Strong CP at the Bare Substrate on $\mathbb{Z}^3 \otimes Q_3$:
Hermitian Code Mass Operators, the Mass-Phase Half, and Two Honest-Negative Results on Continuum Closure

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Abstract

The neutron-EDM–sensitive QCD vacuum angle is the basis-invariant combination $\bar{\theta} = \theta_{\text{gauge}} + \arg\det(M_u M_d)$. The Standard Model provides no mechanism for $\bar{\theta} \lesssim 10^{-10}$; the leading literature alternatives (Peccei–Quinn, Nelson–Barr) require either an as-yet undetected axion or postulated spontaneous CP breaking. We report a two-half closure of $\bar{\theta}$ at the bare substrate of the Holographic Circlette ($\mathbb{Z}^3 \otimes Q_3$) framework. **(I) Gauge half.** The substrate's strong-channel adjacency is a real-symmetric graph Laplacian, which carries no continuous phase degree of freedom; $\theta_{\text{UV}} \equiv 0$ identically (ANCHOR §15 item 93, **Locked at bare**). **(II) Mass half.** The code mass operators $H_{\text{up}}, H_{\text{down}}$ (and lepton analogues) are restrictions of the boundary walk operator $\mathcal{W}$ to the logical codebook. Hermiticity is therefore structural, not fitted. A Hermitian matrix has a real determinant; we verify explicitly that $\det H_{\text{down}} \approx 1.681 \in \mathbb{R}$. Hence $\arg\det(M_u M_d) = 0$ exactly (ANCHOR §15 item 143, **Locked at bare**). **(III) Nelson–Barr realised automatically.** The irreducible CKM phase $\phi \approx 0.85\pi$ enters $\det H_{\text{down}}$ only via the conjugate off-diagonal pair $H_{12}H_{21} = |H_{12}|^2 e^{i\phi} e^{-i\phi}$ and cancels. CP violation lives entirely in the mixing (sourcing the Jarlskog invariant $J \approx 4.3 \times 10^{-5}$); the mass-generating basis is CP-real. **(IV) Continuum closure: pinned residual.** We characterise the bare-to-continuum lift via two parallel routes. The free-field domain-wall fermion's 2D-boundary trace converges exponentially to the overlap operator (a script asserts $\|D_{\text{ov}}^{L_s} - D_{\text{ov}}\| < 10^{-3}$ at $L_s = 32$, measured value 9.5×10^{-5}). Substituting the genuine walk kernel *honestly fails*: the walk's body-diagonal Wilson scalar vanishes at four even corners of the Brillouin zone, leaving 4 species, not 1 the ultralocal-walk closure is ruled out. A second route via symmetric mass generation (CSS X-stabilizer completion of $[8, 4, 4]$ to $[[8, 0, 4]]$) yields a unique gapped symmetry-respecting ground state at the cell level (assert: gap ≥ 1, measured 2.0; entanglement ≥ 1 bit, measured 2.0 bits) with anti-mirror operator $Z_\chi Z_W$ identified; the global dynamically-gauged closure remains open. A 2D strip cutoff scan asserts the $t = 4$ mirror gap *falls* with cutoff ($1.867 \to 1.770$ over $n/q = 3 \to 4$, response $6.4\% \to 10.9\%$), resolving the saturate-vs-track-down fork toward track-down the second honest negative pinning the open frontier. **(V) Perturbative backstop.** Even ignoring substrate continuum matching, a $\bar{\theta} = 0$ boundary condition under SM running generates only $\bar{\theta} \sim 10^{-16}$ via the Ellis–Gaillard / Khriplovich–Vainshtein 7-loop GIM-suppressed effect, three orders below the experimental bound. **Falsifier (Proposition tier, conditional on the open continuum step):** $d_n \sim 10^{-31}\ e\cdot\text{cm}$; any detection above 10^{-30} falsifies the discrete-substrate origin of QCD. All numerical claims in this paper are pinned by assertions in five public scripts (`bulk_domainwall_overlap.py`, `walk_kernel_overlap.py`, `smg_construction.py`, `r2_smg_operator.py`, `smg_mirror_only.py`) plus a cutoff scan (`css_2d_strip_cutoff_scaling.py`); exit-0 of each certifies the asserted bound. All scripts are publicly available at `https://github.com/dgedge/strongCP`.

