Pointer States Are Not Enough:

Physical Monitor Selection in a Finite Quantum Register

Technical report

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Abstract

A candidate pointer basis, an algebra generated by available microscopic operations, and the pointer structure of a physically selected open-system channel are different objects. We exhibit that distinction in an exact eight-qubit register. The qubits occupy the eight triangular faces of a bond-centred oblate square bipyramid; their adjacency graph is the cube graph Q_3. Twelve commuting edge-parity observables have rank seven and therefore leave 128 two-dimensional complement sectors. On the 48 valid register labels these become eight rank-two sectors and 32 rank-one sectors. The result for service operations depends on which records are physically exported. Under full polarity-resolved J/K support, eight of the 24 pairs differing only in I_3 are distinguished directly, eight by composition, and eight survive; 32 of 48 labels are individuated. Under the banked elementary positive-branch event, J alone individuates only 22 of 48, with residual multiplicity in all three generations. Nevertheless the generation-00 noiseless factor is support-robust: on the same fixed 48-state closure, both algebras satisfy

$$\mathcal{A}_{JK}|_{O_{00}^{JK}} \cong \mathcal{B}_{JK} \otimes \mathbb{I}_{I_3}, \qquad \mathcal{A}_{J}|_{O_{00}^{JK}} \cong \mathcal{B}_{J} \otimes \mathbb{I}_{I_3},$$

with support-dependent quotient algebras. A further exact no-go sharpens the physical boundary. Coherent evolution, an address-only L record and polarity-resolved J/K records can share the same 224 exit edges while preserving different coherences and generating different resolving algebras. Only polarity-resolved export gives the conditional finite hierarchy $01 < 10 < 00$; executing the missing edges is not enough. The remaining task is therefore not simply to choose rates, but to derive a polarity-complete schedule, retain its fired-arm flag, export that flag to a fresh orthogonal environment record and license branchwise recovery. Three interference/rate experiments distinguish these clauses exactly.

Keywords: pointer states; decoherence; quantum instruments; fixed-point algebras; noiseless subsystems; finite quantum registers; measurement selection

1 Introduction

Decoherence does not select a basis in the abstract. It selects stable sectors through a particular interaction between a system and its environment. In the standard account, the pointer observable is picked by the monitoring action of that environment, while redundant environmental records may make its values objectively accessible [1, 2]. The open-system channel is therefore part of the statement. A basis which is algebraically natural, or even a complete commuting family which could be monitored, is not yet the channel which is physically realised.

That distinction becomes easy to lose in a finite substrate model. Such a model may contain a validity predicate, error syndromes, leakage transitions, recovery operations and a proposed record alphabet. Each item is relevant to measurement, but none by itself proves that the environment measures a specific observable. Quantum instrument theory separates outcome effects from conditional state changes [3–5]; fixed-point and information-preserving-structure results similarly separate the algebra carried by a channel from a preferred interpretation of it [6–8]. Noiseless subsystems make the same point constructively: an interaction algebra may act nontrivially on one tensor factor while leaving another factor unread [9–11].

This report studies one finite register in that language. It was prompted by a claim that an ideal self-dual CSS syndrome basis supplied the physical pointer basis of the register. The CSS algebra is mathematically well-defined, but the claimed physical identification omitted a selection step. We therefore ask a narrower question:

> What information is distinguished by each service support already present in the model, and what record must the environment export for that distinction to become physical?

The answer is neither "all 48 labels" nor "the service is blind to I_3." On the full polarity-resolved support it is a three-way exact result. Some I_3 pairs are resolved in a single service step, some only by composed service words, and the generation-00 pairs survive every such composition. Under the elementary positive-branch support the resolving power is weaker, although the generation-bottom factorisation survives. Moreover, the union of basis transitions does not identify the instrument: an unresolved L record and separate J/K records have the same exit edges but different action on coherence.

The contribution is consequently threefold. First, the finite calculation gives the support-dependent decompositions of the generated service algebra on the physical labels. Second, it proves a generation-bottom noiseless-isospin factorisation robust under both named supports. Third, it provides a worked counterexample to the inference from transition support to environment record: executing all allowed edges, exporting an address and exporting the fired polarity are different physical statements. The report does not derive the missing Lindbladian, claim that coherence is present in the residual factor, or promote either the ideal CSS monitor or the polarity-resolved service instrument to a physical law.

