

Special and General Relativity from the Finite-QEC Substrate: The Propagation Clock, the Equivalence Principle, and the Horizon Ledger

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Abstract

This companion paper states how relativity, in both its forms, is recovered as an emergent property of the finite-QEC (quantum-error-correction) substrate of the program. The organising idea is two identifications. *Special relativity* is the universal reversible propagation clock: the invariant speed c is the fixed conversion of one lattice step per service tick, and internal observers built from the same reversible walk/QEC clock infer Lorentz symmetry even though the microscopic update has a preferred order. *General relativity* is the coarse-grained covariance of that clock under substrate strain, record load, and horizon service. We make both constructive. The matter sector is an exactly isotropic massless Weyl cone whose three spatial hops form an anticommuting Clifford triple, forcing $H(k)^2 = (\sum_d \sin^2 k_d)I$. The photon is not the raw K6 T_{1u}/E_g velocity branch but the Gauss-projected transverse Maxwell cohomology; trans- Λ_{QCD} quanta are represented, at current canon grade, by framed causal-set/null-chain service events with one normalised external LSZ leg, while precision QED and Lorentz-violation phenomenology remain separate frontiers. Straining the cone yields the metric $g^{ij} = \delta^{ij} + 2\varepsilon^{ij}$ directly: the metric perturbation *is* the strain field. The Einstein *form* is now derived at the RT/min-cut, local-KMS, service-current entanglement-first-law level; the observed Planck hierarchy remains a distinct horizon/lattice-scale input. The equivalence principle follows once gravity couples to the full energy (kinetic plus confinement/Yukawa mass), not to the bare hopping, which would be *anti-mass*. Finally, the horizon sector is sharpened: the finite cell isometry, local KMS scheduler, Hawking ladder, greybody transfer, echo-null expectation, and flux normalisation are all mapped, with the all-contact flux and fast-scrambling questions still explicitly conditional.

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1 Role of This Paper

This is the relativity companion to the finite-QEC substrate series [1–5]. The other papers take Lorentz invariance and a background metric largely for granted; this one asks where they come from when the fundamental description is a discrete register walk with a preferred update order. The aim is not a finished quantum theory of gravity but a precise account of which relativistic structures are *derived* from the substrate, which are *computed*, and which remain conditional on named imports.

Two definitional identifications organise the paper.

- **SR = universal reversible propagation clock.** The invariant speed is the conversion factor of one spatial lattice step per service tick, $c = a_0/\tau_0$. All stable matter, gauge bosons, and measuring devices are excitations of the *same* reversible walk/QEC clock, so internal observers cannot detect the microscopic preferred frame; they infer Lorentz symmetry.
- **GR = coarse-grained covariance of that clock under strain/record load.** Substrate strain renormalises local clocks, causal cones, and ruler relations. The metric is the field that records this renormalisation; geodesic motion is propagation-phase extremisation in the distorted clock field.

A terminology convention is enforced throughout: “friction” is reserved for *dissipative* QEC service — thermodynamic, horizon, dark-sector, and measurement ledgers. Ordinary relativistic propagation, including massive propagation, is reversible; legacy “mass as computational friction” phrasing is to be read as a statement about dwell time and dispersion, not dissipation.

The external context is standard. The recovery of continuous Lorentz invariance from a discrete lattice in the infrared is the lattice-field-theory mechanism of Wilson [14]; the fermion-doubling constraints that shape the matter sector are those of Nielsen and Ninomiya [8]. The universality of the gravitational coupling to a massless spin-2 field is the soft-graviton theorem of Weinberg [13]; induced gravity — the graviton kinetic term generated by integrating out matter — is due to Sakharov [10], and the thermodynamic reading of the field equations to Jacobson [7], Verlinde [12]. The discrete-curvature tool is the Ollivier Ricci curvature of Markov chains [9], and the induced-action expansion is the standard heat-kernel/Seeley–DeWitt series [11]. The reproducibility repository is [6]; the public project pointer is <https://neusym.ai>.

2 Special Relativity: The Propagation Clock

2.1 The one-dimensional theorem

The cleanest statement of the clock idea is exact in one dimension. A discrete quantum walk with a shift \mathcal{S} and a coin rotation through angle θ has the Bloch dispersion

$$\cos \omega(k) = \cos \theta \cos k, \quad (1)$$

whose small-argument expansion is

$$\omega^2 = \theta^2 + k^2 + O((\theta^2 + k^2)^2). \quad (2)$$

This is the relativistic dispersion $E^2 = c^2 p^2 + m^2 c^4$ with the mass equal to the coin angle, $m = \theta$, and $c = 1$ in lattice units (one step per tick). The preferred update order is present in the microscopic walk but absent from the emergent dispersion. Equation (1) is verified to machine precision in the reproducibility scripts.

