An Executable Record Grammar for Quantum Correlations

A finite compression certificate for stabilizer records, Wilson holonomy, magic, detector readout, and bounded residuals

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Abstract

Quantum theory assigns amplitudes to an enormous space of possible correlations. Physical records are much smaller: detector clicks, conserved charges, syndrome bits, stable pointer states, and gauge-invariant loop readouts. This paper presents a finite executable demonstration of a *record grammar*: a disciplined way to separate the full Hilbert-space state from the smaller algebra of detector-readable, stable, reusable records. The demonstration is not a replacement for quantum mechanics and not a proof of the full finite-QEC framework. It is a concrete certificate that the language "what can be said, heard, remembered, and reset" has operator content.

The central example is a four-qubit compression certificate. Two qubits carry Bell/stabilizer endpoint records; one carries a closed-loop T-holonomy record; one is a finite detector branch bit. We define the record algebra generated by the corresponding orthogonal projectors and apply the conditional expectation

$$E_{\mathrm{rec}}(\rho) = \sum_i P_i \rho P_i.$$

The executable check proves that all observables in the record algebra are preserved exactly, while incompatible off-record witnesses are changed but bounded by $\|\rho - E_{\mathrm{rec}}(\rho)\|_1$. The sign witness saturates this bound, so the residual is a measured certificate rather than hidden bookkeeping. This certificate should not be mistaken for a proof of polynomial scalability: a maximal abelian record basis still has 2^n sectors. The scalable target is a generator-form ledger: stabilizer/QEC generators, sparse magic resources, and bounded-correlation tensor-network data. Complementary scripts show that Wilson dressing keeps gauge readouts physical, T-magic is not lost by Clifford-only stabilizer compression but must be retained as a named holonomy resource, noisy detector records degrade an injected T operation by explicit channel laws, and multi-plaquette correlations either factorize, decay with a measured correlation length, or must be promoted to a collective record.

Contents

1 Introduction

Quantum mechanics is often taught as if the state vector, or the density matrix, were the thing that must be understood in full. That is mathematically correct, but it is not how physics is normally read from the world. Detectors do not report arbitrary amplitudes. They report finite records: a click here, a spin result there, a track in a chamber, a pointer position, a syndrome bit, or a macroscopic order parameter. Most of the formal state is never directly addressed by any physical readout.

The record-keeping paradigm begins with that observation. It asks:

> *Of all correlations allowed by Hilbert space, which become stable, readable, reusable records?*

The goal is not to deny the full Hilbert-space state. The goal is to separate three layers that are often blurred:

$$\text{full state capacity}$$
$$\Downarrow$$
$$\text{detector-readable record algebra}$$
$$\Downarrow$$
$$\text{stable records that can be reset and reused}$$

This paper writes up a set of executable toy calculations that make that separation precise. They are deliberately small. They cover Bell and GHZ relations, Wilson dressing, Aharonov-Bohm loop phases, non-abelian path ordering, magic through holonomy, measurement with a finite detector register, noisy feedforward, a plaquette-chain correlation-length test, and finally a record-compression certificate. The scripts are part of the public code record [1].

The most important point is negative as well as positive. A record grammar does *not* license simply ignoring correlations. A correlation outside the current record grammar must be handled in one of three ways:

exactly zero by a selection rule,

bounded by an explicit residual,

or promoted to a named collective record.

This is the standard the demonstration is meant to enforce.

1.1 Why this is not just another compression slogan

Many useful approximations in physics compress the full state: mean fields, effective field theories, stabilizer simulations, tensor networks, pointer bases, hydrodynamic variables, and thermodynamic state variables. The danger is that one can compress too aggressively and lose the important quantum resource. A Clifford-only stabilizer description, for example, can be a powerful skeleton but cannot represent universal quantum computation without non-Clifford resources [2–4].

The demonstration here is therefore built around a stricter rule. Stabilizers and QEC constraints are kept exactly. Gauge phases are kept only when dressed or closed into Wilson data. Magic is not discarded; it is retained as a named closed-loop T-holonomy resource. Detector records are represented by explicit finite registers. Anything else is reported as a residual with a norm bound.

