A Selection-Rule Calculus for Finite Record Physics

Static predicates, monitored recovery instruments, and physics-bearing environment records

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Abstract

Selection rules are usually treated as allowed/forbidden labels: a state is admissible if it obeys the rule and absent if it does not. In a finite record-bearing substrate that is not enough. A static predicate can define a codespace, but it writes no environment record, carries no phase, produces no entropy, and cannot by itself generate a measurable response. A selection rule becomes physics-bearing only when it is implemented as a monitored recovery instrument with syndrome bits, a recovery map, and a Stinespring environment. This paper formulates the resulting selection-rule calculus. The central theorem is a register-access rule: a monitored constraint can generate records only in the registers its syndrome and recovery actually read. Consequently a colour rule cannot supply a generation phase, a sterile repair cannot generate a generation covariance, and a chirality lock cannot repair a colour-counting deficit. The worked example is the framework's Dynamic-R1 mechanism. The forbidden fourth-generation corner $G_0G_1 = 11$ is not merely absent; under monitored recovery it is an active boundary whose allowed generation order ideal $B_2 \setminus \{11\}$ writes Hasse-edge environment records. The symmetric second moment of those records gives the covariance block $K_{\mathrm{R1}} = BB^T$, while the closed oriented cochain Ω_{R1} is the sign-representation carrier for Majorana/CP orientation. Composed with the sector defect inventory, the CP-even Koide row closes in its type-correct form: the absolute contact ledger $(e, \nu, d, u) = (2, 3, 3, 2)$ over $N_{\mathrm{eff}} = (9, 9, 27, 27)$, equivalent to the reduced row $(2, 3, 1, 2)$. The same calculus explicitly forbids using this closure to rescue the baryogenesis magnitude $\eta = (3/14)\alpha_0^4$: the numerator 3 remains an ideal-code count, not a physical $B - L$ source count. The contribution is therefore both constructive and disciplinary: monitored selection rules can become new physics, but only in the registers they actually monitor.

Plain summary

A selection rule normally says something like “this state is allowed” or “that state is forbidden”. This paper adds a second question: *is the rule monitored?* If the answer is no, the rule is only a static boundary. If the answer is yes, the act of maintaining the boundary writes records into the environment. Those records can then carry real physics: counts, covariances, orientation signs, entropy, and response terms.

The key slogan is:

static predicates select states; monitored recovery writes records.

The rule also has a guardrail. A monitored rule can only write what it reads. If a repair channel reads electroweak bits, it cannot be borrowed to generate a generation-space phase. If a confinement rule reads colour bits, it cannot repair a chirality failure. This simple register-access rule prevents many tempting but invalid derivations.

1 Motivation: selection rules are not all the same

Selection rules are among the most useful ideas in physics. Conservation of angular momentum forbids some transitions; gauge charge conservation forbids others; colour confinement removes net-colour asymptotic states; parity, chirality, and flavour assignments control which interaction vertices can exist. In ordinary use the phrase "selection rule" mixes at least two different structures:

- a *predicate*: a statement that a state belongs to an admissible set;
- an *instrument*: a physical process that detects departures from that set and restores or rejects them.

For many calculations the distinction does not matter. If all one asks is whether a transition amplitude is zero, a predicate is enough. In a finite record substrate the distinction is load-bearing. Counts, phases, decoherence, entropy production, and response functions require records. Records are not written by predicates. They are written by monitored instruments.

This paper develops that distinction as a small calculus. It builds on the records/responses programme [1–3]: records say what can be stably known; responses say what experiments measure. Here we apply the same discipline to selection rules. The point is not merely terminological. In the Dynamic-R1 sector of the finite-QEC framework, the distinction separates a genuine new mechanism from a false derivation: the R1 generation rule can carry Koide/CP structure because it reads the generation register; the R4 sterile repair cannot, because it reads electroweak bits instead.

2 Static predicates

Definition 1 (Static selection predicate)**.** *Let the finite substrate register be a Hilbert space*

$$\mathcal{H} = \bigotimes_{a \in \mathsf{Reg}} \mathcal{H}_a,$$

with computational or record labels on the factors $\mathcal{H}_a$. A static selection rule is a projector P_R or Boolean predicate

$$R : \{0,1\}^{|\mathsf{Reg}|} \to \{0,1\}$$

whose valid subspace is

$$\mathcal{H}_R = P_R \mathcal{H}.$$

A static predicate is perfectly real as mathematics. It can define a code, remove a representation, or express a conservation law. But by itself it has no time arrow and no environment channel. It does not say how an invalid state is detected, where the syndrome is stored, or what reset process returns the substrate to reusable form.

