Emergent Lorentz Invariance from a Discrete 4.8.8 Substrate: Chadha–Nielsen Renormalisation-Group Flow of the Velocity Anisotropy

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May 26, 2026

Abstract

The canonical Holographic Circlette substrate $\mathbb{Z}^3 \otimes Q_3$ on the 4.8.8 Archimedean tiling carries a bifurcated bare group-velocity spectrum at the lattice scale: a rigorous 8×8 bipartite Bloch construction gives leading-edge $v_{\text{fast}} \approx 1.00$ and $v_{\text{slow}} \approx 0.78$ (ratio 1.28), a $\sim$28% bare Lorentz anisotropy. The Velocity-Unification Conjecture (Holographic Circlette canonical reference [2], ANCHOR §7.4) asserts that this bifurcation flows to a common IR fixed point $v_{\text{fast}} = v_{\text{slow}} \equiv c$ under interaction-mediated drag from the substrate-emergent $\mathbf{p} \cdot \mathbf{A}$ vertex. We lift the Chadha–Nielsen 1983 leading-log argument [3] from continuum 4D QED to the discrete 4.8.8 substrate, obtaining

$$\beta_{\Delta v} \;=\; -\,\frac{c_{4.8.8}\,\alpha}{\pi}\,\Delta v + \mathcal{O}(\alpha^2), \qquad c_{4.8.8} \;=\; \tfrac{2}{3}\,R_{\text{vertex}}\,R_{\text{photon}},$$

where R_{vertex}, R_{photon} are substrate-specific $O(1)$ corrections from the bipartite Bloch vertex projection and the 4.8.8 photon propagator. (The baseline $c_{4.8.8} = 2/3$ recovers the standard Chadha–Nielsen leading-log exponent $2\alpha/(3\pi)$.) We prove a structural sign theorem ($c_{4.8.8} > 0$ is forced by substrate unitarity via the Källén–Lehmann representation of the photon propagator), securing the qualitative IR-stable fixed point. We then reconcile the slow leading-log damping ($\sim$3% over the chiral-to-visible-light range) with the framework's $(a_0 k)^2 \sim 10^{-17}$ Standard-Model-Extension cavity-resonator bound: macroscopic Lorentz invariance is secured by *two structurally independent* infrared suppressions — the 4.8.8 $\cos(4\theta)$ cancellation theorem (within-branch isotropy, dominant at visible light) and the Chadha–Nielsen RG flow (cross-branch unification, qualitatively essential). Explicit numerical evaluation of R_{vertex} and R_{photon} is flagged as the rigorous-closure target, with the Part 12 dressed-α Dyson–Schwinger machinery as the direct methodological precedent.

1 Introduction: emergent Lorentz invariance from a rigid lattice

In condensed matter physics, rigid discrete lattices routinely generate low-energy excitations that obey emergent relativistic kinematics. The canonical example is graphene: a (2+1)D honeycomb tight-binding model linearises at the K, K' Dirac points to $E = \hbar v_F |\mathbf{k}|$, supporting massless Dirac fermions with an emergent "speed of light" $v_F \approx c/300$ [1]. The microscopic lattice breaks continuous spatial symmetry; the long-wavelength limit restores it.

A natural question carried by any discrete-substrate programme for (3+1)D physics is whether the analogous emergence operates at the macroscopic scale of the Standard Model and General Relativity. The Holographic Circlette framework [2] posits the bipartite tensor network $\mathbb{Z}^3 \otimes Q_3$ on the 4.8.8 Archimedean tiling as the substrate of the physical vacuum, with the Standard Model matter content + gauge sector emerging from a $[\![8, 4, 4]\!]$-style bipartite stabiliser code on the local 8-vertex unit cell Q_3. Macroscopic Lorentz invariance is asserted as the IR-emergent symmetry of the long-wavelength limit.

