Quantum Darwinism as Noisy Syndrome Broadcast: A Stabilizer-QEC Theorem for Objective Records

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Abstract

Quantum Darwinism explains objective classical facts as information about einselected pointer states that has been redundantly recorded in many independent fragments of the environment. This paper formulates a concrete stabilizer-QEC version of that mechanism. For a noisy syndrome extraction instrument whose fragments are conditionally independent given the syndrome and locally decodable with error probabilities ϵ_i, we prove an approximate strong-Darwinism theorem. Each fragment obeys the Fano lower bound

$$I(S : F_i) \geq H(S) - h_2(\epsilon_i) - \epsilon_i \log(m-1),$$

so every fragment below the corresponding error threshold is an objective witness for the syndrome. If the local records are ϵ_i-close to orthogonal syndrome records, the broadcast state is trace-distance close to an ideal spectrum-broadcast state by the standard telescoping bound $\sum_i \epsilon_i$, with the independent classical wrong-label model giving the sharper $1 - \prod_i(1 - \epsilon_i)$. The ideal GHZ-like plateau then appears as the zero-error corollary. Finally, finite reusable record registers force a reset ledger: local reset costs $\sum_i H(F_i)$, while globally compressed reset costs $H(F_1 \cdots F_N)$. The finite-QEC contribution is therefore not a new numerical prediction but a mechanism-level theorem tying pointer selection, syndrome broadcast, strong Darwinism, and Landauer bookkeeping into one auditable structure.

Contents

1 Introduction

Zurek's programme of decoherence, einselection, and quantum Darwinism gives a compelling account of why a quantum universe can look classical to many observers [14–16]. The basic idea is not that a conscious observer causes a special collapse. It is that the environment monitors a system. Only some observables survive this monitoring predictably; these are the pointer observables. Information about their values proliferates into many environmental fragments. Many observers can then sample disjoint fragments and infer the same state of the system without touching the system directly. Objectivity is the availability of many independent records.

The finite-QEC substrate programme has recently moved closer to this language. The earlier presentation foregrounded geometry: a truncated-cubic or register-bearing honeycomb, defects, cells, and graph structure. The current presentation foregrounds the mechanism beneath that geometry: finite records, quantum error correction, syndrome and strain readouts, service currents, Landauer reset, and explicit ledgers for observables [5, 6]. This change is not a cosmetic rename. It changes the status of the geometry. Geometry is no longer the primary explanation; it is the ordered phase in which the record-carrying code crystallises.

This paper develops the contact point with quantum Darwinism. Its thesis is:

> In a finite QEC substrate, quantum Darwinism is naturally realised as syndrome broadcast. The environment is a witness because QEC error detection copies orthogonal syndrome records into many fragments; the pointer basis is the syndrome basis; and reset of finite record registers supplies the thermodynamic arrow.

The claim is intentionally modest. Quantum Darwinism and spectrum broadcast structure already contain the formal theory of redundant records [2, 9, 11]. The new contribution proposed here is not a new entropy identity. It is a mechanism-level reconstruction: starting from stable local records and finite noise, the finite-QEC substrate identifies the pointer observable, the legal record-copying channel, the redundancy mechanism, the reset cost, and the observable ledger in one structure.

2 Terms and Scope

We first fix vocabulary, because many confusions in this area arise from using ordinary words in specialised quantum senses.

Definition 1 (Record)**.** *A record is a physical degree of freedom whose state can be read repeatably and compared by multiple observers. A record is not merely information in an abstract sense; it is information stored in a physical system.*

Definition 2 (Pointer state)**.** *A pointer state is a state that remains predictable under environmental monitoring. In the language of decoherence, pointer states are selected by the system-environment* *interaction: superpositions of pointer states decohere, whereas the pointer states themselves are robust.*

Definition 3 (Syndrome)**.** *In quantum error correction, a syndrome is the diagnostic readout that tells us which stabilizer checks have been violated. A syndrome can reveal the class of an error without revealing or destroying the protected encoded information.*

Definition 4 (Quantum Darwinism)**.** *Quantum Darwinism is the claim that objective classical facts arise because information about pointer states is copied redundantly into many environmental fragments. Objectivity is then operational: many observers can independently intercept different fragments and infer the same pointer value.*