Proposition 1 (No-record lemma for static predicates)**.** *A static predicate P_R defines an admissible subspace but writes no environment record unless it is accompanied by a monitored instrument.*

Reason. The predicate alone is a mathematical constraint. It contains no output register and no completely positive map from system to system plus environment. Hence there is no orthogonal environment label $|r\rangle_{\mathcal{E}}$ that can later be copied or queried. A later observer may infer that a state lies in $P_R \mathcal{H}$, but that inference is not a record written by the predicate itself. □

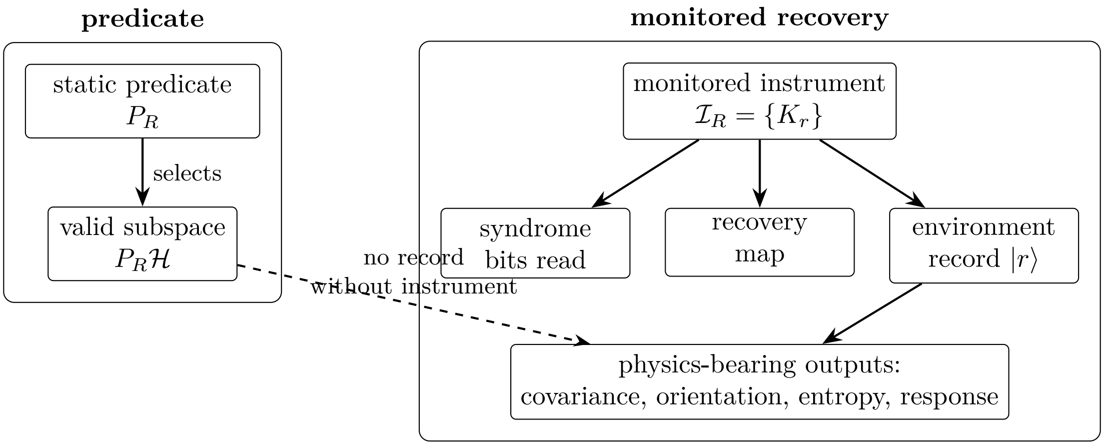

Figure 1: A static predicate selects an admissible subspace. A monitored recovery instrument reads syndrome bits, applies a recovery map, and writes an environment record. Only the second object can generate physics-bearing records.

3 Monitored recovery instruments

Definition 2 (Monitored recovery instrument)**.** *A monitored implementation of a selection rule R is a quantum instrument*

$$\mathcal{I}_R = \{\mathcal{M}_r\}_{r\in\mathcal{R}}, \qquad \mathcal{M}_r(\rho) = K_r \rho K_r^\dagger,$$

with $\sum_r K_r^\dagger K_r = I$, together with a Stinespring dilation

$$V_R|\psi\rangle = \sum_r K_r|\psi\rangle \otimes |r\rangle_{\mathcal{E}}.$$

The orthogonal labels $|r\rangle_{\mathcal{E}}$ are the environment records written by the recovery; their orthogonality is what makes them classically copyable and hence record-grade, in contrast to unknown quantum states [4, 5].

This is the standard structure of a quantum operation [6–8]; syndrome-conditioned recovery is the stabilizer-code instrument [9]. The new claim is how it should be used in a finite record theory. A selection rule is physics-bearing only to the extent that its instrument writes stable records. The instrument has three separate pieces:

piece	notation	physical role
syndrome read	measured bits S_R	what deviation is detected
recovery map	Kraus operators K_r	how validity is restored or enforced
environment record	labels r	what later physics can condition on

4 The register-access theorem

Definition 3 (Read support)**.** *For a monitored rule R, define its read support*

$$\mathsf{Bits}(R) = S_R \cup T_R,$$

where S_R is the set of syndrome bits read by the instrument and T_R is the set of register bits directly acted on by the recovery map.