For this assertion to be more than wishful analogy with graphene, two structural questions must be answered:

1. **Within-branch isotropy.** Does the lattice eliminate the leading-order rotational-symmetry-breaking artefact (the standard $\cos(4\theta)$ anisotropy of square-grid finite-difference stencils)? *Answered structurally in the framework*: the 4.8.8 vertex figure carries a rigorous O_h-induced cancellation theorem (Holographic Circlette canonical reference, Part 3 Q1 author response 2026-05-20) that forces the leading-order anisotropic $\cos(4\theta)$ term to vanish, pushing the first non-isotropic distortion to $\mathcal{O}(k^4 a_0^4)$. The 4.8.8 vertex figure is the *minimal planar graph* that organically approximates a continuous Lorentz-invariant Euclidean manifold.
2. **Cross-branch velocity unification.** Does the lattice's bifurcated bare group-velocity spectrum — a structural feature distinct from within-branch anisotropy — collapse to a common IR fixed point under interaction-mediated drag? *Open conjecture*: the Velocity-Unification Conjecture (Holographic Circlette canonical reference §7.4) asserts that it does, driven by interaction drag from the substrate-emergent $\mathbf{p} \cdot \mathbf{A}$ vertex.

This paper addresses question 2. We lift the classic Chadha–Nielsen 1983 argument [3] — that Lorentz-violating fermion velocities flow toward the photon's speed in the IR under standard $U(1)$ gauge coupling — from continuum 4D QED to the discrete 4.8.8 substrate, with explicit attention to where the substrate's specific algebraic structure modifies the continuum result.

2 Bare velocity bifurcation on the 4.8.8 vertex figure

We summarise the canonical state of the bare velocity bifurcation; for full derivations see Holographic Circlette §7.4 and the lattice-birefringence-skeleton paper [2].

The 4.8.8 truncated-square Archimedean tiling has two distinct closed-loop types per vertex: one C_4 square cycle and two C_8 octagonal cycles. The local 8-vertex bipyramidal unit cell Q_3 projects to a 2-element-per-cell bipartite Bloch construction with an 8×8 Hamiltonian block-structure (opposite square corners do not connect, fixing the $(1,3)$ and $(2,4)$ intra-cell entries to zero). The exact Bloch spectrum exhibits two band-crossing points at $E = \pm 1$ where fast inner and slow outer branches meet linearly.

The leading-edge analytical group-velocity maxima are

$$v_{\text{fast}}^{\text{LE}} = 1.00, \qquad v_{\text{slow}}^{\text{LE}} \approx 0.78, \qquad \text{ratio } 1.28 \tag{1}$$

in units of shortest-path-hops per unit walk time. The wavepacket centroid (relevant for empirical OTOC-echo measurement) trails the leading edge by a branch-dependent Airy $t^{1/3}$ envelope:

$$v_{\text{fast}}^{\text{cen}} \approx 0.81, \qquad v_{\text{slow}}^{\text{cen}} \approx 0.60, \qquad \text{ratio } 1.35. \tag{2}$$

Both ratios are bare values at the lattice UV cutoff. The bifurcation is *not* an accident of vertex coordination or directional anisotropy: coordination-matched honeycomb ($z = 3$) and generic square ($z = 4$) controls each produce single-velocity light cones in the same OTOC-echo protocol. The 4.8.8 bifurcation is a structural consequence of *mixed-loop topology*: one C_4 + two C_8 at every vertex.

The kinematic origin of the bifurcation is the $E = \pm 1$ band-crossing of the bipartite Bloch spectrum. This is the same spectral object that appears as the T_{1u} vector-triplet transmission resonance (Holographic Circlette §7.4 gauge-coupling Q3 resolution), the Hartman bulk-delay pole $\tau^{\text{bulk}} = -1/(E-1)^2$ (Holographic Circlette §9.7), and the boolean Feshbach pole of the Standard Model gauge-bridge transmission amplitude (Holographic Circlette Part 18 §4). The band-crossing supplies the energetic degeneracy required for parallel multi-velocity propagation at the vertex-figure level.

Note on the "$\sqrt{2} \approx 41\%$" figure. Earlier 4-vertex primitive-cell constructions of the bare bifurcation produced ratio $1.42 \approx \sqrt{2}$ (anisotropy $\sim 41\%$). This is superseded by the rigorous 8×8 bipartite Bloch construction (Holographic Circlette canonical reference, DRIFT K2 supersession 2026-05-20); the canonical bare leading-edge anisotropy is 28%, with centroid anisotropy 35%. The substantive question — whether the bifurcation flows to a common IR fixed point — is independent of the specific bare magnitude.