2 The record-keeping paradigm

Definition 1 (Record algebra)**.** *For a finite Hilbert space $\mathcal{H}$, a record algebra $\mathcal{A}_{\text{rec}}$ is the algebra of observables generated by a specified set of compatible record projectors $\{P_i\}$. In the simplest commutative case,*

$$\mathcal{A}_{\text{rec}} = \left\{ \sum_i a_i P_i : a_i \in \mathbb{C} \right\}, \qquad P_i P_j = \delta_{ij} P_i, \qquad \sum_i P_i = I.$$

Definition 2 (Record compression)**.** *The record compression associated with the projectors $\{P_i\}$ is the conditional expectation*

$$E_{\text{rec}}(\rho) = \sum_i P_i \rho P_i.$$

It removes coherences between record sectors while preserving the probabilities inside those sectors.

This is just ordinary quantum mechanics. It is the same mathematical family as projective measurement and dephasing channels [5]. Operator algebra and quantum information theory provide the general language of conditional expectations and completely positive maps [6–8]. The record-grammar claim is not that this map is new. It is that one should use such a map only after declaring the physical record algebra it is meant to preserve.

2.1 Endpoint records and relational records

A Bell pair is the smallest example. For

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle),$$

neither endpoint alone contains a definite local bit:

$$\rho_A = \rho_B = \frac{I}{2}.$$

The relational stabilizers, however, are sharp:

$$X_A X_B = +1, \qquad Z_A Z_B = +1, \qquad Y_A Y_B = -1.$$

This is why Bell correlations [9, 10] are an ideal first example. The fact is not stored as two independent local facts. It is stored as a relational constraint. GHZ/Mermin relations sharpen the lesson: a three-endpoint phase can be invisible to every one- and two-endpoint marginal yet audible to a three-endpoint observable [11, 12].

2.2 Gauge-readable records

Gauge phases add another layer. A naked open phase between two charged endpoints is not a physical record. It changes under a local gauge transformation. A Wilson line or Wilson loop supplies the missing dressing [13]. In a two-route Aharonov-Bohm interferometer [14], individual path phases are gauge bookkeeping, but the closed loop

$$W_{\text{loop}} = U_{\text{upper}} U^*_{\text{lower}} = e^{i\Phi}$$

is gauge-invariant and affects detector probabilities:

$$P_{\text{bright}} = \frac{1 + \cos\Phi}{2}, \qquad P_{\text{dark}} = \frac{1 - \cos\Phi}{2}.$$

The record grammar therefore says: a detector hears dressed endpoint relations or closed loop fluxes, not naked gauge-dependent phases.

2.3 Finite detector records and reset

A projective measurement can be modelled by a reversible write into a finite detector register followed by readout or tracing. If the system is

$$|\psi\rangle = \sqrt{p}\,|0\rangle + e^{i\theta}\sqrt{1-p}\,|1\rangle,$$

a Z-measurement writes the detector branch:

$$\sqrt{p}\,|0_S 0_D\rangle + e^{i\theta}\sqrt{1-p}\,|1_S 1_D\rangle.$$

The irreversible part is not the reversible correlation itself. It is the reset of a finite reusable record. A detached two-outcome detector register with probabilities $p, 1-p$ carries

$$H(p) = -p \ln p - (1-p)\ln(1-p)$$

nats of record entropy. For a fair bit this is $\ln 2$ nats.

This aligns with the general programme of decoherence, einselection, and environment-as-witness records [15–17]. The finite record grammar adds a more mechanical question: which algebra of records is actually being copied, read, protected, and reset?

3 The combinatorial explosion problem

For n qubits, a pure state requires 2^n complex amplitudes before normalisation and global phase are removed. A general density matrix requires

$$4^n - 1$$

real parameters. This grows impossibly fast. Yet bulk matter is usually described by a small set of stable variables: charges, positions, velocities, temperatures, stresses, order parameters, correlation lengths, and response coefficients.

There is an important scaling caveat. A maximal abelian record basis with 2^n sectors has only its diagonal probabilities, but that still requires

$$2^n - 1$$

real numbers. The four-qubit certificate later in this paper compresses 255 density-matrix parameters to 15 record probabilities. That is the ordinary full-to-diagonal reduction

$$4^4 - 1 \longrightarrow 2^4 - 1,$$

not a sub-exponential trend. The scalable goal is therefore different:

the ledger must be represented in generator form, not as a flat list of all record sectors.