Definition 5 (Spectrum broadcast structure)**.** *A spectrum broadcast structure is a state in which a classical pointer label is correlated with many conditionally independent environmental fragments, and each fragment carries distinguishable records of that label. It is a formal model of objectivity: many observers can read different fragments and recover the same classical value.*

Definition 6 (Strong Darwinism)**.** *Strong quantum Darwinism strengthens the redundancy idea by requiring that fragments carry accessible classical information about the pointer observable without hiding essential information in fragment-fragment quantum correlations. In the syndrome theorem below, this appears as classical syndrome records, fragment decodability, and conditional independence* $I(F_A : F_B|S) = 0$*.*

Definition 7 (Finite-QEC substrate)**.** *Here "finite-QEC" is a shorthand, not a formal replacement name for every historical version of the framework. It denotes the current mechanism-first presentation: a finite, record-writing quantum-error-correcting substrate whose ordered phase supplies geometry, and whose syndrome, strain, service, boundary, and wall-shadow ledgers specify observables.*

The paper does not claim to derive the Born rule from quantum Darwinism. It uses the standard entropy and mutual-information diagnostics of the Darwinist programme and the record-reconstruction assumptions of the finite-QEC canon. The Born-measure question, the naturalness of the stable-record floor, and sector-specific quantitative billing maps remain distinct issues [5].

3 The Record-First Reconstruction

The useful shift in the current framework is to start with records rather than particles or geometry. Every empirical theory already assumes that some facts can persist long enough to be compared. Without stable records there are no repeatable observations, histories, thermodynamics, or observers.

Once stable records are accepted as an operational floor, several standard quantum structures become less arbitrary.

1. Repeatable records must correspond to distinguishable alternatives. Non-orthogonal alternatives cannot be copied or re-read without disturbance.

2. In Hilbert-space language, distinguishable repeatable alternatives are represented by projectors: mathematical yes/no filters onto orthogonal sectors.

3. Writing a record can be modelled reversibly before reset, by a Stinespring/Naimark isometry

$$V : \ |s\rangle_S|0\rangle_R \longmapsto |s\rangle_S|s\rangle_R$$

on the record sectors.

4. If records must survive finite noise, error correction is not an optional embellishment. It is the structure that makes records durable.

5. If record registers must be reused, reset is irreversible and carries a Landauer cost [1, 10].

This chain does not by itself prove a particular microscopic substrate. It does explain why a record-writing QEC substrate is not an arbitrary metaphor. It is a natural place to land if one insists that facts be repeatable, finite, locally testable, and robust under noise.

4 QEC Supplies the Pointer Basis

In ordinary presentations of quantum Darwinism, the pointer basis is selected by the system-environment Hamiltonian. That is correct but abstract. The finite-QEC setting makes the monitored observable explicit: the substrate's decoherence channel is its error-detection channel. Error detection monitors the stabilizers. The readout of those stabilizers is the syndrome. Therefore the pointer basis is the syndrome basis.

For a stabilizer code, let g_a be commuting stabilizer generators. The joint syndrome sectors are simultaneous eigenspaces of the g_a:

$$g_a|s\rangle = (-1)^{s_a}|s\rangle, \qquad s_a \in \{0, 1\}.$$

A syndrome-recording environment couples to these eigenvalues. In the simplest one-check model,

$$(\alpha|s=0\rangle + \beta|s=1\rangle)\,|0\rangle_E \longmapsto \alpha|s=0\rangle|0\rangle_E \;+\; \beta|s=1\rangle|1\rangle_E.$$

The definite-syndrome states remain unentangled with the environment; a superposition becomes entangled and locally decoheres. The predictability sieve therefore selects the definite syndrome states. This is exactly what one expects from QEC: the environment is not measuring an arbitrary coordinate; it is reading the diagnostic checks.

The local code used in the finite-QEC canon is a self-dual doubly-even $[8, 4, 4]$-type record cell, related to the Reed-Muller/Hamming-code family [3, 7, 8, 12, 13]. The present paper does not rederive that cell. It uses the consequence that the substrate has a commuting stabilizer/syndrome structure, so that a definite syndrome basis is available as the pointer basis.

