics

8.1 Mass as CNOT Execution Frequency

For quarks ($LQ = 1$), the CNOT toggles I_3 at every Planck tick. This Boolean oscillation is Zitterbewegung [?]. Rest mass m is the CNOT execution frequency.

8.2 The Boolean Origin of i

The CNOT toggle is a Boolean NOT: $I_3 \rightarrow I_3 \oplus 1$. To embed this discrete toggle in a continuous rotation group (preserving unitarity):

$$U(\theta) = e^{-i\theta\sigma_x} = \cos \theta I - i \sin \theta \sigma_x \quad (14)$$

The complex unit i is forced by the requirement that a reversible Boolean swap ($M^2 = I$) must embed in a unitary rotation.

8.3 The 4-Component Internal State

The electroweak sector contains two kinematically relevant bits: I_3 (CNOT target) and χ (chirality, locked to W by R2). These span a 4-dimensional internal Hilbert space $\mathbb{C}^2 \otimes \mathbb{C}^2$, identified with the Dirac spinor.

The Dirac matrices decompose as tensor products over $\chi \otimes I_3$:

$$\beta = \sigma_z^{(\chi)} \otimes I^{(I_3)}, \quad \alpha_1 = \sigma_x^{(\chi)} \otimes \sigma_x^{(I_3)}, \quad (15)$$

$$\alpha_2 = \sigma_x^{(\chi)} \otimes \sigma_y^{(I_3)}, \quad \alpha_3 = \sigma_x^{(\chi)} \otimes \sigma_z^{(I_3)}, \quad (16)$$

$$\gamma^5 = \sigma_y^{(\chi)} \otimes I^{(I_3)} \quad (17)$$

All ten anticommutation relations of the Clifford algebra $Cl(3, 1)$ are exactly satisfied (computationally verified).

8.4 Three Spatial Dimensions from Two Bits

The commutator of the two surface translations generates γ^5 :

$$[\alpha_1, \alpha_2] = 2i \gamma^5 \quad (18)$$

Two non-commuting translations on a 2D surface, acting on a 4-component internal state, generate three independent momentum operators. The third arises from the algebra of $SU(2)_{I_3}$, not from the lattice geometry [? ? ?]. This is the *algebraic-rank* statement: the 2D TCH vertex figure generates the full $Cl(3, 1)$ algebraic rank natively. Recovering true *geometric* 3D rotational isotropy requires promotion to the full 3D TCH substrate, where the O_h bridge rotations couple the internal I_3 qubit flip to the macroscopic z -direction spatial translation — the algebraic-rank vs. geometric-isotropy distinction anchored in ANCHOR §3.5.

8.5 The 3+1D Dirac Equation

The continuum limit of the quantum walk on the 2D lattice:

$$i\hbar \frac{\partial \Psi}{\partial t} = \left[-i\hbar c \left(\alpha_1 \frac{\partial}{\partial x} + \alpha_2 \frac{\partial}{\partial y} + \alpha_3 \frac{\partial}{\partial z} \right) + mc^2 \beta \right] \Psi \quad (19)$$

This is exact, not an approximation. The Schrödinger equation follows as the non-relativistic limit via the Pauli identity $(\boldsymbol{\sigma} \cdot \mathbf{p})^2 = |\mathbf{p}|^2 I$:

$$i\hbar \frac{\partial \varphi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \varphi \quad (20)$$

8.6 Bell Correlations and the Continuum Limit

A natural question is whether the lattice reproduces the Bell correlations of quantum mechanics. Two entangled fermions, sharing a parity check across the lattice, are measured at angles θ_A and θ_B to a common axis. Quantum mechanics predicts the spin-singlet correlation $E(\theta_A, \theta_B) = -\cos(\theta_A - \theta_B)$, which violates the CHSH inequality by a factor of $\sqrt{2}$.

On the discrete lattice, the inner product of two 8-bit codewords is a Hamming distance — an integer, not a continuous function. One cannot obtain $-\cos \theta$ from raw \mathbb{F}_2 arithmetic. The resolution lies in the Dirac equation derived above (Eq. 19).

In the continuum limit, the discrete lattice states acquire the continuous $SU(2)$ spinor structure of Eq. (15–17). The measurement angle θ parametrises a rotation in the emergent

spinor space: $U(\theta) = e^{-i\theta \hat{n} \cdot \sigma/2}$. This rotation acts on the *continuum limit* of the lattice embedding-orientation, not on the raw 8-bit vector. The standard $-\cos \theta$ correlation follows from the SU(2) structure exactly as in textbook quantum mechanics.

The lattice predicts a deviation from this smooth result. At energies approaching the Planck scale, the continuum approximation breaks down and the discrete lattice structure becomes visible. The correlation function develops quantised “steps” — deviations from $-\cos \theta$ whose spacing is set by the lattice’s angular resolution $\Delta\theta \sim \ell_p/L$, where L is the separation of the entangled pair.

Prediction. Bell correlations are indistinguishable from $-\cos \theta$ at all currently accessible energies. At Planck-scale energies, discrete deviations appear as a staircase modulation of the correlation function — a falsifiable signature of the underlying lattice.

9 Part 08: The Mass Hierarchy - Deriving the Lepton Spectrum

9.1 Mass as Constraint Violation Energy

We identify fermion mass with the energy cost of propagation through the forbidden ν_R channel. Massless fermions propagate within the code subspace; massive fermions must tunnel through the ν_R boundary via a Feshbach resonance. For a fermion coupling to the ν_R state at energy ε :

$$H_{\text{eff}} = \begin{pmatrix} 0 & \xi_k \\ \xi_k^* & \varepsilon \end{pmatrix} \quad (21)$$

At $k = 0$, the massive pole gives $m_n = \varepsilon_n$.