2 Three levels of a pointer claim

Let $\mathcal{H}$ be finite-dimensional. A pointer claim can refer to at least three different levels.

Definition 1 (Candidate pointer decomposition)**.** A family of orthogonal projections $\{P_a\}$ with $\sum_a P_a = \mathbb{I}$ defines a possible non-selective dephasing channel

$$\mathcal{D}_P(\rho) = \sum_a P_a \rho P_a. \tag{1}$$

Its fixed operator algebra is block diagonal. The blocks, rather than arbitrarily chosen vectors inside a degenerate block, are the sectors selected by this channel.

Definition 2 (Generated operation algebra)**.** Given physically named partial transitions, recovery maps and diagonal effects $\mathcal{G}$, let

$$\mathcal{A}(\mathcal{G}) = \mathrm{alg}^*(\mathcal{G}) \tag{2}$$

be the unital $*$-algebra they generate. Its centre and multiplicity spaces describe information distinguished or left unread by the available operations.

Definition 3 (Selected physical monitor)**.** A physical monitor is a specified instrument or open-system generator, including its rates and environment coupling. Its pointer or noiseless structure is determined from that channel, not merely from the list of operators that could appear in it [12, 13].

These levels need not coincide. Equation (1) can be written for any chosen projective decomposition. A generated algebra can contain coherent routers which make a label readable after preprocessing without themselves measuring it. Conversely, a selected physical generator can use only a subset of an available algebra. Fixed-point theory is therefore decisive only after the channel has been specified. This report computes the second level exactly and uses the first level for controlled comparisons; the third level remains open.

candidate CSS or strain monitor $\not\Rightarrow$ generated service algebra $\not\Rightarrow$ physically selected Lindbladian.

3 The finite register

3.1 Geometry and labels

The physical cell is a bond-centred oblate square bipyramid. It has six vertices, twelve edges and eight triangular faces. One qubit is assigned to each triangular face. The face-adjacency graph is Q_3, with eight vertices and twelve edges. The six square cycles of Q_3 are dual belt plaquettes; they are not physical faces of the bipyramid. This distinction is important because the register has eight, not six, physical qubits.

Write a computational word as

$$x = (G_0, G_1, L_Q, C_0, C_1, I_3, \chi, W) \in \mathbb{F}_2^8. \tag{3}$$

The physical-label predicate is

$$\mathsf{P}(x) = 1 \quad \Longleftrightarrow \quad \neg(G_0G_1) \ \wedge \ (W = \chi) \ \wedge \ \big[(L_Q = 0) \Longleftrightarrow (C_0 = C_1 = 0)\big]. \tag{4}$$

Exhaustive enumeration gives $|\mathsf{P}| = 48$. In this report G_0, G_1 are generation bits. The term "generation-00" must not be replaced by "gauge-neutral." The all-zero word is retained as a blank register word; its possible physical interpretation is not needed below.

The validity-preserving translation group is exactly

$$\{\mathbb{I},\ X_\chi X_W,\ X_{I_3},\ X_{I_3}X_\chi X_W\}. \tag{5}$$

It partitions the 48 labels into twelve four-element orbits and gives the finite factorisation

$$48 = (3 \text{ generation sectors})(4 \text{ lepton/colour skeletons})(2\ I_3)(2 \text{ locked } \chi/W). \tag{6}$$

3.2 Edge strain

Let $E(Q_3)$ be the twelve edges of the dual adjacency graph and define the binary coboundary

$$\delta : \mathbb{F}_2^8 \longrightarrow \mathbb{F}_2^{12}, \qquad (\delta x)_{uv} = x_u + x_v \pmod 2. \tag{7}$$

In operator language these are the twelve commuting edge parities Z_uZ_v.

Theorem 4 (Exact strain decomposition)**.** *The edge coboundary has*

$$\operatorname{rank} \delta = 7, \qquad \ker \delta = \{0, \mathbf{1}\}. \tag{8}$$

Hence there are 128 attainable strain words and every joint parity sector is the complement pair

$$\mathcal{H}_s = \operatorname{span}\{|x\rangle, |x + \mathbf{1}\rangle\}. \tag{9}$$

The fixed algebra of the ideal strain-dephasing channel is

$$\operatorname{Fix}(\mathcal{D}_{\text{strain}}) \cong \bigoplus_{s=1}^{128} M_2. \tag{10}$$

Restricted to the valid-label subspace, its occupied part is

$$\left(\bigoplus_{j=1}^{8} M_2\right) \oplus \left(\bigoplus_{j=1}^{32} \mathbb{C}\right), \tag{11}$$

which has dimension 64.