Theorem 1 (Register-access theorem)**.** *A monitored selection rule R can write environment records only in the algebra generated by the bits in $\mathsf{Bits}(R)$. It cannot generate a physical covariance, phase, entropy term, or response source in a disjoint register without an additional monitored coupling to that register.*

Proof. In the Stinespring representation

$$V_R|\psi\rangle = \sum_r K_r|\psi\rangle \otimes |r\rangle_{\mathcal{E}},$$

the environment label r is a function of the measurement outcome and recovery branch. Suppose the instrument does not access a register A: the Kraus operators act trivially on it, $K_r = \mathbb{1}_A \otimes M_r$ (mere block-diagonality would not suffice, since A-dependent blocks would themselves constitute a read of A). Then $K_r^\dagger K_r = \mathbb{1}_A \otimes M_r^\dagger M_r$, so two states that differ only in A produce the same distribution of environment labels. Tracing over the system leaves identical environment states for those differences. Therefore no record written by $\mathcal{I}_R$ distinguishes anything in A, and no later response functional depending only on that environment record can acquire an A-dependent covariance or phase. Such a dependence would require a new instrument or coupling whose Kraus operators do not commute with the A-register decomposition. □

Corollary 1 (No cross-register borrowing)**.** *A record written by one monitored rule cannot be reused as the phase or covariance source for another register unless the instrument actually reads that register.*

This is the theorem that blocks several tempting moves. A sterile R4 repair that reads electroweak bits cannot generate a generation-space covariance. An R3 colour/confinement rule cannot repair an R2 chirality failure. An R2 chirality monitor cannot supply a colour triality record. These are not aesthetic restrictions; they are consequences of what information the Stinespring environment actually contains.

5 Symmetric covariance: contact counts from records

Once a monitored rule writes an environment record, the second moment of the record labels is a genuine object. Let v_r be the vector label associated with environment branch r. The symmetric covariance is

$$C_R = \sum_r p_r\, v_r v_r^T - \left(\sum_r p_r v_r\right)\left(\sum_r p_r v_r\right)^T.$$

In a balanced setting the mean term may vanish or be absorbed into a singlet component. What remains is the shape of the record written by the recovery.

The Dynamic-R1 example is especially clean [10]. The allowed generation set is the Boolean order ideal

$$B_2 \setminus \{11\} = \{00, 01, 10\}.$$

With the framework's canonical labels

$$00 = \tau, \qquad 01 = e, \qquad 10 = \mu,$$

the Hasse cover edges are

$$\tau \leftrightarrow e, \qquad \tau \leftrightarrow \mu.$$

The endpoint pair $e \leftrightarrow \mu$ is not a cover; it is a distance-2 pair in the Boolean lattice. If B is the incidence matrix of the two covers, ordered as (τ, e, μ),

$$B = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad K_{\mathrm{R1}} = BB^T = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

This is not fitted. It is forced by Hasse locality and endpoint covariance. A one-edge record breaks the endpoint swap $e \leftrightarrow \mu$. A doubled record is two primitive records, not one. A complete-graph record includes the non-cover $e \leftrightarrow \mu$. Thus one independent sector defect channel deposits exactly one primitive K_{R1} block.

6 Antisymmetric cochains: orientation and CP

The symmetric covariance is not the only record object. A monitored recovery can also write an oriented cochain. In the R1 case the CP-even Hasse path is not itself the full CP-odd object. The sign carrier is the closed oriented generation cochain

$$\Omega_{\mathrm{R1}} = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}.$$

This transforms in the sign representation of S_3: odd relabellings flip its sign. Multiplication by the global substrate orientation line turns it into a scalar Stinespring pointer. In the current lepton branch it supplies Majorana/recovery orientation, not a Dirac PMNS phase. This is why the prediction is a Majorana-only CP branch: long-baseline Dirac CP is null or very small, while $0\nu\beta\beta$ and Majorana interference read the orientation branch [11].

7 Worked theorem: the Dynamic-R1 sector inventory

The selection-rule calculus now gives a compact derivation of the CP-even Koide contact inventory of Elliman [10]. The statement must be read carefully.

Theorem 2 (Dynamic-R1 sector-defect inventory)**.** *Under active R1 boot monitoring, primitive R1 block billing, R3 non-colour single-flip counting, post-EWSB EM closed-pair selection, and colour-inclusive state counting, the CP-even sector inventory is*

$$(e, \nu, d, u) = \frac{(2, 3, 3, 2)}{(9, 9, 27, 27)}.$$

Equivalently, in the reduced display convention it is

$$(e, \nu, d, u) = (2/9,\ 3/9,\ 1/9,\ 2/27),$$

with display numerators $(2, 3, 1, 2)$.

Executable proof sketch. The proof is the composition of five finite checks.