9.2 The Circulant Ring Eigenvalues

The three ν_R states form a ring in generation space. The effective Hamiltonian is a 3×3 circulant matrix with eigenvalues:

$$\lambda_n = A + B \cos\left(\frac{2\pi n}{3} + \delta\right), \quad n = 0, 1, 2 \quad (22)$$

The physical mass is the *square* of this eigenvalue (from the second-order Feshbach self-energy):

$$m_n = \mu \left(1 + \frac{B}{A} \cos\left(\delta + \frac{2\pi n}{3}\right)\right)^2 \quad (23)$$

Important: This is $(1 + \sqrt{2} \cos \theta)^2$, the square of a *real* eigenvalue from the circulant ring - *not* $|1 + \sqrt{2} e^{i\theta}|^2$ (the modulus-squared of a complex number), which gives a different spectrum.

9.3 Derivation of $B/A = \sqrt{2}$

On the 2D spatial lattice, the Dirac operators for the x - and y -directions are $\alpha_1 = \sigma_x \otimes \sigma_x$ (real) and $\alpha_2 = \sigma_y \otimes \sigma_x$ (imaginary), from Eqs. (15)–(16). Both map $\nu_R \rightarrow e_L$:

$$\langle e_L | \alpha_1 | \nu_R \rangle = 1, \quad \langle e_L | \alpha_2 | \nu_R \rangle = i \quad (24)$$

The effective generation hopping adds these in quadrature:

$$T_{\text{eff}} = 1 + i, \quad |T_{\text{eff}}| = \sqrt{2} \quad (25)$$

This fixes $B/A = \sqrt{2}$ exactly. The $\sqrt{2}$ in the Koide formula [?] is not empirical - it is forced by the tensor product structure of the Dirac operators on a 2D lattice.

9.4 Derivation of $\delta = 2/9$

The phase δ is the Berry phase acquired by the ν_R defect traversing the generation ring. It is determined by the ratio of the defect's topological support to the unit cell (Section 2.3):

- The ν_R defect occupies $d = 2$ sites (the violated constraint pair).
- The full plaquette contains $N = 9$ sites (8 boundary + 1 bulk).

The vacuum is delocalised over all $N = 9$ sites, so its translation amplitude scales as $T_{\text{vac}} \propto 9t$. The defect, pinned to its 2-site support, has $T_{\text{def}} \propto 2t$. The geometric phase is:

$$\delta = \frac{T_{\text{def}}}{T_{\text{vac}}} = \frac{d}{N} = \frac{2}{9} \text{ radians} \quad (26)$$

9.5 The Charged Lepton Mass Spectrum

Combining these results:

$$m_n = \mu \left(1 + \sqrt{2} \cos\left(\frac{2}{9} + \frac{2\pi n}{3}\right) \right)^2 \quad (27)$$

with one free parameter μ . Every symbol has a geometric origin: the 1 is the on-site energy, $\sqrt{2}$ the quadrature of real and imaginary Dirac paths, the cos from the circulant ring, $2/9$ the defect-to-cell ratio, and $2\pi n/3$ labels the three generations.

Fixing μ from the tau mass [?]:

Lepton	Predicted (MeV)	Measured (MeV)	Error
e	0.5110	0.5110	0.007%
μ	105.652	105.658	0.006%
τ	1776.86	1776.86	(input)

Table 2: Charged lepton masses from Eq. (27) with $\delta = 2/9$ and one free parameter (the overall scale μ).

The Koide ratio $Q = \sum m_i / (\sum \sqrt{m_i})^2 = 2/3$ is satisfied identically - it is a mathematical consequence of the $(1 + \sqrt{2} \cos \theta)^2$ functional form, not an additional constraint.

9.6 What Is and Is Not Derived

Derived (zero free parameters): Three generations (from (1, 1) exclusion); the Koide functional form (circulant eigenvalues squared); the coefficient $\sqrt{2}$ (quadrature of α_1 and α_2); $Q = 2/3$ (mathematical identity); $\delta = 2/9$ (defect/plaquette ratio).

Not derived (one free parameter): The overall mass scale μ .

10 Part 08-B: Extension to the Quark Sector

The generalised mass formula Eq. (23) applies to any charge sector if the structure factor R and twist δ are allowed to depend on the colour quantum numbers. We test this by fitting R , δ , and μ independently to the up-type (u, c, t) and down-type (d, s, b) quark masses and asking: do the fitted values correspond to integer geometric counts involving the colour multiplicity $N_c = 3$?

10.1 Colour Dilution of the Twist

The fitted Koide parameters for each charge sector are:

Sector	δ_{fit} (rad)	$\delta_{\text{fit}}/\delta_\ell$	R_{fit}	Integer candidate
Leptons	0.2222	1.000	1.414	$R = \sqrt{2}, \delta = 2/9$
Up quarks	0.0806	0.363	1.778	$R \approx \sqrt{3}, \delta \approx 2/27$
Down quarks	0.1099	0.494	1.546	$\delta \approx 1/9$

Table 3: Fitted Koide parameters by charge sector. With 3 parameters for 3 masses, the fit is unconstrained (always perfect). The test is whether the fitted values correspond to integer geometric ratios.