2.2 The isotropic Weyl cone in three dimensions

The non-trivial content is the 3D lift. The matter cell is the eight-bit $[8, 4, 4]$ register on the Q_3 cube (foundations companion); its physical sector is the 48 codewords satisfying the three Standard-Model constraints. The gauge-compatible spatial hops are sharply constrained: the only register hop that preserves all constraints *and* the electric charge Q is the joint chirality flip on the (χ, W) pair. An exhaustive search over register-local Hermitian reflections shows that the maximum mutually *anticommuting* set that is simultaneously charge-neutral (the $U(1)$ Ward condition) and generation-conserving has size exactly three: the Pauli operators $\{\Gamma_x, \Gamma_y, \Gamma_z\}$ of the effective chirality qubit.

For any anticommuting triple the directional Hamiltonian

$$H(k) = \sum_{d=1}^3 \sin(k_d) \Gamma_d, \quad \{\Gamma_i, \Gamma_j\} = 2\delta_{ij}, \quad (3)$$

satisfies the identity

$$H(k)^2 = \left(\sum_d \sin^2 k_d \right) I \implies E(k) = \pm \sqrt{\sum_d \sin^2 k_d} = |k| + O(k^3), \quad (4)$$

an exactly isotropic massless Dirac (Weyl) cone at leading order, with direction dependence appearing only at $O(k^4)$ (the cubic invariant). The existence of the triple and the isotropy (4) are both verified numerically; the residual $[100]/[111]$ velocity ratio is $1 + O(k^2)$.

2.3 Why the cone is canonical: cubic covariance

The triple is not an arbitrary choice of basis. It is a genuine $\mathfrak{su}(2)$ spin triple — it satisfies both $\{\Gamma_i, \Gamma_j\} = 2\delta_{ij}$ and $[\Gamma_i, \Gamma_j] = 2i\varepsilon_{ijk}\Gamma_k$ — so the single register unitary $U_{C_3} = \exp(-i\frac{2\pi}{3}\hat{n}\cdot\vec{\Gamma}/2)$ with $\hat{n} = (1, 1, 1)/\sqrt{3}$ cyclically permutes $\Gamma_x \rightarrow \Gamma_y \rightarrow \Gamma_z$. Hence $U_{C_3}H(k)U_{C_3}^\dagger = H(C_3k)$: the cone is manifestly C_3 - and therefore O_h -covariant, and with $H^2 \propto I$ this fixes the x, y, z identification as canonical rather than conventional.

2.4 No bare mass: the matter is chiral

The chirality triple is the complete $\mathfrak{su}(2)$ of one qubit, and on a single qubit nothing anticommutes with all of $\{X, Y, Z\}$. The exhaustive search confirms it directly: *no* register operator — charge-neutral or charge-changing — anticommutes with the whole triple. There is therefore no Clifford mass term β to add: the gauge-compatible content is a massless chiral fermion, a bare Dirac mass is forbidden on the register, and fermion mass must arise from a Yukawa / symmetry-breaking coupling that pairs the chiralities without being an anticommuting β . This is the Standard-Model situation exactly, and it returns in Section 4 as the resolution of the equivalence-principle puzzle.

2.5 Photon Lorentz invariance

At the Brillouin-zone centre the 6×6 complete-graph Bloch Hamiltonian has a five-fold degenerate level decomposing as $T_{1u} \oplus E_g$, and degenerate $\mathbf{k} \cdot \mathbf{p}$ mixing splits its group velocities with $v_{[100]}/v_{[111]} = \sqrt{2}$, a *marginal* (first-order) 41% anisotropy. Earlier versions of this work identified that T_{1u} band with the photon and asked how a marginal anisotropy could be hidden in the infrared. That identification was the error, and we correct it here.

The physical photon is the Gauss-projected Maxwell mode, not the K6 band. A $U(1)$ photon is the gauge field that couples to conserved charge, so its inverse propagator must be Ward/Gauss transverse, $\Gamma_{ij}(k)k_j = 0$. The raw K6 T_{1u}/E_g branch fails this test: its $T_{1u}-E_g$ mixing $C(\hat{n})$ has longitudinal projection $\|C(\hat{n})\hat{n}\| = 0.816, 0.408, 0$ along $[100], [110], [111]$ — nonzero for generic directions, so the would-be longitudinal mode does not decouple and the branch carries no gauge structure. The physical photon is instead the Gauss-projected transverse mode of the compact- $U(1)$ Wilson field on the dual simple-cubic gauge web: with lattice gradient $g_i(k) = e^{ik_i} - 1$ the Maxwell kernel $M_{ij} = |g|^2\delta_{ij} - g_i g_j^*$ is Ward-transverse ($Mg = 0$), rank two, with eigenvalues $\{0, |g|^2, |g|^2\}$ — two transverse polarisations plus one longitudinal gauge mode. The five K6 labels were over-counted as photon polarisations; correctly $5 = 2_{\text{transverse}} \oplus 1_{\text{gauge}} \oplus 2_{E_g}$, and the marginal T_{1u}/E_g band is auxiliary gauge-web data, never the photon.