For the finite-QEC framework this resolves into a hierarchy whose controlling resource is *magic* (non-Clifford content), not spatial dimension:

1. a stabilizer/QEC tableau or generator list, of size polynomial in the number of cells. This is the substrate's free gift: by the Gottesman–Knill theorem a stabilizer state is classically polynomial in *any* spatial dimension [2], and the framework's record cell is exactly such a state (Sec. 4.1);

2. a sparse magic budget — a count and placement of T-holonomy resources rather than an arbitrary non-Clifford amplitude table. This is the actual cost driver: a state reachable by Clifford operations plus t non-Clifford injections has stabilizer rank $2^{O(t)}$, hence a polynomial ledger when $t = O(\log n)$ [18];

3. a local-correlation ledger (matrix-product or tensor-network) with bounded bond dimension at finite correlation length. This is a *separate* tractable class, not an additional obstruction: it is the route that buys one-dimensional area-law states, whereas stabilizer-plus-sparse-magic needs no correlation-length assumption at all.

The present paper demonstrates the accounting discipline, the first finite certificates, and a direct measurement (Sec. 4.1) placing the record cell in the polynomial regime. It does not claim a polynomial ledger for the full strongly-coupled dynamics — nor could it, since universal polynomial tractability would collapse known complexity separations.

That contrast motivates the central question:

What justifies replacing the full state by a much smaller record description?

The answer cannot be "because the other correlations are inconvenient." The examples below use three legitimate answers:

1. **Exact factorisation or selection rule.** A correlation is zero because the state, stabilizer, gauge rule, charge rule, or projector algebra forces it to be zero.

2. **Explicit bound.** A correlation is nonzero but bounded by a measured correlation length, trace norm, detector threshold, or convergence test.

3. **Promotion.** A correlation is too large or too long-range to be discarded, so it becomes a named collective record.

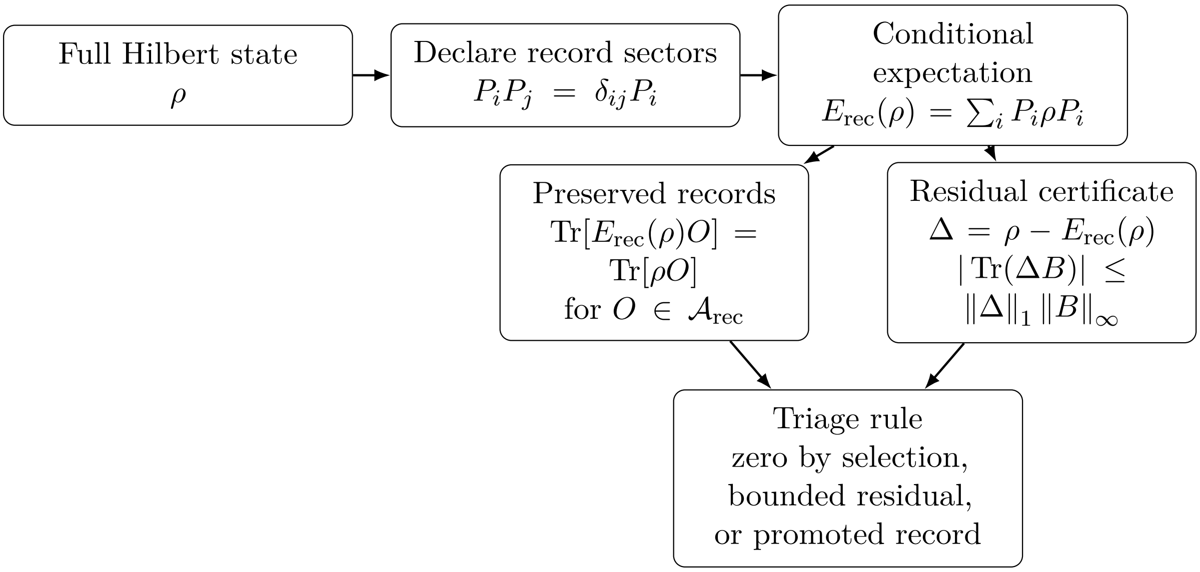

Figure 1: The finite record-compression logic. One does not drop correlations informally. One declares the admissible record projectors, applies the conditional expectation, proves that record observables are preserved, and reports the residual for incompatible witnesses.

3.1 The promotion-termination gap

The word "promote" hides the load-bearing scalability question. If every large correlation is promoted to a new record, and those promoted records generate further long-range correlations without end, the ledger again becomes intractable. Promotion is useful only if it terminates.