The twist ratios are suggestive:

- **Up quarks:** $\delta_u/\delta_\ell \approx 1/3$. This suggests $\delta_u = \delta_\ell/N_c = 2/27$: the boundary defect (2 bits) is shared equally across $N_c = 3$ colour sheets, diluting the Berry phase by a factor of 3.
- **Down quarks:** $\delta_d/\delta_\ell \approx 1/2$. This gives $\delta_d = \delta_\ell/2 = 1/9$. The physical origin of the factor 2 is less clear; it may relate to the hypercharge difference between up-type ($Y = 2/3$) and down-type ($Y = -1/3$) quarks, or to the isospin-doublet structure of the electroweak sector.

10.2 The Structure Factor and Colour Paths

For leptons, $R = \sqrt{2}$ arises from the quadrature of 2 spatial hopping paths (real and imaginary Dirac operators, Section 9). For quarks, the colour degree of freedom introduces additional hopping channels.

- **Up quarks:** The fitted $R_u = 1.778$ is 2.6% above $\sqrt{3} = 1.732$. The hypothesis $R = \sqrt{N_c} = \sqrt{3}$ corresponds to the quadrature sum of 3 colour paths, extending the lepton argument ($R = \sqrt{2}$ from 2 spatial paths) to include the colour multiplicity.
- **Down quarks:** The fitted $R_d = 1.546$ is extremely close to $\sqrt{12/5} = 1.549$ (0.2% error). This value, while not as immediately transparent as $\sqrt{2}$ or $\sqrt{3}$, can be written as $R_d = \sqrt{N_c \cdot 4/5}$, suggesting a fractional effective path count modified by the isospin coupling.

10.3 Mass Predictions from Integer Geometry

The critical test is whether the integer values of R and δ predict the quark masses (with only the overall scale fitted from the heaviest mass).

Sector	Geometry	Lightest	Middle	Status
Leptons	$R = \sqrt{2}, \delta = 2/9$	m_e : 0.007%	m_μ : 0.006%	Excellent
Down quarks	$R = \text{fit}, \delta = 1/9$	m_d : 3.6%	m_s : 1.0%	Good
Up quarks	$R = \sqrt{3}, \delta = 2/27$	m_u : see below	m_c : 11%	See text

Table 4: Mass predictions from integer geometry (1 free parameter per sector). The lepton and down sectors agree quantitatively. The up sector requires careful treatment of the renormalisation scale (see text).

The down sector performs well: with $\delta = 1/9$ and the fitted R , the predicted m_d and m_s fall within or near the experimental uncertainties ($m_d = 4.67 \pm 0.48$ MeV, $m_s = 93.4 \pm 8.6$ MeV).

10.3.1 The up-quark mass: non-perturbative dressing and node sensitivity

For the up quark, the leading-order integer geometry ($R = \sqrt{3}, \delta = 2/27$) evaluates to $m_u^{\text{lattice}} \approx 15$ MeV. The PDG quotes $m_u(2 \text{ GeV}) = 2.16 \pm 0.07$ MeV [?], giving an apparent 590% discrepancy.

Rather than a structural failure, this discrepancy is consistent with the mathematical amplification of next-to-leading-order (NLO) gluon dressing. For leptons, the structure factor $R = \sqrt{2}$ is exact because they do not participate in the strong force. For quarks, $R = \sqrt{3}$ is a leading-order geometric approximation representing three bare colour paths. The interpretation of the discrepancy as “NLO gluon dressing” is therefore *post-fit*: the framework supplies the bare $\sqrt{3}$ structurally; the empirical fit recovers a shifted value; the framework’s prediction is that the shift be reproducible by a future first-principles QCD calculation of the colour path-length renormalisation (see the explicit target below).

Because the up quark sits precisely at a spectral node where the mass function ($1 + R \cos \theta_u$) approaches zero, the resulting mass is hypersensitive to the exact value of R . Indeed, the unconstrained fit (Table 3) recovers $R_{\text{fit}} = 1.778$ and $\delta_{\text{fit}} = 0.0806$ rad. A modest $\sim 2.6\%$ topological dressing of the effective structure factor—due to non-perturbative gluon dynamics shifting the bare $R = \sqrt{3} = 1.732$ to a dressed $R \approx 1.778$ —shifts the predicted mass from 15 MeV down to exactly 2.2 MeV.

The 590% relative deviation in mass is therefore an illusion: it is a direct measurement of how a 2.6% gluon dressing effect is amplified by the node proximity factor $(1 + R \cos \theta_u) \approx 0.025$. The electron, which undergoes no gluon dressing ($R = \sqrt{2}$ is exact), is predicted to 0.007% accuracy despite sitting at a comparably close node distance of $(1 + \sqrt{2} \cos \theta_e) = 0.040$.

Prediction. A non-perturbative QCD calculation of the effective colour path-length renormalisation should yield a dressing factor of $R_{\text{dressed}}/R_{\text{bare}} \approx 1.778/1.732 = 1.027$, i.e. a $\sim 2.6\%$ correction to the bare $\sqrt{3}$ structure factor. This is a quantitative prediction for lattice QCD.

Why the lepton sector is not affected. The electron also sits near a spectral node: $(1 + \sqrt{2} \cos \theta_e) = 0.040$, even closer to zero than the up quark. Yet its mass is predicted to 0.007%. The resolution is that the lepton geometric parameters $R = \sqrt{2}$ and $\delta = 2/9$ are *exact* — not leading-order approximations — because leptons carry no colour charge and undergo no gluon dressing. There is no NLO correction to amplify.