1. R1 monitoring supplies the generation Hasse block K_{R1}.
2. The primitive block-billing theorem proves that one independent sector defect channel deposits one full K_{R1} block.
3. R3 leaves exactly two non-colour single-flip channels at each boot corner (the lepton boot ν_R and the quark boots u_R, d_R): the isolated LQ-flip is always R3-forbidden, so the live base axes are I_3 and χ.
4. The unbroken charge monitor reads $Q = T_3 + Y$. The closed-pair service credit

   $$C_{\mathrm{close}} = (T_3^2 + Y^2) - Q^2 = -2T_3 Y$$

   is positive exactly on $\{\nu, d\}$, adding one channel there.

5. The denominator is the available single-particle state count: $N_{\text{eff}} = 9$ for colourless leptons and $N_{\text{eff}} = 27$ for quarks.

Therefore the absolute contact row in display order (e, ν, d, u) is $(2, 3, 3, 2)$ over $(9, 9, 27, 27)$. Since $3/27 = 1/9$, the reduced display row is $(2, 3, 1, 2)$. □

Remark 1 (Why the old literal reading failed). Literal R1 Hasse contacts are two in every sector. That count is sector-blind. The theorem is not "R1 has d_s edges". The theorem is "sector defect channels bill primitive R1 covariance blocks".

8 Guardrail: why eta is not rescued

The same calculus also blocks an over-extension. The baryogenesis coefficient $\eta = (3/14)\alpha_0^4$ once looked close to closure because the numerator 3 appeared as a colour-silent logical count. The register-access theorem prevents that count being borrowed as a physical $B - L$ source. R3 confinement reads colour/lepton structure. It cannot license two R2-inert, chirality-forbidden logical channels as physical Majorana source channels. The physical colour-silent $\Delta L = 2$ source count is one, not three.

Thus Dynamic R1 can carry fourth-generation exclusion, Koide shape, R-amplitude rank, Majorana orientation, and the Majorana-only CP prediction. It does not derive the eta magnitude. The surviving eta content is only the α_0^4 scale and denominator 14; the numerator 3 remains open unless a new non-confinement source theorem is supplied.

9 Selection-rule calculus

The results above can be summarised as a calculus.

object	what it can do	what it cannot do
static predicate	define admissible states; support counting and representation constraints	write records, phases, entropy, or response terms
monitored syndrome	expose the bits it reads	distinguish registers outside its read support
recovery branch	restore validity; write environment labels	generate physics in a disjoint register
symmetric record covariance	carry shapes, block counts, ellipticities	carry orientation sign by itself
antisymmetric cochain	carry orientation/CP sign when paired with global orientation	set absolute mass scales

For practical use, the rule is:

To promote a selection rule into physics, name the instrument and the record.

One must state the syndrome bits read, the recovery branches, the environment record, the symmetric and antisymmetric record objects, and the registers the instrument does not access. This turns many vague questions into finite checks.

10 Relation to standard quantum mechanics

Nothing in the calculus modifies standard quantum operations. The formal backbone is exactly the usual completely positive instrument/Stinespring picture [6–8], together with its dissipative semigroup limit [12, 13]. The novelty is methodological and interpretive:

- selection rules are split into predicates and monitored instruments;
- only monitored instruments write environment records;
- only written records can be used as sources for later response calculations;
- register access is audited explicitly.

This is also compatible with decoherence and quantum Darwinism [5, 14]. Decoherence explains why some states become stable and copyable. The selection-rule calculus asks a narrower question: which Boolean validity constraints are merely static boundaries, and which are actively maintained boundaries whose recovery process writes records?

Within the finite-QEC substrate programme the calculus is a corollary of the anchored record layer rather than an added axiom. There, the jump operators are projective single-bit recoveries; re-reads of an unchanged configuration repeat deterministically, so the ledger advances only on syndrome increments; an increment commits exactly the address/channel content the instrument reads; and monitored same-address service conserves system coherences exactly, while a committed increment decorrelates with unit probability and carries only the read register [1, 3]. The register-access theorem of Section 4 is the register-level restatement of those facts. The same mechanism, applied to the substrate's repair-line polarisation rather than to constraint rules, produces the screened gravitational response studied elsewhere in the programme — one record-layer theorem, instanced twice.

11 Status and falsifiers

The Dynamic-R1 stack now has a clean tiering:

claim	status	falsifier or boundary
fourth-generation corner $G_0G_1 = 11$ is an R1 recovery boundary	finite monitored-channel grade	direct asymptotic fourth chiral generation kills the order ideal
R1 Hasse covariance K_{R1}	derived from order ideal and primitive block billing	literal edge-count reading remains false and guarded
sector inventory d/N	derived at finite monitored-record grade	depends on closed-pair contact counting and $Q = T_3 + Y$ monitor
Majorana-only leptonic CP	registered prediction branch	robust nonzero intrinsic Dirac PMNS CP kills the branch
eta magnitude 3/14	excluded from this closure	reopen only with new non-confinement source theorem

The cleanest experimental consequence remains the lepton-CP branch: Dirac CP in long-baseline oscillations should be null or very small, while CP lives in Majorana/recovery orientation. That is tested by DUNE/Hyper-K through oscillation CP and by neutrinoless double-beta decay through the Majorana sector [11].