The plaquette-chain example gives the seed of the needed theorem. In the short-range regime, a finite transfer gap gives a finite correlation length ξ, so distant connected correlations can be bounded. In the long-range regime, the aligned mode is too large to discard and must be named as one collective record. The missing many-cell theorem is:

$$\text{bounded } \xi \quad \Longrightarrow \quad \text{only } O(\text{poly}\, n) \text{ promoted records are needed.}$$

Equivalently, the local correlation ledger should have bounded bond dimension or another polynomial-size representation. The magic-budget analysis of Sec. 4.1 sharpens what "terminates" must mean: stabilizer content costs nothing at any range or in any dimension, so the binding requirement is that promoted records inject only *sparse* non-Clifford magic, with the bounded-ξ statement above as the one-dimensional special case. Until that sparse-magic property is established for the actual cell dynamics, tractability at scale remains a target, not a result.

4 Magic, stabilizers, and why Clifford-only compression is not enough

Stabilizer methods are central to quantum error correction and efficient simulation of Clifford circuits [2, 19, 20]. They are also a warning. A stabilizer-only description is not all of quantum theory. Non-Clifford resources, often called magic, are needed for universal quantum computation [3, 4, 21].

The record grammar therefore cannot mean "project everything onto a Clifford stabilizer skeleton." That would lose important physics. The finite scripts make this explicit.

The one-qubit T-magic state can be written

$$|T\rangle = \frac{|0\rangle + e^{i\pi/4}|1\rangle}{\sqrt{2}}.$$

Its Bloch vector is

$$(\langle X\rangle, \langle Y\rangle, \langle Z\rangle) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right),$$

so

$$|\langle X\rangle| + |\langle Y\rangle| + |\langle Z\rangle| = \sqrt{2} > 1.$$

This lies outside the one-qubit stabilizer octahedron. A stabilizer-axis replacement loses

$$1 - \frac{1 + 1/\sqrt{2}}{2} \simeq 0.146$$

of the aligned detector signal.

The Wilson-loop examples show how to keep that resource without making it a naked endpoint phase. The T phase is represented as a closed loop

$$W = e^{i\pi/4}.$$

Open link phases can be moved by gauge transformations and vanish under gauge averaging. The closed loop is gauge-readable. In the magic-injection channel, that loop resource becomes an operation. With a finite branch detector and Clifford feedforward, the two data branches are

$$K_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}, \qquad K_1 = \frac{S}{\sqrt{2}}\begin{pmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix}.$$

At $\phi = \pi/4$,

$$K_0 \rho K_0^\dagger + K_1 \rho K_1^\dagger = T \rho T^\dagger.$$

Thus magic is not lost by the record grammar because it is not silently absorbed into the stabilizer skeleton. It is retained as a named holonomy-sector record, and its detector/feedforward use is explicit.

4.1 The magic budget of the record cell

The previous subsection is qualitative: magic is retained, not absorbed. The script `record_grammar_magic_budget.py` makes it quantitative and locates the polynomial-tractability wall exactly. It computes the *stabilizer nullity* ν — a magic monotone with $\nu = 0$ if and only if the state is a stabilizer state [22] — of the framework's own $[8, 4, 4] = \mathrm{RM}(1, 3)$ record cell.

The bare record cell, the uniform superposition over the self-dual $[8, 4, 4]$ code (the CSS $[[8, 0, 4]]$ "complete stabilizer record"), has

$$\nu = 0.$$

It is exactly a stabilizer state. By Gottesman–Knill this is classically polynomial in any spatial dimension, so the substrate's record, syndrome, and error-correction layer is polynomial *for free*: spatial dimension is not the obstruction. Injecting k non-Clifford T resources raises the nullity by at most one each, saturating at $\nu = N/2 = 4$ because T commutes with Z and the four Z-type generators survive,

$$\nu(k) = 0, 1, 2, 3, 3, 4, 4, 4, 3 \qquad (k = 0, \ldots, 8),$$

the dip at $k = 8$ reflecting that transversal $T^{\otimes 8}$ on this doubly-even self-dual code is a near-logical operation. The classical cost of a state of nullity ν scales as 2^ν [18].

This replaces the vague tractability question with a sharp one. A system of cells has a polynomial ledger if and only if its total magic is *sparse* ($O(\log n)$ injected cells), and is exponential when the magic is extensive. Read this way, the 2^{R-1} transfer bond dimension of the strong-coupling Wilson ledger in the companion confinement study is the extensive non-Clifford content of the non-abelian SU(3) field — not a generic consequence of two-dimensionality, since an abelian (Clifford) Z_N gauge theory would have a polynomial ledger. Magic, not dimension, is the controlling resource, and the record sector lies on the polynomial side of that line.

5 Byte-level bridges to the finite-QEC framework

The examples above are ordinary quantum-information toys unless they are tied back to the finite record cell used by the framework. Three scripts make that bridge explicit.