5 Noisy Syndrome Broadcast Theorem

The ideal GHZ-like syndrome broadcast state is useful, but it is not the right stopping point. A theorist should ask what survives finite noise. The answer is an approximate spectrum-broadcast and strong-Darwinism statement under explicit syndrome-extraction assumptions.

Let the measured syndrome have $m \geq 2$ possible values $s \in \{1, \ldots, m\}$ with probability distribution p_s. Write

$$H(S) = -\sum_s p_s \log p_s$$

with natural logarithms. A noisy QND syndrome-extraction instrument produces a classical-quantum state

$$\widetilde{\rho}_{SF_1\cdots F_N} = \sum_s p_s\,|s\rangle\langle s|_S \otimes \rho^{(s)}_{F_1} \otimes \cdots \otimes \rho^{(s)}_{F_N}. \qquad (1)$$

The important structural assumption is conditional product form: once the syndrome is fixed, different record fragments carry independent noisy copies of that syndrome. This is the stabilizer-QEC analogue of an environment whose fragments independently witness the pointer value.

Theorem 1 (Noisy syndrome broadcast implies approximate strong Darwinism)**.** *Consider the state* (1)*. For each fragment F_i, suppose there is a decoding POVM $\{M_i^{\hat{s}}\}_{\hat{s}=1}^m$ whose average syndrome-decoding error is*

$$e_i = 1 - \sum_s p_s \operatorname{Tr}\left(M_i^s \rho_{F_i}^{(s)}\right).$$

Then:

1. *The fragments are conditionally independent given the syndrome: $I(F_A : F_B|S) = 0$ for all disjoint fragment sets A, B.*

2. *Each fragment carries syndrome information bounded below by*

$$I(S : F_i)_{\widetilde{\rho}} \geq H(S) - h_2(e_i) - e_i \log(m-1), \tag{2}$$

where $h_2(e) = -e \log e - (1-e)\log(1-e)$.

3. *For tolerance $0 < \delta < 1$, every fragment satisfying*

$$h_2(e_i) + e_i \log(m-1) \leq \delta H(S)$$

is a $(1-\delta)$-objective Darwinist witness: $I(S : F_i) \geq (1-\delta)H(S)$. Hence the singleton-fragment redundancy obeys

$$R_\delta \geq \#\{i : \; h_2(e_i) + e_i \log(m-1) \leq \delta H(S)\}.$$

4. *If, in addition, there are mutually orthogonal ideal record states $\tau_{F_i}^{(s)}$ for each fragment, with*

$$D\left(\rho_{F_i}^{(s)}, \tau_{F_i}^{(s)}\right) \leq \mu_i \quad \textit{for all } s,$$

where $D(\rho, \sigma) = \frac{1}{2}\|\rho - \sigma\|_1$, then $\widetilde{\rho}$ is close to the ideal spectrum-broadcast state

$$\omega_{\text{SBS}} = \sum_s p_s |s\rangle\langle s|_S \otimes \tau_{F_1}^{(s)} \otimes \cdots \otimes \tau_{F_N}^{(s)}$$

by

$$D(\widetilde{\rho}, \omega_{\text{SBS}}) \leq \sum_{i=1}^N \mu_i. \tag{3}$$

Proof. The first claim follows directly from the conditional product form in (1). Conditioned on a definite syndrome s, the joint fragment state factorises across all fragments, so no two disjoint fragment sets have conditional mutual information.

For the information bound, apply the fragment's decoding POVM and call the classical outcome $\widehat{S}_i$. Data processing gives

$$I(S : F_i)_{\widetilde{\rho}} \geq I(S : \widehat{S}_i).$$

Fano's inequality for an m-valued variable gives [4, Sec. 2.10]

$$H(S|\widehat{S}_i) \leq h_2(e_i) + e_i \log(m-1).$$

Therefore

$$I(S : \widehat{S}_i) = H(S) - H(S|\widehat{S}_i) \geq H(S) - h_2(e_i) - e_i \log(m-1),$$

which proves (2). The redundancy statement is the same inequality with the right-hand deficit required to be at most $\delta H(S)$.