Methodological note. This paper is structured around an explicit Locked-vs-Proposition tier discipline. Bare-substrate results are Locked. Local cell-level lattice-algebra results are Locked at the cell, with the continuum/global step explicitly open. Demonstrated routes are

Proposition (the supporting evidence is in hand but the route is not yet closed). Two negative results are reported as findings rather than buried failures the ultralocal-walk closure of the overlap and the 2D-strip cutoff stabilisation. Where a script asserts a bound, we quote the asserted bound as the claim and the measured value parenthetically. We do *not* claim "Strong CP is solved". What is closed is the bare-substrate $\bar{\theta} = 0$ with both halves explicitly argued; the continuum residual is pinned to a single concrete obstruction.

1 Introduction: the invariant, and the framework's missing half

The neutron electric dipole moment (EDM) constrains the basis- and renormalisation-group-invariant angle

$$\bar{\theta} = \theta_{\text{gauge}} + \arg\det(M_u M_d), \tag{1}$$

to $|\bar{\theta}| \lesssim 10^{-10}$ (current bound $|d_n| < 1.8 \times 10^{-26}\, e\cdot\text{cm}$ [1]). The Standard Model provides no mechanism for the smallness of this otherwise natural $\mathcal{O}(1)$ parameter — this is the *Strong CP problem.* Leading literature alternatives are the Peccei–Quinn axion [2–4] (no axion detected to date), Nelson–Barr spontaneous CP breaking [5,6] (requires postulated additional CP-violating Higgs structure), and the Vafa–Witten argument [7] (rigorous only in $\mathcal{N} = 1$ SUSY-related cases).

The Holographic Circlette framework's prior canon (ANCHOR §15 item 93) anchored the *gauge half* of (1) at the bare substrate: a real-symmetric graph Laplacian carries zero phase, so $\theta_{\text{UV}} \equiv 0$ identically. The continuum closure of that half was settled-in-form by DRIFT entry K4 (the lattice-chiral Lüscher–Neuberger consequence is sound; what is open is the substrate $\mathcal{W} \to D_{\text{ov}}$ bridge).

The *mass half* $\arg\det(M_u M_d)$ was unargued in the prior canon. A complex quark mass determinant feeds (1) directly, so the previous item-93 "$\bar{\theta} \equiv 0$" claim was incomplete — the framework's CKM matrix has a non-trivial phase $\phi \approx 0.85\pi$, and a Strong-CP problem of the framework's own could in principle have hidden in the mass sector. This paper closes the mass half at the bare substrate, gives the Nelson–Barr-style off-diagonal-phase confinement as a corollary, and reports the present status of the continuum closure together with two honest-negative results that precisely pin the open frontier.

2 The two halves of $\bar{\theta}$

2.1 Gauge half (item 93): real-symmetric graph Laplacian

In continuous Yang–Mills the θ-vacuum arises from instantons: continuous gauge-field configurations mapping the spatial boundary onto $SU(3)$, generating integer winding numbers smoothly interpolated by a phase θ. On the discrete $\mathbb{Z}^3 \otimes Q_3$ substrate, the strong channel runs through an undirected scalar adjacency matrix and its graph Laplacian. *The Laplacian of a finite undirected graph is strictly a real-symmetric matrix* — it has zero complex phase degrees of freedom, and there is no continuous winding parameter that can be tuned.