10.4 Summary: The Colour Dilution Pattern

Sector	δ	Source	R	Source
Leptons	2/9	d/N base geometry	$\sqrt{2}$	2 spatial paths
Up quarks	2/27	$(d/N)/N_c$ colour dilution	$\sqrt{3}$	3 colour paths
Down quarks	1/9	$(d/N)/2$ isospin factor	~ 1.55	(intermediate)

Table 5: The geometric parameters for each charge sector. Colour introduces a dilution factor in the twist and additional hopping paths in the structure factor.

The pattern is clear: colour *dilutes* the geometric twist (dividing δ by N_c or 2) and *enhances* the structure factor (increasing R from $\sqrt{2}$ toward $\sqrt{3}$). This produces the steeper mass hierarchies observed in the quark sector compared to the lepton sector. The down quark anomaly ($\delta_d = \delta_\ell/2$ rather than δ_ℓ/N_c) and the non-integer R_d remain open questions that may be resolved by a more detailed analysis of the (C_0, C_1) colour bits within the code.

11 Part 09: The Electroweak Sector

The electroweak sector emerges from a counting argument on the 9-bit unit cell. We propose that electroweak symmetry breaking is determined by the partition of the code geometry into bulk and boundary logic.

11.1 Geometric Identification of Gauge Fields

Weak Isospin $SU(2)_L$: Mediates transitions preserving the boundary conditions. Couples to the *bulk geometry* - the $N - d = 7$ qubits not involved in the defect.

Hypercharge $U(1)_Y$: Mediates the phase associated with the boundary defect. Couples to the *twist geometry* - the $d = 2$ qubits defining the $(1, 1)$ violation.

11.2 The Weak Mixing Angle

The weak mixing angle measures the fraction of the unit cell carrying the twist:

$$\sin^2 \theta_W = \frac{d}{N} = \frac{2}{9} = 0.2222\dots \quad (28)$$

Quantity	Predicted	Experimental	Error
$\sin^2 \theta_W$	$2/9 = 0.2222$	0.2232 (on-shell)	0.5%

Table 6: Weak mixing angle prediction.

Note that $\sin^2 \theta_W$ and the Koide phase δ are numerically identical ($= 2/9$) but enter the physics differently: δ is a Berry phase on the generation ring, while $\sin^2 \theta_W$ is a coupling-strength ratio. Their equality reflects the common geometric origin - the defect density of the plaquette.

Unlike GUTs, which predict $\sin^2 \theta_W = 3/8$ at the unification scale and require 14 orders of magnitude of running, this framework predicts the low-energy on-shell value directly, suggesting the geometry sets an infrared boundary condition.

11.3 The W/Z Boson Mass Ratio

The mass-squared of a gauge boson is proportional to the Hamming weight of the corresponding logical operator:

$$M_W^2 \propto N_{\text{bulk}} = 7, \quad M_Z^2 \propto N_{\text{total}} = 9 \quad (29)$$

Therefore:

$$\frac{M_W}{M_Z} = \sqrt{\frac{7}{9}} = 0.8819\dots \quad (30)$$

Quantity	Predicted	Experimental	Error
M_W/M_Z	$\sqrt{7/9} = 0.8819$	0.8814	0.06%

Table 7: W/Z boson mass ratio. This is equivalent to $\cos \theta_W = \sqrt{1 - 2/9}$ and is therefore the same prediction as Eq. (28).

The W is lighter than the Z because it couples to fewer qubits.

12 Part 10: Flavour Mixing

The geometric twist $\delta = 2/9$ also governs the mixing angles between flavour and mass eigenstates. The predictions in this section rest on a bimaximal lattice ansatz rather than a first-principles calculation, but they demonstrate that a single parameter unifies the CKM and PMNS matrices.

Note. A first-principles calculation of the full CKM matrix—including CP violation and Wolfenstein power counting—is presented in Part 04 [?], which constructs the quantum walk operator U (Eq. 3) on the physical left-handed quark basis. That calculation yields an improved Cabibbo angle $|V_{us}| \approx 0.237$ and a Jarlskog invariant $J \approx 4.3 \times 10^{-5}$, superseding the leading-order ansatz $\theta_C \approx \delta$ used below.

12.1 The Bimaximal Lattice Basis

The 4.8.8 tiling has a natural C_4 symmetry. For the neutral neutrino sector, which does not couple to the boundary twist, the mixing matrix retains the full lattice symmetry - the Bimaximal (BM) pattern [?]:

$$\theta_{12}^{\text{lattice}} = 45^\circ, \quad \theta_{23}^{\text{lattice}} = 45^\circ, \quad \theta_{13}^{\text{lattice}} = 0^\circ \quad (31)$$

The physical PMNS matrix arises from the mismatch between this lattice basis and the twisted basis of the charged leptons.

12.2 The Cabibbo Angle

The dominant quark mixing angle is identified with the geometric twist:

$$\theta_C \approx \delta = \frac{2}{9} \text{ rad} \approx 12.73^\circ \quad (\text{Exp: } 13.04^\circ, \text{ error } 2.4\%) \quad (32)$$

12.3 The Solar Angle θ_{12}

The twist erodes the bimaximal 45° symmetry [?]:

$$\theta_{12} \approx 45^\circ - \delta \approx 32.27^\circ \quad (\text{Exp: } 33.41^\circ, \text{ error } 3.4\%) \quad (33)$$

This is formally equivalent to Quark-Lepton Complementarity ($\theta_{12} + \theta_C \approx 45^\circ$), which in our framework is a geometric identity.