5.1 The byte endpoint record

The script `record_grammar_logical_bell_cell.py` embeds a Bell relation in a two-record slice of the eight-bit record cell. The slice uses

$$|0_B\rangle = |00000000\rangle, \qquad |1_B\rangle = |11111111\rangle.$$

It then checks that a one-bit physical flip toggles the three cube-edge checks incident on that vertex. Thus the local syndrome is an address: it points to the faulty vertex.

This needs a careful qualification. The framework's $[8, 4, 4]$ byte is a balanced classical/QEC record cell, not a claim that the fully constrained cell stores one unconstrained logical qubit in the usual stabilizer-code sense. The two-state slice above is a pedagogical endpoint-record slice. It is useful for showing how relational records and syndromes sit on the byte, but it should not be conflated with the full matter-codeword use of the byte elsewhere in the framework.

5.2 Gauge data on byte endpoints

The script `record_grammar_byte_plaquette_flux.py` places a closed U(1) Wilson flux on finite byte endpoints. The byte syndrome ledger and the closed flux ledger are different ledgers: a local bit fault is addressed by edge syndromes, while the gauge-readable phase is the closed loop.

The script `record_grammar_byte_nonabelian_plaquette.py` repeats the same lesson for an SU(2) loop. Link matrices are path-ordered; the based loop transforms by conjugation; trace and eigenvalues are the color-blind readable data. Again, local byte faults and non-abelian loop curvature coexist on the same finite endpoint structure without becoming the same object.

These byte-level scripts are the strongest framework-specific bridge in the current demonstration. They do not yet constitute a full TCH-cell ledger, but they show the pieces that such a ledger must combine: syndrome generators, Wilson-loop records, sparse magic resources, detector records, and explicit residual bounds.

6 The compression certificate

The integrated demonstration uses four qubits:

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_R \otimes \mathcal{H}_D.$$

The roles are:

- A, B: endpoint qubits carrying Bell/stabilizer relational records;
- R: a route/loop qubit carrying a T-holonomy record;
- D: a finite detector branch bit.

The record basis is

$$\{\mathrm{Bell}_{AB}\} \times \{T_+, T_-\}_R \times \{D0, D1\}_D.$$

This gives 16 orthogonal rank-one projectors P_i. The record algebra is the diagonal algebra in that basis. The compression is

$$E_{\mathrm{rec}}(\rho) = \sum_{i=1}^{16} P_i \rho P_i.$$

Proposition 1 (Preservation of record observables)**.** *If* $O \in \mathcal{A}_{\mathrm{rec}}$*, then*

$$\mathrm{Tr}[E_{\mathrm{rec}}(\rho)O] = \mathrm{Tr}[\rho O].$$

Proof. Write $O = \sum_i o_i P_i$. Then

$$\mathrm{Tr}[E_{\mathrm{rec}}(\rho)O] = \sum_{ij} o_j \,\mathrm{Tr}[P_i \rho P_i P_j] = \sum_i o_i \,\mathrm{Tr}[P_i \rho P_i] = \sum_i o_i \,\mathrm{Tr}[\rho P_i] = \mathrm{Tr}[\rho O].$$

□

This is the algebraic heart of the demonstration. The compression removes coherences between record sectors, but it preserves every observable in the declared record algebra exactly.

For an incompatible witness B, define

$$\Delta = \rho - E_{\mathrm{rec}}(\rho).$$

Then Holder duality for trace norm and operator norm gives

$$|\,\mathrm{Tr}(\Delta B)| \le \|\Delta\|_1 \, \|B\|_\infty .$$

The executable script also constructs the sign witness

$$B_{\mathrm{sign}} = \mathrm{sgn}(\Delta),$$

which satisfies $\|B_{\mathrm{sign}}\|_\infty = 1$ and

$$|\,\mathrm{Tr}(\Delta B_{\mathrm{sign}})| = \|\Delta\|_1 .$$

Thus the residual bound is sharp.

7 Results

7.1 Executable compression certificate

The script `record_grammar_compression_certificate.py` constructs a positive test density matrix with real off-record coherences and applies E_{rec}. The main numerical outputs are shown in Table 1.

The important lesson is not the particular number 0.16875. That number is state-dependent. The lesson is the form of the certificate. The script does not claim that the omitted coherences vanish. It proves that they are outside the declared detector algebra and then reports the trace-norm residual that bounds their possible effect on incompatible witnesses.