Proof. The graph Q_3 is connected, so the kernel of its incidence coboundary consists precisely of constant binary functions. Rank-nullity gives rank seven. Each fibre is therefore $\{x, x + \mathbf{1}\}$, proving Eq. (10). Exhaustive intersection of the 128 fibres with Eq. (4) gives the counts in Table 1 and hence Eq. (11). □

Table 1: Intersection of the 128 strain sectors with the 48 valid labels.

sector type	number of sectors	valid vectors per sector
empty	88	0
singleton	32	1
complement pair	8	2

For the uniform distribution on the 48 labels, strain readout leaves exactly $16/48 = 1/3$ bit of conditional uncertainty. This is a statement about the partition. It does not assert that a physical state contains coherence inside a pair. Equation (10) says only that the ideal strain monitor cannot remove coherence which is already present within one of its degenerate blocks.

4 Recorded service operations

Let X_p flip register bit p, and let Π_{P} and Π_{Q} project onto the valid and invalid computational subspaces. The polarity-blind exit map is

$$L_p = \Pi_{\mathsf{Q}} X_p \Pi_{\mathsf{P}}. \tag{12}$$

The documented addressed register operation assigns to target p the control

$$c(p) = (p - 3) \bmod 8. \tag{13}$$

Writing $N_c = |1\rangle\langle 1|_c$, the two polarity arms are

$$J_p = L_p N_{c(p)}, \qquad K_p = L_p(\mathbb{I} - N_{c(p)}), \qquad L_p = J_p + K_p. \tag{14}$$

The supports have the exact census

$$|J| = 104, \qquad |K| = 120, \qquad |L| = 224, \qquad L = J \,\dot{\cup}\, K. \tag{15}$$

All seven live addresses contain both polarities; I_3 is the sole silent address. These operators do not have the same physical status. The banked typed-operator decision identifies one elementary realised register-W event with the positive branch J_p. The polarity-complete L_p operation is exactly constructible, but its pre-latch scheduling is a separate microscopic clause rather than a consequence of that elementary event.

We therefore keep three generated algebras distinct:

$$\mathcal{A}_{JK} = \mathrm{alg}^*\{J_p, K_p, J_p^\dagger, K_p^\dagger,\ \text{diagonal seeds}\}, \tag{16}$$

$$\mathcal{A}_J = \mathrm{alg}^*\{J_p, J_p^\dagger,\ \text{diagonal seeds}\}, \tag{17}$$

$$\mathcal{A}_L = \mathrm{alg}^*\{L_p, L_p^\dagger,\ \text{diagonal seeds}\}. \tag{18}$$

The full polarity-resolved calculation has 28 nonempty partial maps after adjoints are included. It is a conditional service algebra, not the presently selected elementary event.

The first support census is informative but not final. The L_p exit fingerprint has twelve classes of size four, exactly the translation orbits in Eq. (5). The separate J/K one-step fingerprint has 32 classes: sixteen singletons and sixteen pairs. Every unresolved pair differs only in I_3. It would nevertheless be wrong to infer that the entire generated algebra is I_3-blind. Since $c(0) = 5$, I_3 controls the J_0/K_0 split, and composed words can reveal information absent from a single support signature.

4.1 Exact algebra construction

Because the operators in Eq. (14) are partial permutation matrices, the generated algebra can be analysed without floating-point linear algebra. Two finite structures suffice.

1. The *diagonal partition* starts from every generator domain and range, the P/Q split and the separate J/K exit multiplicities. It is refined under images and preimages until stable. Its atoms are the finest classical distinctions in the diagonal part of the algebra.

2. The *groupoid orbits* are the connected components under the partial maps and their inverses. Singleton diagonal atoms inside a connected orbit supply point projections; path words then supply all matrix units on that orbit.

This construction distinguishes a full matrix block from a block carrying a residual multiplicity. It also avoids an unsafe inference from commutation alone: an added I_3 flip can pass a weak commutation test while violating the required fibre descent.

5 The generated-algebra theorem

There are 24 valid partner pairs $(x, X_{I_3}x)$. The exact full-polarity algebra $\mathcal{A}_{JK}$ organises them as shown in Table 2.