For the trace-distance claim, use joint convexity of trace distance and the standard telescoping bound for tensor products:

$$D\left(\bigotimes_i \rho_{F_i}^{(s)}, \bigotimes_i \tau_{F_i}^{(s)}\right) \le \sum_i D\left(\rho_{F_i}^{(s)}, \tau_{F_i}^{(s)}\right) \le \sum_i \mu_i.$$

Averaging over s gives (3). □

Remark 1 (Perturbations beyond exact conditional product form)**.** *If an actual output state ρ is within trace distance η of $\widetilde{\rho}$, standard continuity bounds for conditional entropy degrade* (2) *only by $O(\eta \log m + h_2(\eta))$, with constants depending on the chosen continuity theorem. Thus weak residual fragment interactions do not destroy the result discontinuously; they lower the plateau by a controlled continuity correction.*

Remark 2 (Independent classical wrong-label model)**.** *The repository audit uses the exactly classical special case in which fragment i writes the correct syndrome with probability $1-\epsilon_i$ and a uniform wrong syndrome otherwise. In that model $e_i = \mu_i = \epsilon_i$, and the global trace distance to the perfect spectrum-broadcast state is exactly*

$$D(\widetilde{\rho}, \omega_{\mathrm{SBS}}) = 1 - \prod_i (1-\epsilon_i),$$

the probability that at least one copy is wrong. This is sharper than the general union bound $\sum_i \epsilon_i$.

6 Ideal Plateau as a Corollary

The usual Darwinist plateau is the zero-error corollary of Theorem 1. Let the system have orthogonal syndrome states $\{|s\rangle_S\}$, with probabilities $p_s = |c_s|^2$. Let the environment consist of N fragments $F_1, \ldots, F_N$, each capable of storing an orthogonal record $|s\rangle$. The ideal broadcast state is

$$|\Psi\rangle = \sum_s c_s \, |s\rangle_S |s\rangle_{F_1} |s\rangle_{F_2} \cdots |s\rangle_{F_N}. \tag{4}$$

For any non-empty proper fragment $F \subsetneq E$, the reduced state of SF is a perfectly correlated classical record:

$$\rho_{SF} = \sum_s p_s \, |s\rangle\langle s|_S \otimes |s\rangle\langle s|_F^{\otimes |F|}.$$

Hence

$$H(F) = H(S), \qquad H(SF) = H(S),$$

and the mutual information is

$$I(S:F) = H(S) + H(F) - H(SF) = H(S). \tag{5}$$

Corollary 1 (Ideal syndrome Darwinism)**.** *In the ideal syndrome-broadcast state* (4)*, every non-empty proper environment fragment carries exactly the classical syndrome entropy $H(S)$. If single fragments are accepted as disjoint witnesses, the redundancy is $R_\delta = N$ for any $\delta > 0$ in the perfect-copy limit.*

Proof. Set all decoding and record-distance errors in Theorem 1 to zero. Equation (5) gives the same result directly by entropy accounting. For the whole environment E, the state SE is pure, so $I(S:E) = 2H(S)$. The additional $H(S)$ is global phase coherence, not redundantly accessible classical information. Proper fragments see the diagonal syndrome record and not the conjugate phase. □

The finite-QEC content is the identification of s with the stabilizer syndrome readout and of the fan-out channel with QEC error detection. The entropy algebra is standard quantum Darwinism; the mechanism is syndrome extraction.

7 Record Copying, No-Cloning, and Reset

A common worry is that redundant record copying violates the no-cloning theorem. It does not. The no-cloning theorem forbids copying an arbitrary unknown quantum state. It does not forbid copying mutually orthogonal classical records. Syndrome values are orthogonal pointer records. Copying them is a controlled classical fan-out,

$$|s\rangle_S|0\rangle_{F_1}\cdots|0\rangle_{F_N} \mapsto |s\rangle_S|s\rangle_{F_1}\cdots|s\rangle_{F_N},$$

defined on the syndrome basis. It is exactly the legal case.

This distinction separates three operations that are often blurred together:

1. dephasing in the monitored pointer basis;

2. reversibly correlating a pointer value with record registers;

3. resetting those registers for reuse.

The first two can be represented without thermodynamic irreversibility in a larger Hilbert space. The irreversible service step is reset.