Lorentz invariance holds at accessible energies. The transverse Maxwell dispersion is the simple-cubic Laplacian $|g(k)|^2 = 6 - 2\sum_i \cos k_i$, isotropic at leading order with the first anisotropy entering at $O(k^4)$ in the action — a *dimension-six, irrelevant* velocity correction $\Delta v/v \simeq (a_0 k)^2/12$, not the marginal $O(1)$ effect of the K6 band. At $a_0 \simeq 0.595$ fm this is $\sim 5 \times 10^{-18}$ for optical photons, below the SME cavity bound $\sim 10^{-17}$. Photon-band Lorentz invariance is therefore

restored at accessible energies by the ordinary irrelevant-lattice-artifact mechanism; the Collins–Perez–Sudarsky marginal-anisotropy obstruction *does not apply*, because it was a property of the auxiliary K6 vertex band, not of the gauge photon. No infrared log-running mechanism is required, and none is invoked.

The high-energy cutoff, reframed: trans- Λ_{QCD} quanta as framed null chains. A simple-cubic gauge web at spacing $a_0 \sim \hbar c / \Lambda_{\text{QCD}}$ has a Brillouin ceiling $\pi \hbar c / a_0 \simeq 1.04$ GeV, so it carries the irrelevant-anisotropy continuum photon only for $\omega \ll \Lambda_{\text{QCD}}$; a single trans-cutoff (e.g. TeV) photon is *not* a Bloch mode of the condensate. No finer photon UV is admissible: an elementary-oscillator lattice able to support 1 TeV would require $\Lambda_\gamma \simeq 318$ GeV $\simeq 960 \Lambda_{\text{QCD}}$ and would re-inflate the vacuum energy by $\sim 8.5 \times 10^{11}$, destroying the same Λ_{QCD} cutoff that resolves the cosmological constant. The positive representation is therefore not a finer oscillator lattice but the *framed causal-set / null-chain QED* sector: a source-conditioned null chain carries one external detector/LSZ leg with total P^μ (a noncompact link-count/action label, $P^2 = 0$), not N independent soft-photon legs; link holonomies supply the gauge vertex and Ward identities. The current canon closes the support, LSZ, normalised-leg, recoil-vertex, finite- ρ covariance, Lorentzian-Hodge-sign, framed mesoscopic-Maxwell, and framed-Dirac-spin questions at smooth-field EFT grade. Because the high-energy state is source/detector-conditioned it adds no unconditioned UV oscillator tower, so the *same* Λ_{QCD} plays two non-conflicting roles — the vacuum/IR-condensate cutoff of the ρ_Λ ledger, and the service/action unit counted along conditioned null histories. *Scope:* this is *not* an order-only causal-set theorem, *not* a raw-link positive-measure loop sum, and *not* a dynamical gravitational-tetrad equation. The residual is now precision phenomenology: spinor and polarisation sums around the normalised external leg, finite-density corrections, astrophysical transfer, and Lorentz-violation bounds such as the benchmark 1 TeV over 1 Gpc timing constraint $|\zeta_1| \lesssim 3.2 \times 10^{-21}$, $|\zeta_2| \lesssim 1.1 \times 10^{-24}$. (Computational backing: [6]; machine-checked in `python_code/foundations_trans_lambda_closure.py` and `python_code/foundations_trans_lambda_precision_residuals.py`.)

(Tier history: the first preprint claimed an infrared PASS via a Nielsen–Ninomiya log; the second retracted that to a Collins–Perez–Sudarsky open problem on the marginal K6 reading; a third superseded both when the object-identity computation showed the K6 marginal anisotropy was never the photon’s. This version closes the support/LSZ obstruction and moves the remaining high-energy questions into precision QED and propagation phenomenology.)

3 General Relativity: The Metric as Strain/Clock Covariance

3.1 The metric is the strain field

To find the metric, strain the cone. A symmetric substrate strain ε_{ij} distorts bond lengths, mapping the hop momentum $k \rightarrow (I + \varepsilon)k$, so the matter dispersion becomes

$$E^2(k) = |(I + \varepsilon)k|^2 = k^\top (I + \varepsilon)^2 k. \quad (5)$$

Reading $E^2 = g^{ij} k_i k_j$ identifies the effective inverse metric

$$g^{ij}(\varepsilon) = [(I + \varepsilon)^2]^{ij} = \delta^{ij} + 2\varepsilon^{ij} + O(\varepsilon^2), \quad (6)$$

verified directly from the strained dispersion. The metric perturbation $h^{ij} = 2\varepsilon^{ij}$ is the substrate strain field. Geodesic motion is then immediate: a wavepacket’s propagation phase is $\oint k \cdot dx$ evaluated with the local dispersion, and its stationary rays are the geodesics of g . This is the previously-informal “GR clause” made constructive.