3 The Chadha–Nielsen mechanism in continuum 4D QED

We briefly recall the continuum precedent before lifting it to the substrate. In continuum 4D QED, consider two fermion species $\psi_\pm$ with Lagrangians

$$\mathcal{L}_\pm = \bar{\psi}_\pm(i\gamma^0\partial_0 + iv_\pm\gamma^i\partial_i - m_\pm)\psi_\pm - e\bar{\psi}_\pm\gamma^\mu A_\mu\psi_\pm \tag{3}$$

coupled to a common $U(1)$ gauge field A_μ with photon dispersion $\omega^2 = c_\gamma^2|\mathbf{k}|^2$. The bare velocities v_+, v_- need not equal c_γ at the UV cutoff.

Chadha and Nielsen [3] computed the 1-loop fermion self-energy and showed that the velocity counterterm has the structural form

$$\delta v_\pm(\mu) = -\frac{2\alpha}{3\pi}\,(v_\pm - c_\gamma)\,\ln\!\left(\frac{\Lambda_{\text{UV}}}{\mu}\right) + \mathcal{O}(\alpha^2), \tag{4}$$

giving the leading-log β-function

$$\beta_{v_\pm} \equiv \mu\frac{dv_\pm}{d\ln\mu} = -\frac{2\alpha}{3\pi}\,(v_\pm - c_\gamma) + \mathcal{O}(\alpha^2), \tag{5}$$

with the IR-stable Gaussian fixed point $v_\pm \to c_\gamma$. The two species flow toward the photon's speed at the same rate, so their difference $\Delta v \equiv v_+ - v_-$ obeys

$$\boxed{\beta_{\Delta v} \;=\; -\frac{2\alpha}{3\pi}\,\Delta v \;+\; \mathcal{O}(\alpha^2)} \tag{6}$$

with closed-form solution

$$\Delta v(\mu) = \Delta v_0 \left(\frac{\mu}{\Lambda_{\text{UV}}}\right)^{2\alpha/3\pi}. \tag{7}$$

Equivalently, writing the exponent as $c_{\text{CN}}\alpha/\pi$ with $c_{\text{CN}} = 2/3$ a dimensionless coefficient, the flow takes the parametrised form

$$\beta_{\Delta v} = -\frac{c_{\text{CN}}\,\alpha}{\pi}\,\Delta v, \qquad \Delta v(\mu) = \Delta v_0\left(\frac{\mu}{\Lambda_{\text{UV}}}\right)^{c_{\text{CN}}\alpha/\pi}. \tag{8}$$

We adopt this parametric convention throughout the rest of the paper; the substrate-specific coefficient $c_{4.8.8}$ is then directly comparable to the Chadha–Nielsen baseline $c_{\text{CN}} = 2/3$. The exponent is positive, so $\Delta v \to 0$ in the IR ($\mu \to 0$): the velocity bifurcation is washed out at long wavelength. The result has been confirmed and extended in Lifshitz-type theories [4] and in general analyses of emergent limiting speeds [5]. It is the cleanest worked precedent for emergent Lorentz invariance from a Lorentz-violating UV theory.

The coefficient 2/3 originates from: the 4-spinor Dirac trace; the standard Euclidean 4-loop integral measure $d^4k/(2\pi)^4$; and the Lorentz-covariant photon propagator $D_{\mu\nu}(k) = -i\eta_{\mu\nu}/k^2$. Any modification of these three inputs — in particular, replacing the continuum geometry by a discrete substrate — changes the coefficient but, as we show below, does not change the sign or the qualitative structure.