Nor is the important lesson the $255 \to 15$ parameter count by itself. For general n, the corresponding flat diagonal ledger still scales as $2^n - 1$. The certificate is therefore an accounting certificate: it proves which algebra is preserved and measures the residual. A scalable TCH/QEC certificate must replace the flat sector list by stabilizer generators, sparse magic data, and bounded-correlation tensor data.

7.2 Script-level regression status

Each executable script is written as a regression test: it exits with status zero only if its stated identities pass. The current script set passed without stderr in the local run used for this paper. The top-level status is:

Check	Result	Meaning
Projectors P_i	Orthogonal, idempotent, complete to numerical precision	The record sectors form a valid finite projective record basis.
$\mathrm{Tr}\,\rho$, positivity, idempotence	Passed	E_{rec} keeps a valid state and is a true compression map on the test state.
Bell stabilizers $Z_A Z_B$, $X_A X_B$	Preserved exactly	The relational stabilizer content survives compression.
T-loop projector P_{T+}	Preserved exactly	The non-Clifford holonomy record is retained rather than Cliffordized away.
Detector branch Z_D and joint $P_{T+}D1$	Preserved exactly	Finite detector records remain operational records.
Random observables in $\mathcal{A}_{\mathrm{rec}}$	Preserved exactly	The result is algebraic, not a cherry-picked list of observables.
$\lVert\rho - E_{\mathrm{rec}}(\rho)\rVert_1$	0.16875	The omitted off-record part is measured and reported.
Sign witness	Saturates the 0.16875 bound	The residual is a sharp certificate of worst-case lost expectation.
Haar pure-state residuals	median 1.7885, range 1.6513–1.8422	Generic pure states have much larger off-record tails; the selected test state is deliberately tame.
Parameter count	$255 \to 15$	The finite example compresses the full density matrix to its diagonal record ledger; this is not yet a polynomial-scaling claim.

Table 1: Summary of the finite compression certificate. The exact numerical values are produced by `record_grammar_compression_certificate.py`.

Script group	Status	Main check
Bell/GHZ/projective measurement	PASS	Relational records, finite detector registers, and reset ledgers behave as stated.
Wilson and Aharonov-Bohm examples	PASS	Naked open phases are gauge bookkeeping; dressed or closed-loop records are readable.
Byte-level bridge scripts	PASS	Byte syndromes address local faults while U(1)/SU(2) loop data remain separate gauge-readable ledgers.
Magic-through-holonomy and injection	PASS	T-magic is retained as a closed holonomy resource and consumed by a finite feedforward channel.
Noisy magic injection	PASS	Branch-bit flips and erasures obey the stated channel-fidelity laws.
Two-plaquette and chain correlations	PASS	Correlations factorize, decay with measured ξ, or are promoted to a collective mode.
Compression certificate	PASS	Record observables are preserved and off-record residuals are trace-norm certified.

Table 2: Regression status of the executable record-grammar examples.

7.3 Correlation-length compression

The script `record_grammar_plaquette_chain_correlations.py` addresses the other side of the same issue: correlations across many loop records. It uses a periodic compact U(1) chain with action

$$S = \beta \sum_i [1 - \cos \Phi_i] + \kappa \sum_i [1 - \cos(\Phi_i - \Phi_{i+1})].$$

The symmetric transfer matrix is

$$T(\Phi, \Phi') = \exp\left[\frac{\beta}{2}(\cos\Phi + \cos\Phi') + \kappa\cos(\Phi - \Phi')\right],$$

and

$$\langle f(\Phi_0) g(\Phi_r)\rangle = \frac{\mathrm{Tr}[F T^r G T^{L-r}]}{\mathrm{Tr}[T^L]}.$$

The result is the same triage rule in spatial form. With $\kappa = 0$, the chain factorizes. With $\beta = 2, \kappa = 0.5$, the measured correlation length is

$$\xi = 0.523533602,$$

and the sine connected correlation has already fallen to

$$|C_{\sin}(8)| = 6.72 \times 10^{-8}, \qquad |C_{\sin}(16)| = 1.55 \times 10^{-14}.$$

In that regime, long-distance off-grammar correlations are genuinely too small to matter at ordinary detector precision, because the transfer gap gives an explicit bound.

With $\beta = 0, \kappa = 20$, by contrast, the correlation length is

$$\xi = 38.98$$

on a 64-site ring, and the half-chain alignment is still about 0.657. That is not a small tail. The correct record grammar is then to promote the aligned chain mode to a collective record.