3.2 Irreducible content: lapse, and the spin-2 graviton

The symmetric perturbation h_{ij} decomposes under O_h as

$$h_{ij} : A_{1g}(\text{trace}, 1) \oplus E_g(2) \oplus T_{2g}(3), \tag{7}$$

an orthonormal complete basis on symmetric 3×3 matrices. The trace (A_{1g}) is the conformal factor, the *lapse* — the clock rate. The spin-2 graviton is the full $l = 2$ multiplet $E_g \oplus T_{2g}$; the program’s customary “ E_g graviton” is its cubic-visible half, and full rotational (Lorentz) spin-2 invariance requires E_g and T_{2g} to become degenerate in the infrared.

3.3 The clock made literal

A uniform A_{1g} strain s rescales the local light cone,

$$c_{\text{local}} = 1 + s, \tag{8}$$

verified directly. Strain renormalises the tick exactly as the SR-clock picture demands under load: the “metric as coarse clock covariance” is not a metaphor but the computed response of the dispersion to a conformal strain.

3.4 The stress coupling

Matter couples to the metric through the rank-two stress vertex $\partial H/\partial \varepsilon_{ij}$ — not through the $U(1)$ current. The leading-order coupling of a strain mode T_a has the matrix element $\langle -, k | O_a | -, k \rangle = -|k| \hat{k}^T T_a \hat{k}$, giving a stiffness density $|k|^2 (\hat{k}^T T_a \hat{k})^2$ whose angular average is $\frac{2}{15} \text{Tr}(T_a^2)$, equal for all five $l = 2$ modes. The isotropic Weyl loop therefore dresses E_g and T_{2g} equally at leading order — the rank-two analogue of the photon’s δ_{ij} lemma — with the cubic split confined to the Brillouin-zone scale. The closure of the resulting spin-2 infrared degeneracy is taken up in Section 5.

4 The Equivalence Principle

Universal coupling — the statement that all forms of energy gravitate identically — is the subtle target, because the substrate contains a genuine trap.

4.1 The anti-mass trap

If gravity is defined by straining the *bare hopping operators* $\partial H/\partial \varepsilon$ — the natural reading of Section 3 extended to massive matter — the equivalence principle is violated. Under a conformal strain s at fixed momentum, the redshift $\delta E/E$ computed from the bare-kinetic coupling falls with mass: 0.050 at $m = 0$, 0.030 at $m = 0.2$, 0.010 at $m = 0.5$, 0.003 at $m = 1$ (in the units of the reproducibility script). Heavier states couple *less*. The reason is that a heavy, frustrated codeword hops slowly, so bare-kinetic gravity couples to wavefunction *spread*, not mass-energy density — it is structurally anti-mass.

4.2 The resolution: couple to the full energy

The fix is forced and physical: the metric must scale the full energy T^{00} , kinetic *and* mass. The mass-energy of the substrate is the confinement/Yukawa tension (the chiral matter of Section 2 has

no bare Dirac mass), which lives on the lattice and so scales with the strain. With the full-energy coupling the redshift is mass-universal,

$$\frac{\delta E}{E} = s \quad \text{for every mass,} \quad (9)$$

verified to a spread below 10^{-15} across $m \in [0, 2]$. This matches the program's independently-established discrete equivalence principle, the exact linearity of the Ollivier Ricci curvature response in the source mass.

4.3 Universality and the soft-graviton theorem

The full-energy redshift is identical across charge, colour, generation, and mass — gravity couples to energy, not to charge, in sharp contrast to the electromagnetic shift $\delta E = Q A$ which varies by species. Two further ingredients fix the coupling to be unique. First, the substrate's single-entry accounting: each source contributes one strain entry to the boundary ledger, and gravity reads that one recorded boundary strain (the shadow corollary), so the coupling multiplicity is one for every source. Second, the soft-graviton theorem of Weinberg [13]: a massless spin-2 field (Section 5) coupled to a conserved stress tensor *must* couple universally, or Lorentz invariance fails through the non-decoupling of longitudinal polarisations. The substrate supplies the premises; the geometric mechanism (metric = strain) realises them.

5 The Einstein-Equation Form

5.1 The conserved, symmetric stress tensor

The matter strain-response of Section 3 is the symmetrised momentum current

$$T_{ij} = \frac{1}{2}(v_i k_j + v_j k_i), \quad v = \partial H / \partial k, \quad (10)$$

verified symmetric, and for the massless Weyl cone traceless and conformal, $T^\mu{}_\mu = 0$ with the radiation equation of state $E = 3P$ (a mass breaks the trace, the expected control). As the momentum current it is conserved by translation-Noether invariance of the walk. It is exactly the rank-two source that couples to the spin-2 graviton of Section 3.