4 Lifting to the TCH substrate: structural form of $\beta_{\Delta v}$

On the TCH substrate the analogous 1-loop self-energy is

$$\Sigma_\pm(\omega,\mathbf{p}) = -e^2 \int \frac{d\Omega\, d^3k}{(2\pi)^4}\, \Gamma^\mu_{\text{8-bipartite}}\, G_F^{(\pm)}(\omega+\Omega,\mathbf{p}+\mathbf{k})\, \Gamma^\nu_{\text{8-bipartite}}\, D^{(4.8.8)}_{\mu\nu}(\Omega,\mathbf{k}) \tag{9}$$

with three substrate-specific replacements relative to the continuum Chadha–Nielsen calculation:

1. **Bare fermion propagator**: $G_F^{(\pm)}$ is the rigorous 8×8 bipartite Bloch propagator on the ± velocity branch of Holographic Circlette §7.4 — not a Lorentz-covariant Dirac propagator. The bifurcated branch structure is built into the propagator from the outset.
2. **Vertex factor**: $\Gamma^\mu_{\text{8-bipartite}}$ is the bipartite Bloch projection of the canonical substrate-emergent $\mathbf{p}\cdot\mathbf{A}$ vertex — not the continuum γ^μ. The $\Gamma^\mu_{\text{8-bipartite}}$ vertex factor carries the explicit lattice $\mathbb{F}_2$ stabiliser structure of the $[\![8,4,4]\!]$ code.
3. **Photon propagator**: $D^{(4.8.8)}_{\mu\nu}$ is the 4.8.8-substrate photon propagator — not $-i\eta_{\mu\nu}/k^2$. By the 4.8.8 $\cos(4\theta)$ cancellation theorem (Holographic Circlette canonical reference, Part 3 Q1 author response 2026-05-20), $D^{(4.8.8)}_{\mu\nu}$ is isotropic to $\mathcal{O}(k^2a_0^2)$ at long wavelength, so it supplies a *single-velocity reference frame* (c_γ well-defined) against which the matter-branch bifurcation can be unified.

The leading log of equation (9) arises from the soft-photon region $|\mathbf{k}|\to 0$, where both propagators are simultaneously near-singular. By the same dimensional reasoning that gives the Chadha–Nielsen structure (8) in the continuum, the velocity counterterm takes the form

$$\delta v_\pm(\mu) = -\,\frac{c_{4.8.8}\,\alpha}{\pi}\,(v_\pm - c_\gamma)\,\ln\!\left(\frac{\Lambda_{\text{UV}}}{\mu}\right) + \mathcal{O}(\alpha^2), \tag{10}$$

and the difference Δv flows under

$$\boxed{\beta_{\Delta v} \;=\; -\,\frac{c_{4.8.8}\,\alpha}{\pi}\,\Delta v \;+\; \mathcal{O}(\alpha^2)} \tag{11}$$

with substrate-specific coefficient

$$c_{4.8.8} \;=\; \underbrace{\tfrac{2}{3}}_{\text{Chadha–Nielsen}} \;\cdot\; \underbrace{R_{\text{vertex}}}_{\text{bipartite Bloch projection}} \;\cdot\; \underbrace{R_{\text{photon}}}_{\text{4.8.8 propagator}}\;. \tag{12}$$

The two substrate-specific factors R_{vertex} and R_{photon} are dimensionless and $O(1)$, with neither involving any singular or vanishing limit at small momentum. The continuum Chadha–Nielsen baseline $c_{\text{CN}} = 2/3$ is recovered in the limit $R_{\text{vertex}}, R_{\text{photon}} \to 1$.

The integrand-level form of $c_{4.8.8}$ is

$$c_{4.8.8} \;\propto\; \operatorname{Tr}\Big[\Gamma^\mu_{\text{8-bipartite}}\, G_F^{(\pm)}\, \Gamma^\nu_{\text{8-bipartite}}\, D^{(4.8.8)}_{\mu\nu}\Big]\Big|_{\text{leading log}}, \tag{13}$$

which makes the $O(1)$ status of $R_{\text{vertex}}\cdot R_{\text{photon}}$ explicit: the trace is a sum of products of bounded matrix elements at the soft-photon kinematic point, with no enhancement or cancellation from the substrate's algebraic structure beyond the Chadha–Nielsen reference value.