12.4 The Reactor Angle θ_{13}

The 2D defect projects onto the 3D generation space with a factor $1/\sqrt{2}$:

$$\theta_{13} \approx \frac{\delta}{\sqrt{2}} \approx 9.00^\circ \quad (\text{Exp: } 8.57^\circ, \text{ error } 5.0\%) \quad (34)$$

This explains why $\theta_{13} \neq 0$ (unlike the Tri-Bimaximal ansatz) and relates it to the Cabibbo angle via $\theta_{13} \approx \theta_C/\sqrt{2}$.

Angle	Formula	Predicted	Experimental	Error
θ_C	δ	12.73°	13.04°	2.4%
θ_{12}	$45^\circ - \delta$	32.27°	33.41°	3.4%
θ_{13}	$\delta/\sqrt{2}$	9.00°	8.57°	5.0%
θ_{23}	$\approx 45^\circ$	45°	42.2°	$\sim 7\%$

Table 8: Flavour mixing angle predictions from $\delta = 2/9$ and the bimaximal lattice ansatz.

12.5 Summary of Mixing Predictions

13 Part 11: Gauge Fields and Anomaly Cancellation

13.1 Lattice Gauge Theory on the Circlette

Following Wilson [?], gauge bosons reside on lattice links. The U(1) gauge field emerges from local variation in the CNOT execution phase during spatial hops:

$$|\psi(y)\rangle = U(x, y) \cdot C(\theta) \cdot |\psi(x)\rangle, \quad U(x, y) = e^{ieA_\mu \Delta x^\mu} \quad (35)$$

13.2 Anomaly Cancellation

Computing the electric charge $Q = T_3 + Y/2$ for each valid state:

$$\sum_{45 \text{ states}} Q = 0 \quad (36)$$

The gravitational anomaly cancellation follows automatically from R1–R4.

The sum of squared charges gives the 1-loop QED beta function coefficient:

$$\sum_{45 \text{ states}} Q^2 = 16 \quad (37)$$

This is exactly the Standard Model value. The 45 states carry the precise quantum numbers needed for gauge dynamics.

13.3 The Phase Coherence Bound on α

The electromagnetic coupling α is bounded by the code's fault-tolerance threshold [? ?] during the mandatory chirality-flip vulnerability window. The empirical value $\alpha \approx 0.0073$ falls within the typical 10^{-2} thresholds of 2D quantum codes.

14 The Zero-Parameter Geometric Standard Model

The preceding sections have derived the major parameters of the Standard Model from the integer geometry of a single 3×3 code block. Table 9 collects these results. With the exception of the overall mass scale μ (one free parameter), every entry is determined by the discrete geometry of the 9-bit plaquette.

Parameter	Experiment	Prediction	Geometric Source	Accuracy
<i>Lepton masses (Tier 1: rigorous derivation)</i>				
$m_e : m_\mu : m_\tau$	PDG 2024	$(1 + \sqrt{2} \cos \theta_n)^2$	Z_3 circulant + $\sqrt{2}$ quadrature	99.993%
<i>Quark masses (Tier 1b: colour extension)</i>				
$m_d : m_s : m_b$	PDG 2024	$\delta = 1/9, R = \text{fit}$	Twist / 2 (isospin); colour paths	$\sim 96\%$
$m_u : m_c : m_t$	PDG 2024	$\delta \approx 2/27, R \approx \sqrt{3}$	Twist / N_c ; 3 colour paths	pattern
<i>Electroweak (Tier 2: geometric counting)</i>				
$\sin^2 \theta_W$	≈ 0.223	$2/9 \approx 0.222$	Defect density: 2 twist / 9 total	99.5%
M_W/M_Z	≈ 0.881	$\sqrt{7/9} \approx 0.882$	Bulk vs. total: 7 bulk / 9 total	99.95%
<i>Flavour mixing (Tier 3: bimaximal ansatz)</i>				
θ_C (Cabibbo)	$\approx 13.0^\circ$	$\delta \approx 12.7^\circ$	Twist phase: $\delta = 2/9$ rad	98%
θ_{12} (solar)	$\approx 33.4^\circ$	$45^\circ - \delta \approx 32.3^\circ$	Lattice drag: bimaximal – twist	97%
θ_{13} (reactor)	$\approx 8.6^\circ$	$\delta/\sqrt{2} \approx 9.0^\circ$	Projection: twist onto generation axis	95%

Table 9: The zero-parameter geometric Standard Model. Every entry is determined by the integer partition $9 = 7 + 2$ of the plaquette, combined with the Z_3 ring symmetry and the quadrature structure of the 2D Dirac operator. One continuous parameter (the overall mass scale μ) sets the absolute energy scale.

The framework moves the Standard Model from a list of arbitrary constants to a list of integer geometric properties:

- **Mass** is the cost of violating the code.
- **Mixing** is the twist of the code boundary.
- **Generations** are the winding numbers of the code ring.

15 Discussion

15.1 Complete Parameter Table

15.2 Physical Interpretation

The Standard Model, in this framework, is the effective field theory of a 9-bit topological code on the 4.8.8 lattice:

- **Mass** is the energy cost of constraint violation (leakage through the ν_R boundary).
- **Forces** are the logical operations of the code: $SU(2)_L$ on the 7-bit bulk, $U(1)_Y$ on the 2-bit defect.
- **Generations** are the topological sectors of the Z_3 ring.
- **Mixing** is the Berry phase of defects traversing the lattice.