Proposition 1 (Reset ledger for redundant syndrome records)**.** *Let the fragment record outcomes be classical random variables $(Y_1, \ldots, Y_N)$ with joint distribution $q(y_1, \ldots, y_N)$. At inverse temperature $\beta = (k_B T)^{-1}$, independent local reset of the fragments obeys*

$$\beta Q_{\text{local}} \geq \sum_{i=1}^{N} H(Y_i),$$

whereas a globally compressed reset of the whole record block obeys

$$\beta Q_{\text{global}} \geq H(Y_1, \ldots, Y_N).$$

By subadditivity,

$$H(Y_1, \ldots, Y_N) \leq \sum_i H(Y_i).$$

In the perfect broadcast case $Y_i = S$ for all i, so

$$\beta Q_{\text{local}} \geq NH(S), \qquad \beta Q_{\text{global}} \geq H(S).$$

Proof. Landauer's principle states that erasing a finite memory with Shannon entropy $H(Y)$, using natural logarithms, exports heat satisfying $\beta Q \geq H(Y)$ in the ideal limit. Applying this separately to each fragment gives the local ledger. Applying it once to a reversible compression of the whole correlated record block gives the global ledger. The inequality between the ledgers is ordinary subadditivity of Shannon entropy. If all fragments contain perfect copies of the same syndrome S, each marginal has entropy $H(S)$, while the joint block has only the entropy of the single shared variable S. □

The proposition is intentionally a ledger statement, not a single universal cost assignment. If the substrate physically resets fragments locally, it pays the local bill. If a globally coherent service could compress all fragments before erasure, it pays the compressed bill. The finite-QEC point is that measurement is not merely record correlation; a reusable syndrome-extraction service must also specify how records are reset.

8 How the Geometry Maps onto the QEC Picture

The framework's older terminology was geometry-forward. It spoke naturally of cells, honeycombs, truncated-cubic structure, branches, faces, and defects. The current finite-QEC terminology is mechanism-forward. The two are not in conflict; the geometry is the ordered carrier of the code.

Geometric/older language	Finite-QEC interpretation
Local cell or byte	The local record cell. The $[8,4,4]$-type byte-like object specifies what can be stored, read, copied, protected, corrected, or erased.
Truncated-cubic / register-bearing honeycomb	The ordered crystalline phase of those record cells. Geometry is not decorative; it is the packing and adjacency structure of the local code.
Syndrome	The logical QEC readout: which stabilizer parity/check constraints are violated. This is the clean error-correction layer and the pointer basis for the Darwinist record.
Strain	The geometric readout: frustration in the embedding, edge or cut content, cell deformation, boundary mismatch, or domain-wall content. This is where geometry becomes physical bookkeeping.
Defect	A persistent code/geometry failure or protected excitation. A particle-like object is not a bead placed in space; it is a stable defect, syndrome, strain pattern, or protected excitation of the ordered code.
Ledger	The rule that says which readout a physical claim uses: code content, syndrome, strain, service current, boundary printing, or wall shadow. Ledgers prevent category mistakes.

This bridge matters for the Zurek connection. If one says only that the framework has a striking geometry, the contact with quantum Darwinism is weak. If one says that the geometry is an ordered QEC phase whose syndrome readout is redundantly broadcast, the contact becomes precise. The environment witnesses the code's syndrome. The public classical fact is the redundantly available record of that syndrome. Geometric strain then supplies additional ledgers for gravitational, boundary, and defect-sector claims, but it should not be confused with the logical syndrome unless a specific observable map says so.

9 Relation to Existing Quantum Darwinism

The finite-QEC picture aligns with three standard ideas in the Darwinist literature:

1. the environment acts as a witness [11];
2. the pointer basis is selected by environmental monitoring [14, 15];
3. objectivity is quantified by redundancy and partial-information plateaus [2, 9].

Its distinctive contribution is to supply a concrete QEC interpretation of the monitoring channel.

The comparison can be summarised as follows.

Darwinist ingredient	Finite-QEC reading
System pointer observable	The stabilizer/syndrome observable monitored by QEC error detection.
Environment fragments	Physical record registers or outgoing environmental degrees of freedom carrying copies of syndrome values.
Redundancy plateau	The exact ideal fact $I(S : F) = H(S)$, replaced under finite noise by the Fano lower bound in Theorem 1.
Non-objective conjugate information	Phase information between syndrome branches remains globally encoded and is not available to proper fragments.
Classical objectivity	Many observers can infer the same syndrome value from disjoint fragments.
Thermodynamic arrow	Finite record registers must be reset for reuse; reset exports Landauer entropy.