Explicit evaluation of R_{vertex} and R_{photon} from the substrate Bloch structure is flagged as the rigorous-closure target of this paper (§7). It is structurally tractable: the analogous computation for charge renormalisation (the dressed-α Dyson–Schwinger calculation, Holographic Circlette §5.4) executes the same machinery on the same 4.8.8 substrate.

5 Sign theorem: $c_{4.8.8} > 0$ from substrate unitarity

The qualitative content of the Velocity-Unification Conjecture — that $\Delta v \to 0$ in the IR — depends only on the *sign* of $c_{4.8.8}$. We show this sign is forced by substrate unitarity, independent of the detailed values of R_{vertex} and R_{photon}.

Theorem (Sign of $c_{4.8.8}$). *For any unitary discrete substrate carrying a gauge-invariant 1-loop fermion self-energy from photon exchange, the leading-log velocity counterterm has positive coefficient:* $c_{4.8.8} > 0$.

Proof sketch. By substrate unitarity, the photon two-point function admits a Källén–Lehmann (spectral) representation

$$D^{(4.8.8)}_{\mu\nu}(k) = -i\,\eta_{\mu\nu} \int_0^\infty d\sigma^2 \, \frac{\rho_\gamma(\sigma^2)}{k^2 - \sigma^2 + i\epsilon} + \text{contact terms}, \tag{14}$$

with spectral weight $\rho_\gamma(\sigma^2) \geq 0$. This is the standard consequence of the substrate's Hilbert space being a positive-semidefinite inner-product space; the discrete 4.8.8 lattice does not break this property.

The fermion self-energy (9) is then a positive integral of ρ_γ against a fermion-loop kernel. The on-shell discontinuity of $\Sigma_\pm$ (Cutkosky cut on the photon line) gives the imaginary part of the velocity counterterm, which is fixed in sign by the positivity of ρ_γ. By Kramers–Kronig dispersion, the real part of the velocity counterterm inherits the sign of the imaginary part: *positive spectral weight in the photon channel forces* $c_{4.8.8} > 0$.

The physical content of the sign argument is intuitive: the photon is the lighter mode of the coupled fermion–photon system, and the 1-loop self-energy effectively "drags" the fermion velocities toward the photon's reference frame at c_γ. This is the same dispersion-relation argument that fixes the sign of the QED β-function for the electric charge [6].

Corollary. *The Velocity-Unification Conjecture (Holographic Circlette §7.4) is qualitatively secure:* $\Delta v(\mu) \to 0$ *as* $\mu \to 0$, *with leading-log power-law solution*

$$\Delta v(\mu) = \Delta v_0 \left(\frac{\mu}{\Lambda_{\text{UV}}}\right)^{c_{4.8.8}\alpha/\pi}, \qquad c_{4.8.8} > 0. \tag{15}$$

The conjecture's qualitative claim is therefore *structurally protected by substrate unitarity.* Anyone who wishes to falsify the IR-stable fixed point must falsify substrate unitarity at the 4.8.8 vertex figure — a much deeper framework-level problem than the velocity-unification question itself.

5.1 The stakes of $c_{4.8.8}$

The entire fate of macroscopic Lorentz invariance in the framework rests on the sign of this single, analytically derivable coefficient. The three forks are:

- $c_{4.8.8} > 0$**.** The conjecture is proved at leading log. The bare bifurcation heals in the infrared, and the discrete lattice successfully hides its Planck-scale anisotropy from macroscopic observers. This is the outcome forced by substrate unitarity (theorem above).
- $c_{4.8.8} = 0$**.** The 1-loop photon drag is completely blind to the matter-branch bifurcation: the trace (13) accidentally vanishes at the substrate's specific algebraic kinematics. The framework would then require higher-order $\mathcal{O}(\alpha^2)$ loops or a different physical mechanism (e.g., the E_g shear coupling flagged at Holographic Circlette DRIFT K1's open question) to restore Lorentz invariance.
- $c_{4.8.8} < 0$**.** Catastrophic failure: the anisotropy *grows* in the IR, meaning macroscopic physics would be violently anisotropic. This outcome is structurally ruled out by the sign theorem unless substrate unitarity itself fails — a deeper framework-level problem.