5.2 Bianchi \Leftrightarrow QEC conservation

The current canon separates two questions that older drafts conflated. The *form* of the field equation is the covariance statement; the observed Planck hierarchy is a separate horizon/lattice-scale normalisation. At the linearised level the consistency condition is an identity. The linearised Einstein tensor is transverse,

$$q^\mu G_{\mu\nu}[h] = 0 \quad \text{identically for all symmetric } h, \quad (11)$$

verified numerically to 10^{-15} ; the matter source obeys the same transversality $q^\mu T_{\mu\nu} = 0$. Both are the single diffeomorphism invariance of the induced action $S[g]$, since matter couples covariantly through $g = \delta + 2\varepsilon$. In substrate terms this diffeomorphism invariance is the strain-ledger continuity — the shadow corollary that gravity reads recorded boundary strain, together with single-entry accounting. The field equation $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ is therefore consistent in form rather than postulated from outside: the geometric Bianchi identity and the substrate's QEC conservation law are the same statement.

5.3 Entanglement first law to Einstein form

The stronger current result uses the holographic part of the substrate rather than only the strained cone. The verified RT/min-cut area law supplies $\delta S_{\text{bulk}} = \delta A/(4G_{\text{eff}})$ for local balls, the local horizon service-current steady state supplies the KMS modular weight, and the entanglement first law $\delta S = \delta \langle K \rangle$ identifies the linearised geometric variation with the service-current energy variation. This is the Jacobson/Faulkner route in substrate variables: it derives the Einstein-equation *form* at leading order from RT entanglement plus a local boost/KMS modular Hamiltonian. It does *not* by itself derive the observed M_{Pl} hierarchy. The direct entanglement normalisation is a bare emergent value of order GeV; the observed hierarchy remains the separate horizon/lattice selector problem handled in the gravity and constants ledgers. The remaining mathematical residuals are the full nonlinear/discrete derivation, modular-Hamiltonian locality beyond the local-ball limit, and cutoff corrections to the service-current stress tensor.

5.4 Graviton infrared closure via induced gravity

The spin-2 degeneracy left open in Section 3 closes once the graviton kinetic term is recognised as *induced* [10]: the metric has no bare action; its dynamics is the matter loop’s effective action. The leading two-derivative term in the heat-kernel expansion [11] is the Seeley–DeWitt coefficient $a_1 \propto R$, a scalar and therefore isotropic, because it is the determinant of the isotropic Weyl cone. The cubic E_g/T_{2g} anisotropy lives in the higher-derivative $a_2 \propto R^2$ sector, which is dimension six or higher and hence genuinely irrelevant by power counting. This is the regime in which the Wilsonian shortcut *works*: induced-gravity anisotropy is born higher-derivative, so there is no marginal anisotropy to screen — unlike the bare K6 gauge bands of Section 2.5, whose marginal T_{1u}/E_g splitting is auxiliary gauge-web data, not the physical photon. The leading-isotropic stress stiffness computed in Section 3 is the numerical witness. This warrants the same scrutiny that reopened the photon, and the result is informative. The bare K6 E_g band *is* marginally anisotropic ($v_+[100]/v_+[111] = \sqrt{2}$, like the bare K6 T_{1u} band), so the graviton is not safe by default. What rescues it is parametric: the induced graviton kinetic term ($1/G$, Einstein–Hilbert) is a *relevant*, quadratically cutoff-sensitive operator ($1/G \sim \Lambda^2$), whereas the induced photon term ($1/e^2$) is only *marginal* (logarithmic). The structural root is exact: the stress vertex $T_{ij} \sim k_{(i}v_{j)}$ carries one extra power of momentum relative to the current vertex $j_i \sim v_i$, so the stress loop is two powers more UV-divergent. A relevant induced term *is* the leading kinetic term — gravity is genuinely induced — so the bare anisotropic E_g seed is replaced rather than corrected, provided that seed is removed; canonically it is the Γ -point E_g tachyon stabilised by the period-4 vacuum into the induced graviton. The graviton closure is therefore conditional on that stabilisation specifically: if a healthy anisotropic bare E_g kinetic term survived, the graviton would inherit a Collins–Perez–Sudarsky marginal-anisotropy problem.

6 The Horizon Ledger

The program treats four horizon objects separately: the Bekenstein area law (a record-rate statement with channel count $C = 55/8$), the discrete Hawking ladder, the firewall-resolving isometry V_{cell} , and the gravitational source (the shadow corollary). They are one object read four ways. The single object is the twelve-edge strain syndrome δ on the horizon Q_3 register — the 3-cube coboundary $\delta : \mathbb{F}_2^8 \rightarrow \mathbb{F}_2^{12}$ — and the following are computed map-independently.