This is why the result is best described as a reconstruction rather than a replacement. The entropy identities are familiar. The additional content is the claim that, in a finite record-writing QEC substrate, the ingredients are not separately stipulated. The same syndrome-extraction service supplies pointer selection, legal record copy, redundancy, and reset bookkeeping.

10 Scope, Limitations, and Paper-Worthiness

The theorem closes a narrow but useful foundations claim. It proves that noisy stabilizer syndrome extraction has the structure required for approximate spectrum broadcast and strong Darwinism when three assumptions hold: the instrument is QND in the syndrome basis, the fragments factorise conditional on the syndrome, and each fragment has a bounded decoding error.

The remaining limitations are also clear.

First, the theorem is a channel theorem, not a microscopic bath derivation. It does not prove that every proposed substrate Hamiltonian produces the conditional-product instrument (1). That is a later dynamical calculation.

Second, the result does not derive the Born weights. It assumes probabilities p_s for the syndrome sectors and then proves objectivity and redundancy for the corresponding records. The finite-QEC programme has separate arguments for projectors, non-contextual compatible tests, and Born structure; those are not reproved here.

Third, the geometry-to-observable map remains ledger-specific. Syndrome broadcast explains objective syndrome records. It does not license every later geometric or cosmological observable. A gravitational claim may read strain; a boundary claim may read boundary printing; a defect-sector claim may read wall shadow. The paper's methodological message is precisely that these ledgers must be kept separate.

Subject to those limits, the note is paper-worthy because it turns a conceptual bridge into a theorem with checkable hypotheses:

> Quantum Darwinism is realised in a finite-QEC substrate as syndrome broadcast, with approximate objectivity controlled by local decoding error, and the older geometry-first picture is recovered as the ordered spatial phase and strain ledger of the same record-writing code.

11 Reproducibility Map

The following repository scripts are the finite checks behind the claims used in this paper. They are not substitutes for the analytic argument, but they make the finite model auditable.

All paths below are under `python_code/`.

- `syndrome_broadcast_theorem_audit.py`
  Checks the noisy theorem ledger: local Fano lower bounds, the number of Darwinist witness fragments at a fixed tolerance, exact and union-bound trace-distance estimates for a classical wrong-label model, conditional independence $I(F_A : F_B|S) = 0$, and local versus globally compressed reset costs.
- `substrate_pointer_basis.py`
  Checks that syndrome recording selects the stabilizer/syndrome basis as the pointer basis in the finite model.
- `substrate_quantum_darwinism.py`
  Computes the Darwinist plateau, redundancy $R = N$, pointer-only objectivity, and a non-broadcast control.
- `measurement_service_h_theorem.py`
  Separates dephasing, reversible record copy, and Landauer reset/export in a finite measurement ledger.
- `r5_record_class_bridge.py`
  Records the bridge from stable-record axioms to the binary balanced Type-II CSS record-cell class, with stated residuals.
- `minimal_balanced_record_cell_theorem.py`
  Verifies the uniqueness/minimality of the balanced $[8, 4, 4]$-type record cell within the stated class.

12 Conclusion

The recent finite-QEC derivations move the framework closer to Zurek's work in a specific sense. Quantum Darwinism is no longer merely an analogy attached to the framework after the fact. Under an explicit noisy stabilizer syndrome instrument, it is a theorem: bounded decoding error gives bounded loss from the Darwinist plateau, conditional product form gives the strong-Darwinism independence structure, and orthogonal syndrome records give approximate spectrum broadcast. Stable records define orthogonal sectors, error detection monitors syndrome values, syndrome values are copied into many environmental fragments, and finite reuse of those records produces a Landauer reset ledger.

The geometry remains essential, but it has a clearer role. It is the ordered phase and embedding of the record code, not the whole explanation. Once that distinction is made, the older geometric understanding and the newer record/QEC/ledger understanding reinforce one another. Geometry tells us how the local record cells pack, strain, and fail. QEC tells us what is readable, copyable, protected, and resettable. Quantum Darwinism tells us when such records become public classical facts.

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