Theorem 1 (Bare gauge half, item 93)**.** *At the bare substrate level, $\theta_{gauge} = \theta_{UV} \equiv 0$ identically. The bare UV lattice has no continuous winding degeneracy; the strong sector contributes nothing to $\bar{\theta}$ at the substrate.* ***[Locked at bare substrate.]***

The continuum closure of Theorem 1 — that the emergent RG flow does not develop an anomalous Jacobian measure inducing a continuum θ-term — is the substrate-to-overlap $\mathcal{W} \to D_{\text{ov}}$ bridge of DRIFT K4, addressed in §5 below.

2.2 Mass half: the missing piece

The physical invariant (1) also receives a contribution from the argument of the quark mass determinant. Because the framework's CKM mass matrix $H_{\rm down}$ carries an irreducible phase $\phi \approx 0.85\pi$ in its off-diagonal entries (CKM paper §3.1–3.2), one cannot *a priori* assert $\arg\det(M_u M_d) = 0$ without argument. We now provide that argument.

3 Mass-phase half from Hermiticity of the code mass operators

3.1 Promoted ANCHOR: code mass operators are Hermitian by construction

Theorem 2 (Hermiticity from walk-restriction, item 143)**.** *The code mass operators H_{up} and H_{down} (and the charged-lepton analogues) are restrictions of the boundary walk operator $\mathcal{W}$ to the logical codebook. The walk operator $\mathcal{W}$ is a real, uniform depolarising transition operator with a real eigenspectrum (§3.1 of the canonical encoding; boundary walk: $\lambda_0 = 1$, spectral gap $\Delta_1 = 1/28$). A $\mathcal{W}$-restriction inherits self-adjointness; therefore H_{up}, H_{down} are Hermitian* structurally, not by fitting choice.

Hermiticity is manifest in the explicit operators of the CKM paper:

- $H_{\rm up}$: real-symmetric (imaginary parts $< 10^{-18}$);
- $H_{\rm down}$: $H_{21} = 0.034\, e^{-i\cdot 0.85\pi} = (H_{12})^*$; diagonal entries real.

3.2 Derivation: the mass term vanishes exactly

A Hermitian matrix has a real spectrum and hence a real determinant. For $H_{\rm down}$ the irreducible phase ϕ enters the determinant *only* through the conjugate off-diagonal pair, where it cancels:

$$\begin{aligned}\det H_{\rm down} &= H_{11}H_{22}H_{33} - \underbrace{H_{12}H_{21}}_{=|H_{12}|^2 e^{i\phi}e^{-i\phi} = |H_{12}|^2} H_{33} - H_{22}H_{13}H_{31} \\ &= 1.169 \cdot 1.152 \cdot 1.249 - (0.034)^2 \cdot 1.249 - 1.152 \cdot (0.002)^2 \\ &\approx 1.681 \in \mathbb{R}. \end{aligned} \tag{2}$$

The same computation on $H_{\rm up}$ gives a real (positive) determinant trivially because $H_{\rm up}$ is already real-symmetric. Therefore:

Corollary 1 (Mass-phase half, item 143)**.** *At the bare substrate,* $\arg\det(M_u M_d) = 0$ *exactly (a theorem, not a tuning). The result is robust to whether the mass operator is taken as H or $H^\dagger H$: both are Hermitian when H is, with the same conclusion.* ***[Locked at bare substrate.]***

Combining Theorem 1 (gauge half) and Corollary 1 (mass half):

$$\boxed{\bar\theta = \theta_{\rm gauge} + \arg\det(M_u M_d) = 0 + 0 = 0 \quad \text{at the bare substrate.}} \tag{3}$$

4 Nelson–Barr realised automatically

In the Nelson–Barr class of Strong-CP solutions [5, 6], one postulates that CP is a fundamental symmetry broken spontaneously by additional scalar VEVs, with the breaking confined to off-diagonal terms of the mass matrix so that $\arg\det M = 0$ while CKM mixing carries the irreducible phase. This is normally installed by hand.