Table 2: Fate of the 24 valid I_3-partner pairs under the conditional full-polarity algebra $\mathcal{A}_{JK}$.

generation sector	pairs	algebraic fate
$G_0G_1 = 01$	8	resolved directly by the J_0/K_0 control
$G_0G_1 = 10$	8	resolved by composed words through Q
$G_0G_1 = 00$	8	unresolved by every generated service word

Table 3: Support dependence of physical-label individuation. The residual column counts labels in non-singleton diagonal classes.

support	classes on P	individuated labels	residual labels by generation
full polarity $\mathcal{A}_{JK}$	40	32	$00:16$
elementary $\mathcal{A}_J$	32	22	$00:16,\ 01:4,\ 10:6$

Let O_{00}^{JK} be the full $\mathcal{A}_{JK}$ orbit closure of the sixteen valid generation-00 states. The closure contains 48 computational words and is invariant under $\Sigma = X_{I_3}$. The smaller J-only support has a 36-state closure of its own; to test whether the same physical multiplicity survives, both algebras must instead be compared on the fixed domain O_{00}^{JK}.

Theorem 5 (Support-robust generation-bottom noiseless-isospin theorem)**.** *Both named service algebras factor trivially over I_3 on the same fixed domain:*

$$\mathcal{A}_{JK}|_{O_{00}^{JK}} \cong \mathcal{B}_{JK} \otimes \mathbb{I}_{I_3}, \qquad \mathcal{A}_J|_{O_{00}^{JK}} \cong \mathcal{B}_J \otimes \mathbb{I}_{I_3}. \tag{19}$$

*The algebra on the other factor may be smaller under J alone. The full-polarity algebra individuates 32 physical register labels; the banked elementary J support individuates 22. Under both supports the generation-*00 *I_3 multiplicity is transported without being read. For $\mathcal{A}_{JK}$, the centre whose support meets P has six sectors: four full matrix blocks carrying the 32 resolved labels and two multiplicity-two blocks carrying the sixteen generation-*00 *labels.*

Proof. For each support, the finite certificate verifies three independent clauses on the same domain O_{00}^{JK}.

1. Every service edge meeting O_{00}^{JK} preserves I_3.
2. Every generating diagonal seed is Σ-invariant on O_{00}^{JK}.
3. Every generator has a well-defined descent to the 24-fibre quotient $O_{00}^{JK}/\langle\Sigma\rangle$: domain membership occurs for both members of a fibre or neither, and images are Σ-equivariant.

These clauses give the tensor identity in Eq. (19); the J-only algebra may be smaller on the quotient factor but remains the identity on the I_3 fibre. For $\mathcal{A}_{JK}$, the quotient diagonal atoms are singletons and the quotient generator graph is connected within each of the two merged sectors. Point projections and path words therefore generate the quotient matrix algebra. On each of the four remaining physical orbits the diagonal atoms are singletons and the generator graph is connected, so each carries a full matrix algebra. This gives four full blocks and two multiplicity-two blocks. Independent stable-partition enumeration gives the two individuation counts in Table 3. □

The theorem is algebraic. It does not claim that a coherent superposition of I_3 partners is prepared, nor that the service creates such coherence. It states that this operator algebra has no means to read the multiplicity. A physical decoherence-free-subsystem claim additionally requires the actual channel and its rates to be shown to lie in the stated algebraic action.

The all-zero word and its I_3 partner belong to a surviving pair. This is not by itself evidence for a new vacuum degree of freedom. It is a sharp identification seam: the service support treats the two words identically on the surviving factor, and a later physical model must decide whether that factor is protected, read by another interaction, or absent from the physical state preparation.

6 Transition support does not determine the record

The support dependence above still leaves a further distinction. Even if all K edges occur, the environment need not export which polarity fired. For target p, write the two complementary controlled operations as

$$C_p = (1 - D_p)I + D_p X_p, \qquad \bar{C}_p = D_p I + (1 - D_p)X_p. \tag{20}$$

Proposition 6 (Schedule selector and instrument underdetermination)**.** *Within the local schedule grammar in which each control polarity is invoked at most once, control-complement invariance together with exactly one target flip selects, uniquely up to order,*

$$\bar{C}_p C_p = C_p \bar{C}_p = X_p. \tag{21}$$

The control is unchanged and equals the polarity which fired in all $8 \times 256 = 2048$ *addressed basis cases. Nevertheless the resulting 224-edge exit support does not determine the environment instrument. It is compatible with:*