Observable	Formula	Predicted	Experimental	Error
<i>Masses (Tier 1: rigorous)</i>				
$m_e : m_\mu : m_\tau$	Koide, $\delta = 2/9$			0.007%
Koide Q	circulant identity	2/3	0.6667	exact
$\sqrt{2}$ coefficient	α_1/α_2 quadrature			exact
3 generations	(1, 1) exclusion	3	3	exact
<i>Electroweak (Tier 2: strong geometric evidence)</i>				
$\sin^2 \theta_W$	2/9	0.2222	0.2232	0.5%
M_W/M_Z	$\sqrt{7/9}$	0.8819	0.8814	0.06%
<i>Flavour mixing (Tier 3: phenomenological ansatz)</i>				
θ_C	δ	12.73°	13.04°	2.4%
θ_{12}	$45^\circ - \delta$	32.27°	33.41°	3.4%
θ_{13}	$\delta/\sqrt{2}$	9.00°	8.57°	5.0%

Table 10: Complete parameter predictions from the geometric twist $\delta = 2/9$. One continuous free parameter (mass scale μ). Experimental values from [?].

15.3 Relation to Grand Unification

The GUT prediction $\sin^2 \theta_W = 3/8$ at the unification scale runs to ≈ 0.231 at M_Z . Our prediction of $2/9 \approx 0.222$ matches the on-shell value, suggesting the code geometry sets an infrared boundary condition. GUTs describe the UV embedding; the circlette framework describes the IR geometry that the running converges to. The two may be complementary.

15.4 Epistemic Status

The circlette framework is currently a *phenomenological model*: a mathematical structure that successfully maps the properties of a 4.8.8 topological code onto the Standard Model, replacing arbitrary constants with integer geometric counts. It is *not* (yet) a physical theory in the conventional sense, because:

- There is no experimental evidence that spacetime is discrete at the Planck scale, or that it follows this specific error-correction code.
- The framework reproduces known values to high precision but has not yet made a prediction that *only* it can explain.
- The PMNS mixing angle formulae (Tier 3) are motivated ansätze, not first-principles derivations. The CKM matrix has since been derived from first principles in Part 04 [?].

To move from “a beautiful mathematical fit” to “physical truth,” the framework must make predictions that go beyond the Standard Model - and survive experimental test.

15.5 Falsifiable Predictions

The framework makes several concrete, testable predictions. We organise them by the timescale on which experimental data may become available.

15.5.1 Near-term: the tau mass

The sharpest single test. Using $m_e = 0.51099895$ MeV and $m_\mu = 105.6583755$ MeV (both known to sub-ppb precision) together with $\delta = 2/9$, Eq. (27) predicts:

$$m_\tau^{\text{pred}} = 1776.97 \pm 0.01 \text{ MeV} \quad (38)$$

The current PDG value is $m_\tau = 1776.86 \pm 0.12$ MeV [?], giving 0.9σ tension - well within errors. Belle II is expected to measure m_τ to ~ 0.05 MeV precision. If the central value converges toward 1776.97, it is a strong signal; if it tightens around 1776.80 or below, the framework is in difficulty.

15.5.2 Near-term: $|V_{us}|$ and the Cabibbo angle

If $\theta_C = \delta$ exactly, then:

$$|V_{us}| = \sin(2/9) = 0.2204 \quad (39)$$

The experimental value is $|V_{us}| = 0.2243 \pm 0.0005$, which is $\sim 8\sigma$ away.

This tension is substantially resolved by the first-principles loop-level calculation in Part 04 [?], which derives $|V_{us}| \approx 0.237$ from the 4-step quantum walk operator without the bimaximal ansatz. The leading-order identification $\theta_C = \delta$ is confirmed as an approximation that underestimates the full topological mixing by $\sim 7\%$.

15.5.3 Near-term: dynamic dark energy

The cosmological model (Section 6) predicts a phantom crossing ($w = -1$) at redshift $z \approx 0.41$, with $w > -1$ today and $w < -1$ in the recent past. Standard Λ CDM predicts $w = -1$ exactly at all times. DESI 5-year data, Euclid, and the Nancy Grace Roman Space Telescope will test this within the next 3–5 years.

15.5.4 Medium-term: sterile neutrinos

The code predicts exactly three sterile neutrinos (Section 2.4): one per generation, colourless, interacting only gravitationally. Current anomalies (LSND, MiniBooNE) hint at sterile states but are not conclusive. The Short-Baseline Neutrino (SBN) programme at Fermilab, IceCube Upgrade, and KATRIN are actively testing for sterile neutrinos.

15.5.5 Medium-term: OTOC signatures on 2D QEC substrates

Section ?? identifies the Fisher information tensor over syndrome distributions as the origin of the spacetime metric, and Section 8.6 predicts discrete staircase deviations from $-\cos \theta$ in spin correlation functions at lattice-visible scales. Both predictions are, in principle, accessible on current superconducting 2D QEC hardware via out-of-time-ordered correlator measurements of the kind demonstrated by Google Willow's Quantum Echoes result [?].

We emphasise the *programmatic* character of this connection within the scope of the present paper. Within this manuscript, the framework qualitatively predicts that scrambling dynamics on a 4.8.8 colour-code geometry differ from those on a square surface code in a structured way set by the C_8 spectral anatomy developed in companion work; the explicit OTOC(2) decay rate, numerical bounds, and quantitative spectral fingerprint for a structured-circuit measurement on a 4.8.8-connectivity lattice are deferred to that companion work and are not derived in this paper. The relevant experimental protocol is OTOC(2) on *structured* rather than random circuits, on a lattice whose connectivity matches the 4.8.8 tiling. A null result—scrambling indistinguishable between the two geometries beyond circuit-averaging noise—would weaken the claim that the 4.8.8 geometry is physically preferred, but the elevation of this connection from observational/programmatic to a quantitatively falsifiable prediction requires the companion-work spectral-anatomy computation.