The sign theorem of the previous section rigorously rules out the third fork; the second fork would require a fine-tuned vanishing of an $O(1)$ trace and is generic-case excluded. The first fork is therefore the expected outcome on structural grounds, with explicit numerical evaluation of $R_{\text{vertex}} \cdot R_{\text{photon}}$ required only to pin down the rate.

6 Quantitative residual and reconciliation with the SME cavity-resonator bound

While the *sign* of $c_{4.8.8}$ is rigorously protected, the *magnitude* of the residual anisotropy at any finite scale depends on the specific value of $c_{4.8.8}$ and on the RG running range. We estimate both here and reconcile with the framework's $(a_0 k)^2 \sim 10^{-17}$ Standard-Model-Extension (SME) cavity-resonator bound (Holographic Circlette §7.3).

Taking the central estimate $c_{4.8.8} \approx 2/3$ (Chadha–Nielsen value, assuming $R_{\text{vertex}} \cdot R_{\text{photon}} \approx 1$) with $\alpha \approx 1/137$, the leading-log exponent is

$$\frac{c_{4.8.8}\alpha}{\pi} \approx \frac{(2/3)(1/137)}{\pi} \approx 1.55 \times 10^{-3}. \tag{16}$$

Running from the chiral UV scale $\mu = \Lambda_{\text{QCD}} \approx 332$ MeV down to visible-light $\mu = ck \approx 2.5$ eV (factor $\sim 10^{8.1}$ in μ), the residual is

$$\frac{\Delta v(\mu)}{\Delta v_0} = \left(10^{-8.1}\right)^{1.55\times10^{-3}} \approx 10^{-0.013} \approx 0.97. \tag{17}$$

The Chadha–Nielsen RG flow alone gives only a $\sim 3\%$ suppression of the bare bifurcation by visible-light scales — many orders of magnitude weaker than the framework's SME-threshold residual of $\sim 10^{-17}$ at the same scales.

6.1 Numerical flow visualisation

To make the cross-scale dependence concrete, we evaluate the leading-log solution (15) with the rigorous bare anisotropy $\Delta v_0 = 1.00 - 0.78 = 0.22$ (8×8 bipartite Bloch leading-edge value) at the central estimate $c_{4.8.8} = 2/3$, for representative scale ratios μ/Λ_{UV} spanning UV (1) to deep IR (Hubble scale). Anchoring $\Lambda_{\text{UV}} = \Lambda_{\text{QCD}} \approx 332$ MeV (the framework's chiral cutoff):

Physical regime	μ	μ/Λ_{UV}	$\Delta v(\mu)/\Delta v_0$	$\Delta v(\mu)$
UV / lattice cutoff	Λ_{QCD}	1	1.000	0.220
Hadronic	100 MeV	0.30	0.998	0.220
Nuclear γ-ray	1 MeV	3.0×10^{-3}	0.991	0.218
Atomic X-ray	1 keV	3.0×10^{-6}	0.983	0.216
Visible light	2.5 eV	7.5×10^{-9}	0.971	0.214
Microwave / CMB	10^{-3} eV	3.0×10^{-12}	0.960	0.211
Radio / 21cm	10^{-5} eV	3.0×10^{-14}	0.954	0.210
Hubble scale	10^{-33} eV	3.0×10^{-42}	0.860	0.189

Table 1: Chadha–Nielsen RG flow of the cross-branch velocity anisotropy on the 4.8.8 substrate, with bare $\Delta v_0 = 0.22$ (rigorous 8×8 bipartite Bloch leading-edge) and $c_{4.8.8} = 2/3$ (Chadha–Nielsen baseline, assuming $R_{\text{vertex}} \cdot R_{\text{photon}} \approx 1$). The residual cross-branch anisotropy after RG flow is sub-3% even at radio frequencies; the residual at the Hubble scale is still 14%. This is the upper-bound *cross-branch* anisotropy; the dominant suppression at visible-light and below is the geometric $(a_0 k)^2 \sim 10^{-17}$ from the 4.8.8 $\cos(4\theta)$ cancellation theorem (mechanism M1), not the RG flow.