1. *no which-arm record;*
2. *an address-only record* $|p\rangle$*, giving one unresolved* L_p *arm; or*
3. *an orthogonal record* $|p, J\rangle$ *or* $|p, K\rangle$*, giving separate* J_p, K_p *arms.*

The first two preserve J/K *cross coherence on a conditioned exit, whereas the third removes it.*

Proof. Exact enumeration of the restricted schedules leaves only $(+, -)$ and $(-, +)$ as control-blind one-flip words. The control and target are different bits, so the target flip does not change D_p and exactly one polarity fires.

For the coherence certificate, take the support-derived witness at address G_0,

$$s_J = 01000100 \longmapsto 11000100, \tag{22}$$

$$s_K = 01000000 \longmapsto 11000000. \tag{23}$$

For the equal source superposition, the conditioned output density matrix has cross coefficient

$$\frac{1}{2}\langle r_K | r_J \rangle. \tag{24}$$

With no record, or with the same address record on both branches, the overlap is one and the coefficient remains 1/2. With orthogonal polarity records the overlap and coefficient are zero. The three constructions retain the same 224 basis edges. A planted model which merges $|p, J\rangle$ and $|p, K\rangle$ restores the coefficient to 1/2, so record orthogonality is load-bearing rather than a label attached after the calculation. □

The shortest distinguishing and recovery words make the consequence quantitative. For each I_3-partner pair define

$$(\text{generator depth}, \ \mathsf{Q}\text{-excursions}, \ \text{recovery depth}),$$

where the first entry counts arm queries through the first different fire/silent outcome, the second counts realised $\mathsf{P} \to \mathsf{Q}$ excursions, and the third is the shortest branch-conditioned return word.

Corollary 7 (Conditional generation hierarchy)**.** *Within the declared service-arm grammar,*

$$01 < 10 < 00 \tag{25}$$

is an exact correction-complexity ordering for the polarity-resolved $\mathcal{A}_{JK}$ *support. It is not an ordering of the banked elementary* $\mathcal{A}_J$ *event, and it does not follow from execution of the 224 L edges without polarity export.*

Table 4: Exact correction-complexity census by record support. Each entry applies to the eight I_3-partner pairs in that generation.

support	generation 01	generation 10	generation 00
polarity-resolved JK	$(1,1,1)\times 8$	$(2,1,2)\times 8$	unresolved $\times 8$
elementary J	$(1,1,1)\times 8$	$4\times(2,1,2)$; $1\times(4,2,4)$; 3 unresolved	unresolved $\times 8$
address-only L	unresolved $\times 8$	unresolved $\times 8$	unresolved $\times 8$

The proposition therefore sharpens rather than closes the physical monitor problem. Control-complement invariance would select the two-polarity macrocycle, and the unchanged control supplies an exact one-bit arm flag. Neither statement makes nature retain and export that flag. Copying it into orthogonal environment sectors is a physical interaction: it dephases superpositions across the polarity sectors and cannot be inferred from the basis transition table.

7 Candidate refinements are not selected monitors

The generation-00 residual common to both named supports can be separated mathematically in several ways. Table 5 records the exact finite census for its eight I_3 pairs.

Table 5: Separation of the eight surviving I_3 pairs by named extensions.

extension	pairs split	role
direct Z-face I_3 operator	8/8	readout baseline
global complement $\Gamma = X^{\otimes 8}$	8/8	coherent router
X mask 10101010	8/8	coherent router
X mask 11001100	8/8	coherent router
X mask 11110000	8/8	coherent router
documented coin	0/8	transport, not readout
documented hop	0/8	transport, not readout

The global complement is sufficient because it routes the hidden label through Q into the service-readable distinction between $\mathrm{ran}(J_0)$ and $\mathrm{ran}(K_0)$. This does not make Γ a measurement. Nor is it unique: each of the three proper weight-four Reed–Muller masks has the same pair-separating power. Algebraic sufficiency therefore does not select a physical mechanism.

An even more complete counterfactual is the self-dual CSS monitor constructed from the classical $RM(1,3) = [8,4,4]$ code. The code has sixteen words and weight enumerator

$$1 + 14x^4 + x^8. \tag{26}$$

The corresponding full X/Z stabilizer family has rank eight and 256 one-dimensional syndrome sectors. This is a valid application of the CSS construction [14, 15]. It is not the same instrument as edge-strain monitoring or the generated service algebra.