Regime	Numerical result	Record-grammar verdict
$\kappa = 0$	Connected covariances vanish to numerical precision	Exact factorisation.
$\beta = 2, \kappa = 0.5$	$\xi = 0.5235$, $C_{\sin}(8) = 6.72 \times 10^{-8}$	Bounded tail; safe compression after the stated error budget.
$\beta = 0, \kappa = 20$	$\xi = 38.98$, half-chain alignment 0.657	Not discardable; promote to a collective aligned-loop record.

Table 3: Correlation triage from the plaquette-chain transfer-matrix script.

7.4 Non-abelian Wilson-cluster certificates

The scripts `record_grammar_nonabelian_wilson_cluster.py` and `record_grammar_su3_wilson_cluster.py` begin the strong-sector extension. They declare a two-dimensional axial-gauge plaquette record-action measure and use rectangular Wilson loops as detector-readable records.

For SU(2) the independent plaquette records are distributed as

$$dP(U_p) \propto d\mathrm{Haar}(U_p) \exp\left[\frac{\beta}{2}\mathrm{Re}\,\mathrm{Tr}(U_p)\right],$$

and for SU(3) as

$$dP(U_p) \propto d\text{Haar}_{SU(3)}(U_p)\exp\left[\frac{\beta}{3}\text{Re}\,\text{Tr}(U_p)\right].$$

The loop record is the normalized trace of the ordered product inside the rectangle. Since the one-plaquette measure is central, $\mathbb{E}[U_p] = a(\beta)I$, and independent plaquettes give

$$\langle W(C)\rangle = a(\beta)^A.$$

At $\beta = 2$, the SU(2) script finds

$$a = 0.433127431614, \qquad \sigma = -\log a = 0.836723294938,$$

with the fitted perimeter term and constant zero to numerical precision. At $\beta = 5$, the SU(3) script gives

$$a = 0.355744245 \pm 0.002364, \qquad \sigma = 1.033543219,$$

again with a zero perimeter term in the factorized area-law ledger. In both cases the flat list of rectangular loops grows like $O(L^4)$, while the generator ledger uses local plaquette records plus one promoted area-law record, scaling as $O(L^2)$ in this controlled setting.

This is not a proof of four-dimensional Yang-Mills confinement or of the full TCH/SU(3) dynamics. It is a deliberately bounded certificate: the same Wilson-loop object is both the detector interface and the tractable generator record under an explicit record-action measure.

8 Discussion

8.1 What the demonstration establishes

The demonstration establishes a useful finite standard:

> *A record-grammar compression is acceptable only when it specifies its record projectors, preserves all observables in the declared record algebra, and reports an explicit residual for everything outside that algebra.*

That is the main result. It is not a philosophical slogan. It is implemented as finite matrices and self-checking scripts.

The record-keeping paradigm also clarifies the relationship between stabilizer methods and more general quantum behaviour. Stabilizers and QEC give the protected skeleton. Wilson loops give gauge-readable phase records. Magic phases must be retained as named non-Clifford resources, not silently projected away. Finite detector registers make measurement and reset operational rather than merely interpretive.

8.2 Why the full correlations are not just ignored

The phrase "full correlations are too small to be significant" must be used with care. It is true only after a calculation has shown it. The plaquette-chain example gives such a calculation in the short-correlation regime: the transfer gap produces a measured ξ, and distant correlations fall below a stated numerical scale. But the collective regime gives the opposite result: the correlations remain large and must be promoted.

The compression certificate says the same thing algebraically. The off-record residual is not zero. It is

$$\|\rho - E_{\text{rec}}(\rho)\|_1 = 0.16875$$

for the chosen test state. That is not swept away. It is reported as the sharp worst-case effect on incompatible witnesses. What is exact is the preservation of the declared record algebra.

The residual is also state-dependent. To check that the selected test state is not being used to hide a generic problem, the script samples 64 Haar-distributed pure states in the same 16-dimensional Hilbert space. Their trace-norm residuals have

$$\min = 1.6513, \qquad \text{median} = 1.7885, \qquad \max = 1.8422.$$

This is the expected behaviour: a generic pure state is mostly off-diagonal in any fixed record basis. The record grammar does not claim those coherences are small. It claims only that the declared detector algebra is preserved exactly and that the omitted part is measured.

This is the discipline that makes the approach scientifically useful. It can say when a reduced description is legitimate, and it can also say when it is not.