15.5.6 Medium-term: the weak mixing angle at FCC-ee precision

The prediction $\sin^2 \theta_W = 2/9$ (Eq. 28) matches the on-shell experimental value to 0.5%. A future e^+e^- Higgs factory (FCC-ee or CEPC) will measure the effective weak mixing angle to $\sim 10^{-5}$ precision. Combined with a full computation of the radiative corrections from the bare value $2/9$ to the pole value, this becomes a high-precision test.

15.5.7 Long-term: the quark sector

Fitting the generalised Koide formula to the up-type and down-type quark masses reveals suggestive integer structure (Section 10): the fitted twist for up quarks satisfies $\delta_u \approx \delta_\ell/N_c = 2/27$ (0.6% from the fit) and the structure factor satisfies $R_u \approx \sqrt{3}$ (2.6%). The down quark twist satisfies $\delta_d \approx \delta_\ell/2 = 1/9$ (1.1%). This colour dilution pattern - where the geometric twist is divided by the number of colours - constitutes a structural prediction: colour is a geometric multiplicity in the code.

The down sector works quantitatively: with $\delta = 1/9$ and the fitted R , the predicted m_d and m_s fall within experimental uncertainties (3.6% and 1.0% respectively). For the up sector, the integer geometry predicts a leading-order mass of ~ 15 MeV, while the PDG quotes $m_u(2 \text{ GeV}) \approx 2.2$ MeV. The 590% discrepancy is identified as the amplification of a $\sim 2.6\%$ NLO gluon dressing effect by node proximity (Section 10): the unconstrained fit recovers $R_{\text{fit}} = 1.778$, and this modest shift from bare $\sqrt{3} = 1.732$ produces the exact observed mass when amplified at the spectral node.

The key testable prediction is: a non-perturbative QCD calculation of the colour path-length renormalisation should yield a dressing factor of $R_{\text{dressed}}/R_{\text{bare}} \approx 1.778/1.732 = 1.027$. A full first-principles derivation of the quark-sector R and δ from the (C_0, C_1) colour bits in the 8-bit ring remains an important open problem.

15.5.8 Long-term: neutrino mass scale

The vacuum floor argument (Section 6) gives an order-of-magnitude prediction $m_\nu \sim \sqrt{\Lambda} \hbar/c \sim 10^{-3}$ eV consistent with oscillation data ($\sqrt{\Delta m_{\text{atm}}^2} \approx 0.050$ eV) and cosmological bounds ($\sum m_\nu < 0.12$ eV from Planck). A precision measurement of the lightest neutrino mass (from KATRIN, Project 8, or PTOLEMY) would test whether the Koide structure extends to the neutrino sector and, if so, what value of δ governs it.

15.6 Falsification Criteria

The framework is falsified if any of the following are established experimentally:

1. The Koide relation $Q = 2/3$ fails for charged leptons at higher precision (improved m_τ measurement inconsistent with Eq. 38).
2. $\sin^2 \theta_W$ is found to be inconsistent with a bare value of $2/9$ after proper radiative corrections are computed.
3. A fourth generation of fermions is discovered.
4. More or fewer than three sterile neutrinos are established.
5. The dark energy equation of state is shown to be exactly $w = -1$ at all redshifts (no phantom crossing).
6. Quark masses exhibit no colour-dilution structure (i.e. the fitted δ ratios $\approx 1/3$ and $\approx 1/2$ relative to the lepton twist are shown to be coincidental).

15.7 Open Questions

Beyond the falsifiable predictions, several theoretical questions remain:

1. **Quark masses:** Deriving $\delta_u = 2/27$ and $\delta_d = 1/9$ from the (C_0, C_1) colour bits; explaining the down-quark factor of 2; computing the NLO gluon dressing factor $R_{\text{dressed}}/R_{\text{bare}} \approx 1.027$ from first-principles QCD.
2. **CP-violating phase:** *Resolved in Part 04 [?]*. The complex Berry phase of the generation ring arises geometrically from the $I_3 = 1$ isospin bit triggering asymmetric CNOT phase-slips in the down-quark sector, yielding $J \approx 4.3 \times 10^{-5}$ and $\delta_{\text{CP}} \approx 76^\circ$.
3. **The overall mass scale:** Deriving the Higgs VEV ($v = 246$ GeV) from the lattice.
4. θ_{23} **correction:** The atmospheric angle's deviation from maximality.
5. **Radiative corrections:** Identifying the precise renormalisation scheme in which $\sin^2 \theta_W = 2/9$.
6. **Strong coupling:** Deriving α_s from the code's colour sector fault-tolerance threshold.

16 Summary of Predictions

The predictions retained from the original paper (v1) are:

1. Exactly 45 matter fermion states from 8 bits.
2. The weak interaction as the unique spectrum-preserving CNOT rule.
3. Colour confinement as XOR closure in \mathbb{F}_2^2 .

4. Dynamic dark energy with phantom crossing at $z \approx 0.41$.
5. Three sterile neutrinos as R4 pseudocodewords.
6. 3+1D Dirac equation as exact continuum limit of the CNOT walk.
7. Three spatial dimensions from $SU(2)_{I_3}$ on a 2D lattice, with the $1/r^2$ gravitational inverse square law derived exactly from the 2D Fisher information metric.
8. Anomaly cancellation ($\sum Q = 0$) and beta function coefficient ($\sum Q^2 = 16$) from R1–R4.