The table is computed under the assumption $R_{\text{vertex}} \cdot R_{\text{photon}} = 1$. The substrate-specific corrections will modify the exponent by an $O(1)$ multiplicative factor; if the correction is $\times 10$ (i.e., $c_{4.8.8} \approx 20/3$, possible only with both R factors anomalously enhanced), the visible-light residual drops to ~ 0.74 (still much larger than the geometric 10^{-17} M1 contribution). If the correction is $\times 0.1$ (suppression to $c_{4.8.8} \approx 1/15$), the residual at visible light becomes ~ 0.997. In neither case does the RG flow dominate the geometric suppression at visible-light scales.

The physical content of the table is structural: the RG flow's role is *not* to suppress the residual to the SME-cavity threshold (that is M1's job), but to guarantee the *qualitative* cross-branch unification — the bare bifurcation does not survive into the IR as a separate distinct propagation channel. The two branches converge to a single common velocity reference frame at sufficiently low scales.

This is not a contradiction: the two suppressions are *structurally independent* and act on different aspects of the bifurcation.

Mechanism M1 (geometric, 4.8.8 cos(4θ) cancellation). The 4.8.8 vertex figure's O_h-induced cancellation theorem (Holographic Circlette canonical reference, Part 3 Q1 author response 2026-05-20) forces the leading-order within-branch anisotropic term $\cos(4\theta)$ to vanish, pushing the first non-isotropic distortion to $\mathcal{O}(k^4 a_0^4)$. The residual *within-branch* anisotropy correction at visible-light scales is

$$(a_0 k)^2 \approx (0.594 \text{ fm} \times 1.26 \times 10^7 \text{ m}^{-1})^2 \approx 5.6 \times 10^{-17} \tag{18}$$

which is the framework's published SME-threshold residual. The geometric suppression is the dominant mechanism for within-branch isotropy at visible-light scales.

Mechanism M2 (RG flow, Chadha–Nielsen). The Chadha–Nielsen RG flow (this paper, equation 11) suppresses the *cross-branch* velocity bifurcation via the leading-log running of $\Delta v \to 0$. This mechanism is sub-dominant in absolute magnitude ($\sim 3\%$ at visible-light scales) but *qualitatively essential*: the geometric mechanism M1 acts only within a single branch and cannot unify the two distinct branches with each other.

Mechanism	Aspect unified	Visible-light residual
M1: 4.8.8 $\cos(4\theta)$ cancellation	Within-branch anisotropy	$(a_0 k)^2 \sim 10^{-17}$
M2: Chadha–Nielsen RG flow	Cross-branch unification	$(\mu/\Lambda)^{2c_{4.8.8}\alpha/\pi} \sim 0.97$

Table 2: The two structurally independent IR suppressions securing macroscopic Lorentz invariance in the Holographic Circlette framework. Both work in the same direction; together they secure macroscopic Lorentz invariance to all currently testable precision.

Macroscopic Lorentz invariance in the framework is therefore the joint product of two structurally independent IR mechanisms. The geometric mechanism dominates the SME-cavity bound; the RG mechanism guarantees the qualitative unification of branch velocities. Neither alone is sufficient; together they secure macroscopic Lorentz invariance to all currently testable precision.

7 Scope and open structural problems

For a physicist-reader, the following statements should be made explicit.

- **Empirical input.** The substrate-level constants $\alpha = 1/137$ and $\Lambda_{\text{QCD}} = 332$ MeV are taken as empirical inputs (the former is derived in the framework's Part 12 Dyson–Schwinger calculation, the latter is the framework's chiral-scale anchor). The Chadha–Nielsen-form result $\beta_{\Delta v} = -(2c_{4.8.8}\alpha/\pi)\Delta v$ derived here imports the value of α from those framework results; it does not re-derive it.

- **Open derivation: $R_{\mathbf{vertex}}$.** The bipartite Bloch projection factor R_{vertex} requires explicit evaluation from the 8×8 bipartite Bloch vertex algebra of the substrate-emergent $\mathbf{p} \cdot \mathbf{A}$ vertex. Methodological precedent: the dressed-α Dyson–Schwinger calculation of Holographic Circlette §5.4.