12 Conclusion

The selection-rule calculus is a small conceptual move with large consequences. It says that "allowed" and "forbidden" are not the end of the story. In a finite record substrate a selection rule can be static, in which case it defines a boundary, or monitored, in which case maintaining the boundary writes records. Those records can carry covariance, orientation, entropy and response. They can also fail to carry them, if the relevant register was not read.

Dynamic R1 is the first substantial worked example. The forbidden fourth-generation corner becomes an active recovery boundary; the allowed generation order ideal writes Hasse-edge records; the symmetric record covariance gives the Koide contact inventory; the antisymmetric oriented cochain gives the Majorana/CP sign carrier. At the same time, the calculus prevents overreach: eta's numerator 3 is not rescued, because the relevant logical count is not a physical $B - L$ source record.

That is the discipline this framework needs. Monitored selection rules can become new physics, but only through the records they actually write.

Executable checks

The following local gates implement the paper's claims:

- `python_code/selection_rule_recordability_theorem_gate.py`
- `python_code/item87_dynamic_r1_sector_defect_inventory_theorem.py`
- `python_code/item87_clause_b_sector_count.py`
- `python_code/item87_r1_block_billing_map_theorem.py`
- `python_code/item87_neutrality_selector_closure.py`
- `python_code/item87_deltaL2_portal_record_action_theorem.py`
- `python_code/leptonic_cp_majorana_only_prediction.py`
- `python_code/eta_r3_confinement_dynamics_verdict_gate.py`

References

[1] David Elliman. Records and responses: Dressing-blind theorems for monitored quantum instruments. Zenodo preprint, 2026. doi: 10.5281/zenodo.21189012.

[2] David Elliman. From counts to observables: A measurement discipline for discrete record-based physics. Zenodo preprint, 2026. doi: 10.5281/zenodo.21251758.

[3] David Elliman. Records say what can be known: Empirical access, response functionals, and severity in finite information physics. Zenodo preprint, 2026. doi: 10.5281/zenodo.21219214.

[4] William K. Wootters and Wojciech H. Zurek. A single quantum cannot be cloned. *Nature*, 299:802–803, 1982. doi: 10.1038/299802a0.

[5] Wojciech H. Zurek. Quantum darwinism. *Nature Physics*, 5:181–188, 2009. doi: 10.1038/nphys1202.

[6] W. Forrest Stinespring. Positive functions on C*-algebras. *Proceedings of the American Mathematical Society*, 6(2):211–216, 1955. doi: 10.2307/2032342.

[7] Karl Kraus. *States, Effects, and Operations: Fundamental Notions of Quantum Theory*. Springer, Berlin, 1983. doi: 10.1007/3-540-12732-1.

[8] Michael A. Nielsen and Isaac L. Chuang. *Quantum Computation and Quantum Information*. Cambridge University Press, Cambridge, 10th anniversary edition, 2010. doi: 10.1017/CBO9780511976667.

[9] Daniel Gottesman. Stabilizer codes and quantum error correction. *arXiv preprint quant-ph/9705052*, 1997.

[10] David Elliman. The lepton mass spectrum from r1-recovery contact counting: Koide's q=2/3 and its phase derived, a neutrino floor prediction, and a mixing no-go. Zenodo preprint, 2026. doi: 10.5281/zenodo.21284174.

[11] David Elliman. Leptonic cp is majorana-only: A registered prediction addendum. Zenodo preprint, 2026. doi: 10.5281/zenodo.21251753.

[12] Goran Lindblad. On the generators of quantum dynamical semigroups. *Communications in Mathematical Physics*, 48:119–130, 1976. doi: 10.1007/BF01608499.

[13] Vittorio Gorini, Andrzej Kossakowski, and E. C. G. Sudarshan. Completely positive dynamical semigroups of N-level systems. *Journal of Mathematical Physics*, 17:821–825, 1976. doi: 10.1063/1.522979.

[14] Wojciech H. Zurek. Decoherence, einselection, and the quantum origins of the classical. *Reviews of Modern Physics*, 75:715–775, 2003. doi: 10.1103/RevModPhys.75.715.