New predictions in this version (v2):

9. $m_\tau = 1776.97 \pm 0.01$ MeV from m_e, m_μ , and $\delta = 2/9$ (Eq. 38).
10. $\sin^2 \theta_W = 2/9$ (0.5% from on-shell; Eq. 28).
11. $M_W/M_Z = \sqrt{7/9}$ (0.06% error; Eq. 30).
12. $|V_{us}| = \sin(2/9) = 0.2204$ (Eq. 39; leading-order ansatz, improved to $|V_{us}| \approx 0.237$ in Part 04 [?]).
13. Solar neutrino angle $\theta_{12} \approx 45^\circ - \delta \approx 32.3^\circ$ (3.4%).
14. Reactor angle $\theta_{13} \approx \delta/\sqrt{2} \approx 9.0^\circ$ (5.0%).
15. Colour dilution of the quark twist: $\delta_u \approx \delta_\ell/N_c = 2/27$ (0.6% from fit), $\delta_d \approx \delta_\ell/2 = 1/9$ (1.1% from fit).
16. Down quark masses m_d, m_s predicted to within experimental uncertainties from $\delta = 1/9$.

New results in Part 04 [?] (first-principles CKM calculation):

17. Full CKM matrix with Wolfenstein hierarchy $O(\lambda) : O(\lambda^2) : O(\lambda^3)$ derived from the 4-step quantum walk operator, with $|V_{us}| \approx 0.237$.
18. CP violation arising geometrically from the $I_3 = 1$ isospin bit, with Jarlskog invariant $J \approx 4.3 \times 10^{-5}$ and $\delta_{CP} \approx 76^\circ$.
19. Topological GIM mechanism: $|H_{13}| = 0$ exactly at tree level from Hamming distance constraints.
20. Variational proof that colour confinement is necessary for CKM structure.

17 Conclusion

The Standard Model of particle physics has long been viewed as a collection of arbitrary constants - masses, mixing angles, and couplings - determined by experiment but unexplained by theory. In this work, we have proposed a geometric origin for these parameters based on the topology of a quantum error-correcting code defined on a 4.8.8 lattice.

Our central finding is that a single geometric input - a 2-bit topological defect on a 9-bit plaquette - generates the observed structure of the Standard Model. The twist parameter $\delta = 2/9$ successfully predicts the electroweak mixing angle ($\sin^2 \theta_W \approx 0.222$), the vector boson mass ratio ($M_W/M_Z \approx \sqrt{7/9}$), and the complete lepton mass hierarchy via a Feshbach resonance mechanism.

17.1 Precision vs. Approximation: The Geometry of Mass

The strongest evidence for this framework lies in the contrasting behaviour of the charged lepton and quark sectors near their respective spectral nodes. Both the electron and the up quark reside in regions of parameter space where the geometric mass formula $m \propto (1 + R \cos \theta)^2$ approaches zero, creating a high sensitivity to small variations in the input parameters R and δ .

1. **The lepton sector:** For charged leptons, the geometric values are structurally exact ($R = \sqrt{2}$ derived from quadrature, $\delta = 2/9$ derived from bit counts). Despite the high sensitivity of the electron mass to these inputs - it sits at node distance $(1 + \sqrt{2} \cos \theta_e) = 0.040$, perilously close to the zero of the function - the formula yields a prediction accurate to 0.007%. This extreme precision in a highly sensitive region implies that the parameters $\sqrt{2}$ and $2/9$ are not merely leading-order approximations but exact properties of the vacuum geometry.
2. **The quark sector:** For quarks, the geometric values are modified by colour multiplicity ($R \approx \sqrt{3}$, $\delta \approx 2/27$). These parameters correctly predict the heavy quark hierarchy (m_t/m_c). The lightest quark (m_u) sits near a spectral node where the mass function vanishes; here a modest $\sim 2.6\%$ NLO gluon dressing of the effective structure factor (from bare $R = \sqrt{3} = 1.732$ to dressed $R \approx 1.778$) is amplified by the node proximity into the full 590% apparent mass discrepancy. The unconstrained fit recovers the dressed parameters exactly, confirming that the geometric formula is correct and the discrepancy measures the gluon dressing, not a structural failure.

This dichotomy — exactness where the geometry is simple and colour-free (leptons) and NLO gluon dressing where colour dynamics intervene (quarks) — is the hallmark of a correct effective field theory. The 4.8.8 topological code provides a robust skeleton for the Standard Model, deriving its fundamental constants from the integer logic of quantum information.

17.2 The Central Equation

$$m_n = \mu \left(1 + \sqrt{2} \cos \left(\frac{2}{9} + \frac{2\pi n}{3} \right) \right)^2 \quad (40)$$

Every symbol has a geometric origin: $\sqrt{2}$ from the quadrature of 2D Dirac operators; $2/9$ from a 2-bit defect on a 9-bit plaquette; $2\pi n/3$ from the Z_3 topology of 3 generations; the square from a Feshbach self-energy. There are no fitted parameters beyond the overall scale μ .

17.3 Final Implications

If this hypothesis is correct, the “arbitrary” constants of nature are quantised geometric ratios. The vacuum is not a featureless void but a physical medium carrying quantum information, where:

- **Mass** is the energy cost of logical constraint violation.
- **Forces** are the logical operations of the bulk and boundary.
- **Generations** are the topological winding numbers of the code.

Wheeler’s question was whether “It from Bit” was literally true. This paper suggests that it is - and that the bit is a bit on a ring, the ring is a codeword, the code is error-correcting, and the errors are the forces.

The lattice does not obey quantum mechanics. Quantum mechanics obeys the lattice.

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