1. $\ker \delta = \{0, \text{ALL}\}$: a single blind \mathbb{Z}_2 degree of freedom; the syndrome has rank 7, image 128, every value with exactly two preimages $\{c, c + \text{ALL}\}$.

2. The firewall isometry’s “128 complement classes” *are* the 128 syndrome values; its vacuum latch restores the blind bit.
3. The -1 in the severing count $55/8 = (8 \cdot 7 - 1)/8$, the isometry’s vacuum latch, and $\ker \delta$ are the same degree of freedom.
4. The Hawking energy is the syndrome weight $F = |\delta|$, with spectral gap $F_{\min} = 3$ forced by the cube’s 3-regularity and top line 12:2 forced by bipartiteness.

The decisive check is a falsifiable bridge: the program’s Hawking degeneracy ladder is *literally* the bare 3-cube cut-weight distribution minus a 48-state valid set, with the difference non-negative at every level and summing to exactly 48. This confirms $F = |\delta|$ is the genuine cube coboundary rather than a separately-postulated spectrum. Horizon thermodynamics — entropy, spectrum, firewall isometry, gravitational source — is therefore four functionals of δ , collapsing four separate premises into one, the horizon’s readout-channel settlement. This unifies; it does not re-derive the area-law coefficient.

The observable black-hole map has also sharpened. The finite V_{cell} isometry lifts to a finite shell channel $V_{\text{Sch}}(M)$ at current finite-cell grade; the localized source obeys a half-Boltzmann KMS service current; the Hawking ladder is finite; the proper freeze shell and escape-cone factor give the expected M^{-2} scaling; and standard Schwarzschild spin/partial-wave greybody factors can be applied outside the source shell. The current-channel echo amplitude is null in the local service model. The flux normalisation is close: the all-contact Moore alphabet transfer gives the 10/27 latch and $P/P_{\text{SB}} = 0.9971$ for the scalar benchmark, but this remains conditional on proving that horizon severing uses closed-cell Landauer erasure through every non-empty nearest contact. Species/polarisation factors and the full partial-wave ledger remain bookkeeping work. Finally, a strictly local horizon-cell service graph has spectral gap $O(1/N)$ rather than an expander gap, and the current separator/topology audit strengthens this into a bounded-degree local-topology obstruction. Fast scrambling would require extensive nonlocal horizon topology or a non-spatial address-space expander; it is not supplied by the local finite-cell map alone.

7 Status Ledger

Target	Status	Current reading
SR dispersion (matter)	COMPUTED	Anticommuting chirality triple $\Rightarrow H^2 = (\sum \sin^2 k_d)I$, isotropic Weyl cone; no bare Dirac mass.
Photon Lorentz	COMPUTED	Physical photon = Gauss-projected transverse SC-Maxwell mode (not the K6 T_{1u}/E_g band, which fails Ward); irrelevant $O((a_0k)^2)$ anisotropy \Rightarrow Lorentz-invariant for $\omega \ll \Lambda_{\text{QCD}}$ ($\sim 5 \times 10^{-18}$ optical, below SME). Trans-cutoff support/LSZ/normalised leg closed at framed causal-set grade; precision QED, finite-density corrections, and Lorentz-violation phenomenology remain.
Metric variable	COMPUTED	$g^{ij} = \delta^{ij} + 2\varepsilon^{ij}$ from the strained cone; h = strain; geodesics = phase extremals; lapse \oplus spin-2 ($E_g \oplus T_{2g}$).
Equivalence principle	COMPUTED / CONDITIONAL	Full-energy coupling gives mass-universal redshift; anti-mass trap avoided via confinement tension; universality forced by Weinberg.
Einstein-equation form	COMPUTED / CONDITIONAL	Conserved symmetric stress tensor and Bianchi/QEC continuity; RT/min-cut plus local KMS entanglement first law gives the Einstein form at leading order. The observed M_{P1} hierarchy remains horizon/lattice input; non-linear/discrete modular-Hamiltonian rigor remains open.
Horizon ledger	COMPUTED	Four horizon objects = one syndrome δ ; Hawking ladder = cube cut-distribution minus the 48 valid states (bridge verified); local KMS, finite ladder, greybody transfer, and echo-null sharpened. Flux normalisation remains conditional on all-contact horizon severing; local service graph is not a fast scrambler.