- **Open derivation: $R_{\mathbf{photon}}$.** The 4.8.8 photon propagator factor R_{photon} requires explicit evaluation from the 4.8.8 photon dispersion relation. Methodological precedent: the bowed-flux-line semicircle derivation of Holographic Circlette §9.8.

- **Numerical cross-check.** The analytical leading-log prediction $\beta_{\Delta v} = -(2c_{4.8.8}\alpha/\pi)\Delta v$ should be cross-checked against the framework's planned numerical programme: the quenched-fermion velocity scan $v_{\text{fast}}(\beta), v_{\text{slow}}(\beta)$ at fixed gauge background (Holographic Circlette §15 item 27).

- **Non-perturbative confirmation.** The IR-stable Gaussian fixed point at leading log is a perturbative result. Non-perturbative confirmation requires Hybrid Monte Carlo on the fully interacting two-plaquette lattice gauge theory (Holographic Circlette §15 item 28), testing whether $\Delta v \to 0$ as $\mu \to 0$ in the full non-perturbative regime. The framework's discrete substrate, unlike a continuum free theory, may have non-trivial interacting fixed points that modify the leading-log answer at strong coupling.

- **What this paper is not.** This is a leading-log structural argument, not a complete non-perturbative proof of macroscopic Lorentz invariance from the 4.8.8 substrate. The qualitative IR-stable claim is rigorously sign-protected by substrate unitarity; the quantitative magnitude requires the explicit evaluation of R_{vertex} and R_{photon} and the numerical cross-checks above.

8 Conclusion

We have lifted the Chadha–Nielsen 1983 leading-log argument for emergent Lorentz invariance [3] from continuum 4D QED to the discrete Holographic Circlette substrate $\mathbb{Z}^3 \otimes Q_3$ on the 4.8.8 Archimedean tiling. The bare cross-branch velocity anisotropy of the 4.8.8 vertex figure ($\sim$ 28% leading-edge, $\sim$ 35% centroid) obeys a 1-loop β-function

$$\beta_{\Delta v} \;=\; -\,\frac{2\,c_{4.8.8}\,\alpha}{\pi}\,\Delta v + \mathcal{O}(\alpha^2), \qquad c_{4.8.8} \;=\; \tfrac{2}{3}\,R_{\text{vertex}}\,R_{\text{photon}}$$

with sign-protected $c_{4.8.8} > 0$. The sign theorem follows from substrate unitarity via the Källén–Lehmann positivity of the photon spectral weight; it requires no detailed loop algebra. The conjecture's qualitative claim — that the bare velocity bifurcation flows to a common IR fixed point — is therefore rigorously secure modulo substrate unitarity.

The quantitative leading-log damping is slow ($\sim$ 3% over the chiral-to-visible-light range), but it complements the framework's much stronger geometric $(a_0 k)^2 \sim 10^{-17}$ within-branch isotropy from the 4.8.8 $\cos(4\theta)$ cancellation theorem. Macroscopic Lorentz invariance in the framework is the joint product of two structurally independent IR suppressions: geometric within-branch and RG-flow cross-branch.

Explicit evaluation of the substrate-specific coefficients R_{vertex} and R_{photon} is the natural next step, with the Part 12 dressed-α Dyson–Schwinger machinery on the same 4.8.8 substrate as direct methodological precedent. Numerical cross-checks against the framework's planned quenched-fermion velocity scan and dynamical-fermion HMC programme will then promote the leading-log structural argument to a non-perturbative result.

The substrate-level take-away for the broader emergent-Lorentz-invariance literature is structural: a rigid discrete lattice with finite-dimensional local Hilbert space and a stabiliser-code parity-check algebra is sufficient to drive bifurcated UV velocities to a common IR limiting speed under standard $U(1)$ gauge coupling, with the qualitative claim sign-protected by lattice unitarity alone. The continuum 4D QED Chadha–Nielsen result extends to the discrete substrate without structural loss.

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