Here it follows for free. The framework's irreducible phase $\phi \approx 0.85\pi$ is a substrate-level physical input from the discrete walk operator (the CP-requires-$I_3 = 1$ structure, §6.7 of the

CKM paper; ANCHOR §15 item 23). Hermiticity *forces* the phase to live in the off-diagonal conjugate pair — the same pair where the cancellation $e^{i\phi} \cdot e^{-i\phi} = 1$ kills the phase contribution to $\det H_{\text{down}}$ in (2). The phase therefore:

- sources the CKM mixing matrix and the Jarlskog invariant ($J \approx 4.3 \times 10^{-5}$, ANCHOR §15 item 23);
- contributes *nothing* to arg det, hence nothing to $\bar{\theta}$;
- lives in a basis that is not the mass-generating basis: the mass basis is CP-real.

The structural separation a Nelson–Barr model installs by hand is automatic in the framework. Cross-link: this is the same LQ control distinction as the Bipartite Grassmann Trace structure (ANCHOR §15 item 79).

5 Continuum closure: pinned residual via two parallel routes

The continuum closure of (3) asks whether the substrate-to-continuum matching can induce a $\bar{\theta}$-term — the mass-side analogue of K4's gauge-side "anomalous-Jacobian" worry. The answer combines four results: a free-field reduction that works (§5.2), an ultralocal walk-kernel attempt that explicitly fails (§5.3), a parallel symmetric-mass-generation route whose local pieces are exact but whose global closure is open (§5.4), and a cutoff scan that resolves the saturate-vs-track-down fork toward track-down (§5.5). A perturbative SM backstop independent of all four (§5.6) caps the worst-case residual at $\bar{\theta} \sim 10^{-16}$.

5.1 Reformulation: shared residual, not two separate problems

$\bar{\theta}$ is the RG-invariant; $\arg\det M$ and θ_{gauge} are basis halves traded by the chiral anomaly under field redefinitions. "Continuum-preserve $\arg\det M = 0$" is meaningful only together with θ_{gauge} — i.e. as continuum-preserve $\bar{\theta} = 0$. Both halves vanish at the substrate (3); a Ginsparg–Wilson Dirac operator realises exact lattice chiral symmetry, reproduces the chiral anomaly, and keeps $\bar{\theta} \equiv 0$ at all RG scales (Lüscher–Neuberger [8, 9]). DRIFT K4 explicitly certifies this consequence as sound; what it leaves open is only the antecedent — the $\mathcal{W} \to D_{\text{ov}}$ bridge.

Mechanism on the mass side: the overlap $D_{\text{ov}} = 1 + \gamma_5 \,\text{sgn}(\gamma_5 D_W)$ is exactly γ_5-Hermitian. A script (`bulk_domainwall_overlap.py`) asserts γ_5-Hermiticity to better than 10^{-9} (measured residual 4×10^{-15}). With Hermitian M (§3), $\gamma_5(D_{\text{ov}} + M)\gamma_5 = (D_{\text{ov}} + M)^\dagger$, so $\det(D_{\text{ov}} + M) \in \mathbb{R}$ flavour-block by flavour-block. The fermion measure induces *no* determinant phase; the continuum mass sector contributes nothing to $\bar{\theta}$.

5.2 Free-field route: domain-wall → overlap reduction Demonstrated

The framework's holographic dimensional reduction (item 77 / Part 11 Q5) localises matter on a 2D octagonal domain wall with the 3D bulk integrated out. Domain-wall fermions are equivalent to the overlap (Kaplan [10] / Shamir [11] / Neuberger [9]); the boundary trace of the local 3D-bulk operator is the (exp-local) D_{ov}. The free-field reduction is demonstrated:

Proposition 1 (Domain-wall to overlap, free field)**.** *With the framework's Clifford set ($\gamma_5 = \sigma_y \otimes I$) and a free-field D_W, the local finite-L_s domain-wall operator's 2D-boundary trace converges exponentially to D_{ov}. The verification script (`bulk_domainwall_overlap.py`) asserts $\|D_{ov}^{L_s} - D_{ov}\| < 10^{-3}$ at $L_s = 32$ (measured value 9.5×10^{-5}), monotone decay from $L_s = 4$, and explicit chirality-localised zero modes (mid-amplitude < 0.2; opposite chiralities on the two walls). **[Demonstrated — Proposition; exit-0.]***

5.3 Walk-kernel route: ultralocal closure ruled out (honest negative #1)

We then attempt the framework's strongest claim — closing the bridge with the genuine walk kernel rather than a free-field D_W. The walk's body-diagonal kinetic operator is $D_{\text{kin}} = i\sum_\mu V_\mu \alpha_\mu$ with the walk-sourced finite-range Wilson scalar $W_{\text{body}} = 8(1 - \cos k_x \cos k_y \cos k_z)$.

Proposition 2 (Walk-kernel overlap, honest negative)**.** *The walk-sourced overlap is still exactly Ginsparg–Wilson and γ_5-Hermitian (assert: GW residual $< 10^{-9}$, γ_5-Hermiticity residual $< 10^{-9}$). However W_{body} vanishes at four even corners of the Brillouin zone ($k = 0$ and $(\pi, \pi, 0)$-type), so the walk-sourced overlap keeps 4* ***species, not*** *1 (`walk_kernel_overlap.py` asserts: number of nodes = 4, line minimum > 1.0). A single species needs a face-direction Wilson term, which is not sourced by the body-diagonal shift and is forbidden in finite-range form by Poincaré–Hopf $\chi(T^3) = 0$ (gw_nogo).* ***[Honest negative; exit-0 certifies the failure direction.]***

This result is reported as a finding. It rules out the ultralocal-walk closure of the $\mathcal{W} \to D_{\text{ov}}$ bridge. The continuum closure therefore settles at the *standard overlap (exp-local, external-Wilson)* tier — defensible by ordinary lattice-QCD standards of locality, but not the framework's strongest "ultralocal" ideal.

5.4 SMG second route: local pieces exact, global open

A distinct continuum route is symmetric mass generation (SMG) [12,13]: gap the mirror sector rather than lifting doublers. The framework's internal Q_3 space provides all the structural preconditions:

- **Anomaly cancellation**: the 16 fermions per cell including ν_R are exactly the $\mathbb{Z}_{16}$-anomaly-free Wang–Wen $SO(10)$ spinor; ANCHOR §2.11 $\sum(B - L) = +1/\text{gen}$ is the ν_R-necessity.
- **Nielsen–Ninomiya obstruction absent on the internal space**: the Q_3 walk is on 8 sites, gapped, bipartite-chiral.
- **Anti-mirror operator identified**: $Z_\chi Z_W$ is the gauge-invariant mirror-flipping operator; `r2_smg_operator.py` asserts $\|[Z_\chi Z_W, G]\| < 10^{-10}$ for all SM gauge generators. The R2 stabiliser of the canonical encoding is the W-blind weak coin (§5.3 of the encoding paper), *not* this operator.
- **Local symmetric gap proved on the cell**: completing the self-dual $[8, 4, 4]$ code to $[[8, 0, 4]]$ via the CSS X-stabiliser (item 13's Z/X unification) yields a unique gapped symmetry-respecting entangled ground state. The construction is verified (`smg_construction.py`) with asserts: spectral gap ≥ 1 (measured 2.0), entanglement ≥ 1 bit (measured 2.0 bits), unique zero of the full stabiliser projector. [**Locked at the cell level. This is the local symmetric gap on one Q_3 cell — it is not the continuum mirror gap.**]
- **Spatial mirror-selectivity (necessary condition, leading order)**: with the item-77 domain-wall providing exponential physical-mirror separation, the CSS X-SMG interaction couples to the mirror at $O(1)$ and to the physical mode at $O(e^{-L})$. The leading-order check is a 1D SSH toy model (`smg_mirror_only.py`) with asserts: physical decay < 0.8 (measured 0.5), physical-mode matrix element $< 10^{-3}$, mirror matrix element > 0.3. [**Proposition; illustrates the mechanism, not the spatial theory.**]

What is open: the non-perturbative dynamically-gauged global closure — the chiral-lattice-gauge frontier — is unconstructed. The local cell-level results above are exact and Locked at the cell; the spatial extension to a continuum gapped mirror sector is the open frontier and is *not* established by these scripts.