8.3 How this supports intuition about bulk matter

The full many-body wavefunction is generally intractable. Bulk matter is not. The reason is not that the full state is simple. The reason is that most macroscopic probes couple to a much smaller set of stable records: conserved charges, local densities, order parameters, fluxes, symmetry sectors, quasiparticles, hydrodynamic modes, and response functions.

The finite examples in this paper show the shape of a possible explanation: protected records form a stable algebra, gauge-readable phases live in Wilson data, magic is a sparse named resource, detector records are copied and reset, and off-grammar tails are either short-ranged or promoted to collective variables. That does not prove the universe is literally this finite model. It does show how a vast Hilbert space can present a much smaller stable interface to detectors.

8.4 What remains to be assembled

The present paper deliberately keeps the algebra small enough to check by direct matrix calculation. The decisive next demonstration is larger and more framework-specific. It should take an eight-bit cell, or a small connected cluster of cells, and report the ledger in generator form:

$$\begin{aligned}\text{ledger size} = {} & \text{stabilizer generators} + \text{sparse magic records} \\ & + \text{bounded-correlation tensor data} + \text{detector/reset records}.\end{aligned}$$

It should then apply one step of the framework walk or service map $\mathcal{W}$ and report:

1. which generators are preserved or updated symbolically;
2. which magic records are created, moved, or consumed;
3. the residual leakage outside the record algebra per step;
4. whether any promoted collective records terminate after polynomially many additions.

That would turn the current static certificate into a dynamic TCH/QEC ledger test. Without that step, large-scale tractability is plausible but not yet demonstrated.

8.5 Limitations

The current demonstration is finite and pedagogical. It does not derive the Born rule, Maxwell theory, Yang-Mills dynamics, relativistic scattering, or a many-body continuum limit. The compression certificate uses a chosen record algebra; a deeper theory must explain why that algebra is selected by the dynamics, environment, QEC structure, or detector coupling. The plaquette-chain example is one-dimensional and compact U(1), not a realistic field theory. Most importantly, the flat record-basis certificate still scales as 2^n. Polynomial tractability is controlled by magic, not dimension: the record cell is measured to be an exact stabilizer state (Sec. 4.1), so the substrate layer is polynomial in any dimension and scale tractability reduces to whether the dynamics injects only sparse non-Clifford magic.

Those limitations are not cosmetic. They define the next research tasks.

9 Conclusions

The record-keeping paradigm can be made executable. In the finite examples studied here, the words "endpoint", "relation", "Wilson dressing", "magic", "detector", "feedforward", "reset", and "residual" all have operator meanings.

The key conclusions are:

1. Bell and GHZ examples show that records can be relational rather than local endpoint facts.
2. Wilson examples show that detector-readable phase is dressed or closed loop data, not naked gauge bookkeeping.
3. Stabilizer compression is useful but incomplete; non-Clifford magic must be retained as a named resource.
4. A closed T-holonomy can be consumed by a finite detector/feedforward channel to implement a T operation.
5. Detector record failure gives explicit noisy-channel laws, so record reliability has operational meaning.
6. Plaquette-chain correlations are either exactly zero, bounded by a measured correlation length, or promoted to collective records.
7. SU(2) and SU(3) Wilson-cluster scripts show an explicit area-law loop ledger under a declared two-dimensional record-action measure.
8. The integrated compression certificate preserves all declared record-algebra observables exactly and reports the trace-norm residual for everything outside the algebra.
9. The certificate is an accounting result, not a scaling theorem; but the record cell is measured to be an exact stabilizer state (nullity $\nu = 0$), so large-scale tractability is controlled by the magic budget — sparse non-Clifford content — not by spatial dimension.

The resulting message is simple:

do not drop correlations and hope;   preserve the record algebra and certify the residual.

That is the standard needed if a record-based view of quantum physics is to be more than metaphor.

Code availability

The executable scripts described here are in the project code base under the `python_code` directory, with names beginning `record_grammar_`. The public repository for accessible project code is `https://github.com/dgedge/itfrombit`. The central scripts for this paper are:

- `record_grammar_compression_certificate.py`
- `record_grammar_plaquette_chain_correlations.py`
- `record_grammar_magic_injection_channel.py`

- record_grammar_magic_injection_error_model.py
- record_grammar_magic_through_holonomy.py
- record_grammar_logical_bell_cell.py
- record_grammar_byte_plaquette_flux.py
- record_grammar_byte_nonabelian_plaquette.py
- record_grammar_nonabelian_wilson_cluster.py
- record_grammar_su3_wilson_cluster.py

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