8 Open Frontiers

1. **Precision QED and Lorentz-violation phenomenology.** The support and LSZ problem for trans- Λ_{QCD} quanta is closed at framed causal-set/service-GNS grade. What remains is the ordinary phenomenological interface: spinor and polarisation sums around the normalised external leg, finite-density corrections, astrophysical propagation, and translation into SME/Lorentz-violation bounds.
2. **Framed causal-set dynamics.** The mesoscopic Maxwell action and Dirac spin-frame lift are closed at framed smooth-field grade. A raw order-only causal-set theorem, a positive-measure raw-link loop action, and a dynamical gravitational tetrad/spin-connection equation remain genuinely open.
3. **Einstein equation beyond leading order.** The RT/min-cut plus KMS first-law argument gives the Einstein form at local linearised grade. A full nonlinear/discrete proof, modular-Hamiltonian locality away from local balls, and cutoff corrections to the service-current stress tensor remain to be built.
4. **Planck hierarchy.** The entanglement route gives the form of the field equation, not the observed M_{P1} . The observed hierarchy still belongs to the separate horizon/lattice selector and dimensionful-anchor problem.

5. **Confinement-tension stress tensor.** The equivalence principle uses the full energy including the confinement/Yukawa mass; an explicit construction of the confinement-tension contribution to $T_{\mu\nu}$, matching the bare-kinetic part smoothly, is open.
6. **Black-hole observables.** Local KMS, the finite Hawking ladder, Schwarzschild greybody transfer, and echo-null are substantially sharpened. The all-contact horizon-severing theorem, species/polarisation flux factors, and any fast-scrambling nonlocal service graph remain open.
7. **Manifest covariance beyond leading order and geodesics.** The cone is O_h -covariant and isotropic at leading order; a manifestly covariant formulation controlling $O(k^4)$ cubic terms, and a full geodesic-equation derivation including connection terms from the substrate walk, would sharpen the SR/GR interface.

A Detailed Mathematics

A.1 The Clifford triple and the dispersion identity

Let the effective chirality qubit carry Pauli operators $\Gamma_1, \Gamma_2, \Gamma_3$ with $\Gamma_i^2 = I$, $\{\Gamma_i, \Gamma_j\} = 2\delta_{ij}$, and $[\Gamma_i, \Gamma_j] = 2i\varepsilon_{ijk}\Gamma_k$. For $H(k) = \sum_d \sin(k_d)\Gamma_d$,

$$H(k)^2 = \sum_d \sin^2(k_d)\Gamma_d^2 + \sum_{d<e} \sin(k_d)\sin(k_e)\{\Gamma_d, \Gamma_e\} = \left(\sum_d \sin^2 k_d\right)I, \quad (12)$$

the cross terms vanishing by anticommutation. Hence $E(k) = \sqrt{\sum_d \sin^2 k_d}$, with $\sum_d \sin^2 k_d = |k|^2 - \frac{1}{3}\sum_d k_d^4 + \dots$, so the leading dispersion is the isotropic $|k|$ and the first anisotropy is the cubic invariant $\sum_d k_d^4$ at $O(k^4)$, suppressed by $(a_0 k)^2$. The body-diagonal rotation $U_{C_3} = \cos(\pi/3)I - i\sin(\pi/3)\hat{n}\cdot\vec{\Gamma}$, $\hat{n} = (1, 1, 1)/\sqrt{3}$, satisfies $U_{C_3}\Gamma_x U_{C_3}^\dagger = \Gamma_y$ and cyclically, giving $U_{C_3}H(k)U_{C_3}^\dagger = H(C_3 k)$.

A.2 Strain, the metric, and the stress vertex

With the strained hop $H_\varepsilon(k) = \sum_d \sin([(I + \varepsilon)k]_d)\Gamma_d$ and $E^2 = k^\top(I + \varepsilon)^2 k$, the inverse metric is $g^{ij} = \delta^{ij} + 2\varepsilon^{ij} + (\varepsilon^2)^{ij}$. The stress vertex is

$$S_{ij}(k) = \left.\frac{\partial H_\varepsilon}{\partial \varepsilon_{ij}}\right|_{\varepsilon=0} = \cos(k_i)k_j\Gamma_i, \quad (13)$$

and a strain mode T_a (symmetric, traceless) couples through $O_a(k) = \sum_i \cos(k_i)(T_a)_i\Gamma_i$. On the lower band, $\langle -, k|O_a|-, k\rangle = -|k|\hat{k}^\top T_a \hat{k}$, so the stiffness density is $|k|^2(\hat{k}^\top T_a \hat{k})^2$ with angular average $\int d\Omega (\hat{k}^\top T_a \hat{k})^2 = \frac{2}{15}\text{Tr}(T_a^2)$, identical for all traceless T_a — the leading-order isotropy of the spin-2 dressing.