5.5 Cutoff scan: 2D mirror gap tracks down (honest negative #2)

The remaining numerical question is whether, in the genuine 2D charge-block strip representation (the smallest spatial realisation of the mirror-selective interaction), the mirror gap stabilises with increasing hopping cutoff t or tracks down toward gaplessness as in 1D. The cutoff scan (`css_2d_strip_cutoff_scaling.py`) addresses this directly:

Proposition 3 (2D-strip cutoff scaling, honest negative)**.** *Over the achievable cutoff range* $n/q = 3 \to 4$*, the* $t = 4$ *mirror gap* falls *from* 1.867 *to* 1.770 *and the t-response* grows *from* 6.4% *to* 10.9%*. The script asserts the direction explicitly:* `by_cutoff[4][4.0] < by_cutoff[3][4.0]` *(exit-0 certifies "gap falls with cutoff"). **[Honest negative; the 2D strip shows no stabilisation; the saturate-vs-track-down fork resolves toward track-down, mirroring the 1D behaviour.]***

This second negative result, orthogonal to Proposition 2, pins the open frontier precisely: *the global continuum mirror gap is not exhibited by either the ultralocal-walk route or by direct 2D cutoff scaling.* The local symmetric gap (§5.4) and the leading-order spatial selectivity (`smg_mirror_only.py`) remain Locked-at-cell and Demonstrated respectively; what is open is exactly the dynamically-gauged global step.

5.6 Perturbative SM backstop: $\bar{\theta}_{\mathbf{rad}} \sim 10^{-16}$

Independent of all continuum-matching considerations above, perturbative SM running from a $\bar{\theta} = 0$ boundary radiatively generates only a tiny $\bar{\theta}$:

Proposition 4 (Ellis–Gaillard / Khriplovich–Vainshtein backstop)**.** *A boundary condition* $\bar{\theta}(bare) = 0$ *runs under Standard Model RG flow to* $\bar{\theta}_{rad} \sim 10^{-16}$*, a 7-loop, fully GIM-suppressed effect [14, 15]. This is three orders below the current experimental upper bound on* $|\bar{\theta}|$ *and is independent of the substrate-to-continuum matching of (§5.2–5.5).*

Combining the four routes:

- **Bare substrate**: $\bar{\theta} = 0$ *[Locked]*.
- **Free-field overlap reduction**: passes *[Demonstrated]*.
- **Ultralocal-walk closure**: ruled out *[honest negative #1]*.
- **SMG local pieces**: exact *[Locked at cell]*; spatial leading-order *[Demonstrated]*; global dynamical-gauge closure *[open]*.
- **2D-strip cutoff stabilisation**: ruled out *[honest negative #2]*.
- **Perturbative SM backstop**: $\bar{\theta}_{\text{rad}} \sim 10^{-16}$ *[Locked, external]*.

The net statement: $\bar{\theta} = 0$ at the bare substrate is Locked; the continuum closure settles at the standard overlap (exp-local, external-Wilson) tier, with two ruled-out routes and one explicitly-open route (dynamically-gauged global SMG), backed by an order-of-magnitude perturbative safety net.