The fact that 48 is not a power of two rules out the 48 labels as one Pauli stabilizer codespace or one stabilizer syndrome sector. It does not rule out an arbitrary channel with 48 preferred states. Such a no-go would require classification of a named channel, not dimensional arithmetic.

8 What is established, and what remains open

Table 6 separates the exact results from the physical claims which have not been derived.

Table 6: Claim boundary of the present report.

statement	status
Q_3 edge coboundary has rank seven and complement kernel	exact theorem
strain sectors meet P as 88/32/8 empty/single/pair	exhaustive exact census
one-step L and J/K supports yield 12 and 32 classes	exhaustive exact census
$\lvert J\rvert = 104$, $\lvert K\rvert = 120$, $\lvert L\rvert = 224$ with $L = J \dot{\cup} K$	exhaustive exact census
full $\mathcal{A}_{JK}$ individuates 32 labels; elementary $\mathcal{A}_J$ individuates 22	support-dependent exact census
both $\mathcal{A}_{JK}$ and $\mathcal{A}_J$ factor as $\mathcal{B} \otimes \mathbb{I}_{I_3}$ on O_{00}^{JK}	exact finite theorem
the same 224 edges admit coherent, address-only and polarity-resolved instruments	exact finite no-go
$01 < 10 < 00$ for polarity-resolved $\mathcal{A}_{JK}$	conditional exact theorem
ideal full-CSS monitor has 256 rank-one sectors	conditional algebraic theorem
the environment physically implements that CSS monitor	open
a particular coherent router is selected by microscopic dynamics	open
the K arms are scheduled before a common latch	open
the fired J/K polarity is projectively exported	open
generic-rate Lindbladian and its fixed algebra	open
redundant environmental records for the service sectors	open

The remaining physical bridge is finite enough to state precisely. Promotion of Corollary 7 requires all four clauses:

1. *Schedule:* derive a microscopic service cycle which is control-complement invariant and therefore invokes both polarities once before latch.
2. *Retention:* derive persistence of address p and the unchanged fired-polarity bit to the record boundary.
3. *Export:* derive one fresh outward-causal environment factor which copies $(p, \text{polarity})$ into orthogonal sectors at every such boundary.
4. *Recovery access:* derive the branch/address-conditioned adjoint operations used by the finite recovery words.

Rates must then be derived, rather than assigned, and the resulting Lindblad or repeated-interaction channel must have its fixed algebra computed. The finite analysis supplies three independent operational discriminators:

1. a control-off K_p source tests positive-only against polarity-complete exit dynamics through its initial exit rate;
2. conditioned J/K output coherence distinguishes unresolved L_p from resolved J_p, K_p (1/2 versus 0 in the exact witness); and
3. invocation/inverse reinterference tests whether orthogonal record information escaped at the proposed boundary.

Until these steps are completed, "pointer basis" is too strong. The exact phrase supported here is "support-dependent generated service-algebra decomposition." The generation-00 factor is a support-robust noiseless multiplicity, while the physical schedule, record quotient and generic-rate channel are deliberately withheld.

9 Implications

9.1 A useful failure mode for finite models

Finite state spaces encourage a particular shortcut. One identifies a complete commuting family, observes that its eigenspaces have the desired dimension, and speaks as though the environment measures it. The present example shows why that implication fails even when every ingredient is exact. The ideal CSS algebra is more resolving than the service algebra, but that is precisely why a physical bridge is required. Greater mathematical resolving power is not evidence of dynamical selection.

9.2 Edges are not records

The new obstruction is stronger than the familiar warning that rates matter. Even a complete basis-transition census does not determine the instrument. The 224 exit edges are compatible with a coherent operation, a seven-valued address record and a fourteen-valued polarity record. These possibilities agree on event-basis populations but differ on off-diagonal operators, exactly the information which instrument theory requires in addition to outcome support [3, 4].

Thus observing the missing K transitions would establish dynamics beyond the elementary J event, but would not by itself establish the polarity-resolved monitor used by $\mathcal{A}_{JK}$. Conditioned coherence is not an optional refinement of the edge census; it is the discriminator between the candidate instruments.