A.3 Conservation and the linearised Bianchi identity

The symmetrised momentum current $T_{ij} = \frac{1}{2}(v_i k_j + v_j k_i)$, $v = \partial H/\partial k$, is conserved by the translation invariance of the walk. For the massless cone $E = |k|$, the spatial trace equals the energy density, $T^i_i = E$, giving $T^\mu_\mu = 0$ ($E = 3P$); a mass term restores a non-zero trace $-m^2/E$. The linearised Einstein tensor,

$$G_{\mu\nu}[h] = -\frac{1}{2}(\partial_\alpha\partial_\mu h^\alpha_\nu + \partial_\alpha\partial_\nu h^\alpha_\mu - \square h_{\mu\nu} - \partial_\mu\partial_\nu h - \eta_{\mu\nu}\partial_\alpha\partial_\beta h^{\alpha\beta} + \eta_{\mu\nu}\square h), \quad (14)$$

satisfies $\partial^\mu G_{\mu\nu} \equiv 0$ for arbitrary symmetric h (the linearised Bianchi identity), matching the matter transversality $\partial^\mu T_{\mu\nu} = 0$; both express the diffeomorphism invariance of the induced action.

A.4 The horizon coboundary

On the cube Q_3 (8 vertices, 12 edges) the coboundary $\delta_e(s) = s_i + s_j \pmod{2}$ has kernel $\{0, \text{ALL}\}$ and rank 7, image of size $2^7 = 128$. The cut-weight distribution over the 256 colourings and the Hawking ladder are

$$D(F) = \{0:2, 3:16, 4:30, 5:48, 6:64, 7:48, 8:30, 9:16, 12:2\},$$

$$g(F) = \{0:1, 3:11, 4:22, 5:38, 6:54, 7:41, 8:25, 9:14, 12:2\}.$$

They satisfy $D(F) - g(F) \geq 0$ for all F with $\sum_F(D - g) = 48$. The spectral gap $F_{\min} = 3$ is the cube's regular degree; the top line $F = 12$ with degeneracy 2 is the bipartition and its complement.

B Reproducibility Starter Table

All scripts listed here live under `python_code/` in [6].

Script	Sector	Purpose
<code>relativity_integration_programme.py</code>	SR clock	Registers the SR/GR identifications; verifies the 1D walk dispersion theorem.
<code>item115_clifford_triple_search.py</code>	SR matter	Exhaustive search: max anticommuting Ward+G0 clique is the chirality triple (3).
<code>item115_clifford_triple_confirm.py</code>	SR matter	Confirms $H^2 \propto I$ isotropy and the no-bare-mass (Weyl) result.
<code>relativity_TR1_3D_dispersion_lift.py</code>	SR matter	Scalar-hop vs Clifford-triple dispersion; locates the isotropy condition.
<code>relativity_TR2_which_photon.py</code>	Photon identity	Gauss-projected transverse Maxwell mode versus raw K6 T_{1u}/E_g branch.
<code>relativity_TR2_cutoff_residual.py</code>	Photon cutoff	Shows why the simple-cubic photon band cannot itself carry TeV quanta without reopening the vacuum-energy cutoff.
<code>foundations_trans_lambda_closure.py</code>	Photon (trans-cutoff)	Framed causal-set null-chain QED representation of trans- Λ_{QCD} quanta; scoped closure of support/LSZ.
<code>foundations_trans_lambda_precision_residuals.py</code>	Photon phenomenology	Normalised-leg precision-QED, finite-density, and Lorentz-violation residual map.
<code>foundations_causet_dirac_spin_closure.py</code>	Dirac spin	Framed causal-set/QEC-service-frame spinor closure; order-only no-go.
<code>relativity_TR3_metric_construction.py</code>	Metric	$g = \delta + 2\epsilon$ from the strained cone; lapse/ E_g/T_{2g} split; clock $c = 1 + s$.
<code>relativity_TR5_einstein_form.py</code>	Einstein	Conserved symmetric/conformal stress tensor; linearised Bianchi transversality.
<code>emergent_einstein_from_entanglement.py</code>	Einstein	RT/min-cut plus local KMS entanglement-first-law derivation of the Einstein form and the Planck-hierarchy residual.
<code>relativity_TR4_universal_coupling.py</code>	Equivalence	Anti-mass trap; full-energy mass-universal redshift; species universality.
<code>relativity_TR6_horizon_ledger_identity.py</code>	Horizon	The four horizon objects as one syndrome; Hawking-ladder/cube-cut bridge.
<code>bh_observable_residual_map.py</code>	Black holes	Local KMS, finite ladder, freeze-shell scaling, greybody transfer, echo-null, and flux residual map.
<code>bh_flux_alphabet_selection.py</code>	Black holes	Tests the all-contact Moore alphabet route to the 10/27 scalar flux normalisation.
<code>bh_fast_scrambling_expander_test.py</code>	Black holes	Shows the local horizon service graph is not a fast scrambler; expander control.
<code>bh_fast_scrambling_topological_obstruction.py</code>	Black holes	Shows bounded-degree local horizon topology is separator-limited; fast scrambling needs nonlocal topology or address-space expansion.

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