6 Falsifiable consequence: $d_n \sim 10^{-31}$ $e{\cdot}$cm

Because the strong-sector contribution to $\bar{\theta}$ vanishes at the bare substrate and the perturbative backstop caps the worst-case continuum residual at $\bar{\theta} \sim 10^{-16}$, the neutron EDM receives *strictly zero* contribution from the strong sector in the framework. The physical d_n must arise exclusively from higher-order weak CKM loop-leakage (the $I_3 = 1$ spatial routing asymmetry of §6.7 of the CKM paper), establishing a strict upper bound

$$d_n \sim 10^{-31}\, e\cdot\text{cm}. \tag{4}$$

Tier and conditions. This bound is *Proposition tier, conditional on the open continuum step* of §5.4. The bare-substrate $\bar{\theta} = 0$ result is Locked; lifting it to a continuum statement that suffices for (4) currently relies on the overlap (exp-local, external-Wilson) tier or, equivalently, on the perturbative backstop of Proposition 4.

Falsifier. Any experimental detection of $d_n > 10^{-30}\, e\cdot\text{cm}$ (forthcoming sensitivity: SNS nEDM [16], n2EDM [17]) would falsify the discrete-substrate origin of QCD as proposed here. Current bound $|d_n| < 1.8 \times 10^{-26}\, e\cdot\text{cm}$ [1] is consistent with (4).

A practical note on color: the substrate's color structure at the level of these results is the $\mathbb{Z}_3$-center / qubit-proxy mapping, not full $SU(3)$. The $F_2 \to \mathbb{Z}_3$ continuum map (§2.8 of the encoding) is itself open. This caveat does not affect the $\bar{\theta} = 0$ result, which is a statement about the determinant of the mass operator and the symmetry of the gauge Laplacian, both of which carry over at the center / qubit proxy level.

7 Comparison with alternatives

Mechanism	Core ingredient	Status / signature
Peccei–Quinn axion [2–4]	Postulate $U(1)_{PQ}$; QCD-anomalous breaking gives axion pseudo-Nambu–Goldstone boson canceling $\bar{\theta}$.	Axion not detected (ADMX, IAXO bounds).
Nelson–Barr [5, 6]	Postulate CP as fundamental, spontaneously broken by additional CP-violating Higgs. Phase confined off-diagonal by construction.	Requires postulated extra Higgs sector with engineered couplings.
Vafa–Witten [7]	Rigorous $\bar{\theta} = 0$ in $\mathcal{N} = 1$ SUSY-related contexts.	Not directly applicable to non-SUSY SM.
This work	Hermitian code mass operators from walk-restriction (structural, not fitted) + real-symmetric gauge Laplacian. Nelson–Barr realised automatically as a corollary.	Bare $\bar{\theta} = 0$ Locked; continuum at standard overlap tier (two negative routes ruled out, SMG global open); $d_n \sim 10^{-31}\, e\cdot\text{cm}$ falsifiable. No axion required.

Table 1: Mechanisms addressing Strong CP. This work is closest in spirit to Nelson–Barr, but does not require postulating a CP-breaking Higgs sector — the off-diagonal-phase confinement follows from Hermiticity of the walk-restricted mass operators.

8 Open work and status table

The single explicitly-open step is the global dynamically-gauged closure of the SMG mirror gap in the spatial theory — equivalently, the construction of a chiral lattice gauge theory in which the substrate's CSS X-stabiliser is dynamically gauged. Two routes have been ruled out (ultralocal-walk closure of $\mathcal{W} \to D_{\text{ov}}$; 2D-strip cutoff stabilisation); the route that remains is the chiral-lattice-gauge frontier itself, which the community is actively pursuing for SMG-class arguments. We do not claim to close this here; we claim to have pinned it.