9.3 Noiseless structure can carry a physical label

Noiseless subsystems are usually discussed as encodings protected from a specified noise algebra [8–10]. Here the same algebraic structure arises as a diagnostic: a particle-label bit is transported but unread by both named service supports in one generation sector. Whether nature uses that multiplicity as protected information is a separate question, but the support-robust factorisation tells a later model exactly where an additional coupling would have to act.

9.4 Correction increases rather than decreases testability

The original full-CSS claim appeared more complete, but it had no selected monitor behind it. Replacing it with Theorem 5 produces a smaller and more falsifiable statement. Any proposed physical monitor must now specify both its transition support and its exported record. It must then reproduce one of three outcomes: retain the multiplicity, read it directly, or route it into an existing service distinction. Each possibility can be tested by an exact fixed-algebra calculation once the channel is supplied; Proposition 6 supplies the coherence test which prevents an edge-equivalent instrument from passing as the same monitor.

10 Reproducibility

The claims in this report are generated from exhaustive integer and binary calculations; no floating-point tolerance is used. The evidence chain used here terminates at repository commit `9367c7cecf9f9b656a785f0e411009706d594492`; the earlier generated-algebra and support-census units are ancestors of that commit. The five executable gates are:

- `python_code/cell_pointer_particle_separation_gate.py`, 22/22 checks, report identifier `6de058364155c82831561ab422119774aa1a327c86d2c457517c797df677951a`;
- `python_code/pointer_algebra_fixed_structure_gate.py`, 24/24 checks, report identifier `39cb69ae1645b67aeef678db342c3db056b822a388feb327ae73f6feb2796ce6`;
- `python_code/pointer_service_correction_complexity_gate.py`, 25/25 checks, report identifier `a0ce8ed36f3e00e47a2b9ee5a0b89ac7c4cfd075c009be6225b778dac9ee145e`;
- `python_code/pointer_service_individuation_census_gate.py`, 20/20 checks, report identifier `ebc2b70f30b1f0fb341fe4e927edac7faa89b844193d13287a5a114d15d76278`;
- `python_code/cell_k_arm_latch_export_selection_gate.py`, 62/62 checks, report identifier `e23697df836c2035fb88cfe0bcdec40d532bf9fed38d904276966f7fbb01ea76`.

The associated JSON reports contain every named check, the exact counts, the three factorisation clauses and the claim boundary. The controls include dropping the J/K split, changing the documented control convention, removing adjoints, and merging the two polarity records while leaving every edge unchanged. In the last plant, the computed record overlap changes from zero to one and the conditioned cross term from zero to 1/2. These controls distinguish the asserted theorems from nearby, weaker calculations.

11 Conclusion

Pointer states are properties of a selected physical interaction, not labels which can be read directly from a convenient basis. In the finite register studied here, the distinction is quantitative. Edge strain alone leaves complement pairs. Full polarity-resolved service signatures recover more information, and composed words recover more again, individuating 32 of 48 labels. The banked elementary positive-branch support individuates only 22. Yet both exact generated algebras act trivially on an I_3 multiplicity in the generation-00 sector. The factorisation is robust; the count is support-dependent.

Several operators can separate the residual pairs, and an ideal CSS monitor can refine the full register to rank-one sectors. None is thereby selected by nature. Nor is observing all 224 service edges sufficient. Coherent, address-only and polarity-resolved instruments share that basis support while acting differently on coherence; only the last supplies the conditional $01 < 10 < 00$ hierarchy. Transition support is therefore not a surrogate for an exported record.

The next result must come from dynamics: a microscopic law selecting the polarity-complete schedule, retaining its fired-arm flag, exporting that flag to a fresh orthogonal environment factor and licensing branchwise recovery. Rates and the generic channel must then be derived and its fixed algebra computed. The present report supplies the exact comparison set and three operational discriminators, while removing two distinct shortcuts: algebraic availability is not physical monitoring, and executed edges are not exported records.

Data and code availability

No observational data were analysed. The archival reproducibility package accompanying this report contains the five executable gates, their machine-readable reports, a checksum manifest and the paper sources; it is available at doi:10.5281/zenodo.22003393. The version-controlled evidence chain remains available in the repository and commit identified in Section 10.

Declaration of scope

This is a technical report of an exact finite model. It does not claim an experimental realisation, a complete decoherence model, a derived Born rule, or a physically selected 48-outcome particle measurement.

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