Item	Status	Evidence
$\theta_{\rm UV} = 0$ (gauge half, item 93)	**Locked at bare**	Real-symmetric Laplacian theorem
$\arg\det(M_u M_d) = 0$ (item 143)	**Locked at bare**	Hermiticity from walk-restriction; explicit $\det H_{\rm down} \approx 1.681$
Nelson–Barr automatic	**Locked at bare** (corollary)	Off-diagonal phase cancels in det
Domain-wall $\to$ overlap (free field)	Demonstrated (Proposition)	`bulk_domainwall_overlap.py`: $< 10^{-3}$ at $L_s = 32$
Walk-kernel ultralocal closure	**Honest negative**	`walk_kernel_overlap.py`: 4 species not 1
SMG anti-mirror operator $Z_\chi Z_W$	Locked at cell	`r2_smg_operator.py`: commutes with all G
SMG local symmetric gap	Locked at cell	`smg_construction.py`: gap ≥ 1 (measured 2.0), entanglement ≥ 1 bit (measured 2.0)
SMG spatial mirror-selectivity	Demonstrated leading order (1D SSH toy)	`smg_mirror_only.py`: decay < 0.8
2D-strip cutoff stabilisation	**Honest negative**	`css_2d_strip_cutoff_scaling.py`: gap falls with cutoff
Global dynamically-gauged closure	**Open**	Chiral-lattice-gauge frontier; not closable by numpy-ED
SM perturbative backstop	Locked (external)	Ellis–Gaillard / Khriplovich–Vainshtein: $\bar\theta_{\rm rad} \sim 10^{-16}$
$d_n \sim 10^{-31}$ $e{\cdot}$cm falsifier	Proposition, conditional	Combines bare result + backstop / continuum routes

Table 2: Tier and status of every claim in this paper.

A Assert audit: scripts and exit codes

All numerical claims in this paper are pinned by assertions in six public scripts. Exit-0 of each certifies the asserted bound. Where the measured value is quoted in the body, it appears parenthetically alongside the asserted bound.

Code availability. The six scripts referenced below, together with a `README` documenting reproduction, are publicly archived at

`https://github.com/dgedge/strongCP`

The repository can be cloned with `git clone https://github.com/dgedge/strongCP.git`; each script is self-contained and can be re-run with `python3 <name>.py` to verify exit-0.

Dependencies and runtime. Each script requires only `numpy` (and `scipy.sparse` for the cutoff scan), and runs in seconds on a laptop.

Script	Asserted bound (measured value)
`bulk_domainwall_overlap.py`	GW residual $< 10^{-9}$ (4×10^{-15}); γ_5-Hermiticity $< 10^{-9}$; $\|D_{\rm ov}^{L_s} - D_{\rm ov}\| < 10^{-3}$ at $L_s = 32$ (9.5×10^{-5}); single species; chirality-opposite walls.
`walk_kernel_overlap.py`	GW + γ_5-Hermiticity preserved; 4 species, not 1; line min > 1.0. *Honest negative #1: walk-kernel ultralocal closure ruled out.*
`smg_construction.py`	16 codewords; pairwise commuting stabilisers; unique zero of full projector; spectral gap ≥ 1 (2.0); entanglement ≥ 1 bit (2.0).
`r2_smg_operator.py`	$\|[Z_\chi Z_W, G]\| < 10^{-10}$ for all SM gauge generators; H_{R2} is W-blind weak coin (*not* $Z_\chi Z_W$).
`smg_mirror_only.py`	Physical decay < 0.8 (0.5); $\lvert\langle \text{phys}\rvert H_{\rm SMG} \lvert\text{phys}\rangle\rvert < 10^{-3}$; $\lvert\langle\text{mirror}\rvert \cdot \lvert\text{mirror}\rangle\rvert > 0.3$.
`css_2d_strip_cutoff_scaling.py`	`by_cutoff[4][4.0] < by_cutoff[3][4.0]` (gap falls with cutoff: $1.867 \to 1.770$ at $t = 4$ over $n/q = 3 \to 4$). *Honest negative #2: 2D-strip cutoff stabilisation ruled out.*

Table 3: Assert-on-every-quoted-number audit. All scripts are numpy-only and self-asserting; exit-0 certifies the bound. Measured values quoted parenthetically.

References

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