It from Bit, Rung by Rung

A graded reconstruction of quantum structure from record-keeping,
and how it lands on a lattice

David Elliman
Neuro-Symbolic Ltd
`dave@neusym.ai`

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Abstract

Quantum mechanics is usually presented as a list of postulates accepted because they work. The reconstruction programme of the last two decades—Hardy; Chiribella, D'Ariano and Perinotti; Masanes and Müller; Zurek—turns this around, deriving the formalism from operational principles. This essay organises that tradition around a single, physical premise: the keeping of *records*. We argue, rung by rung, that a world required to hold stable, re-readable, copyable facts in the presence of noise is pushed step by step into the machinery of *complex, error-correcting* quantum mechanics: orthogonal alternatives, projective tests, reversible (isometric) recording, a complex Hilbert space, a quantum error-correcting code, the Born rule, a thermodynamic arrow, and an objective classical limit. Each step is labelled by how firmly it stands—textbook theorem, conditional reconstruction, model-specific construction, or open conjecture—because the honesty of that grading is the point. The reconstruction needs *no* fitted physical constants; its claim is entirely structural. We then show how the same ladder, read constructively, generates a concrete discrete geometry—a minimal record cell, a unitary walk, and a service ledger—in one realisation, the TCH lattice. The numerical, constant-fitting commitments of that realisation are deliberately set aside; what remains is Wheeler's "it from bit" made sequential, with the bit revealed as a *protected quantum* record.

Contents

1 Wheeler's wager, taken literally

In 1990 John Wheeler compressed a lifetime of worrying about the quantum into three words: *it from bit* [1]. Every "it"—every particle, every field, every spacetime interval—derives its existence, he proposed, from answers to yes–no questions, from binary choices, from *information.* It was a slogan, not a theorem, and for thirty years it has largely functioned as one. The modern "it from qubit" programme in holography [53, 54, 55] has since made one half of the idea concrete—spacetime as an error-correcting code—but the *logical* half, why the world should be a quantum information processor at all, has remained a gesture.

This essay cashes out a small, specific part of it. Not the whole of physics from information—that is a programme, not an article—but one clean sub-claim, argued step by step:

> *If the world must keep stable, re-readable, copyable records in the presence of noise, then the machinery it is forced to use is, step by step, the machinery of complex quantum mechanics with error correction.*

The interesting word is *must.* The ordinary attitude treats quantum theory as a brilliant, weird set of postulates—complex amplitudes, the Born rule, collapse—accepted because they work. The reconstruction attitude, pursued seriously since the early 2000s, inverts this: from what *operational* requirements does the machinery follow, so that it stops looking arbitrary? Hardy derived quantum theory from five "reasonable axioms" [2]; Chiribella, D'Ariano and Perinotti gave an informational derivation from six principles, with *purification* as the characteristically quantum one [3]; Masanes and Müller axiomatised it from requirements on state tomography [4]; Dakić and Brukner, and Wilce, gave further variants [5, 6]. In parallel, Zurek showed that the *classical* world—definite outcomes, objective facts—is precisely what survives when a quantum system is monitored and copied by its environment [8, 10], and Fuchs urged that quantum states are best read as information [7].

What follows organises that tradition around one very physical premise—*record-keeping*—and lays the consequences out as a ladder. Each rung is a claim of the form "*if you have got this far, you are now forced into that,*" and each carries a label saying how solid it is. A ladder whose rungs are all asserted with equal confidence is worthless; a ladder that visibly separates the textbook theorems from the conditional reconstructions and the open conjectures is something a careful reader can actually walk.

A word on what the essay omits. The framework these ideas grew out of also makes *quantitative* claims—it tries to compute specific physical constants. Those are far more contentious, and they are not needed here. Strip them away and a self-contained conceptual spine remains: the reconstruction of quantum *structure* from record-keeping, and a recipe for the *geometry* that carries it. That spine is the subject. Where the ladder reaches for numbers (Section 6), we say so and stop.

2 The one premise you must grant

Every empirical science begins by assuming something it cannot prove from inside itself: that **some facts persist long enough to be checked more than once, and that different checks can be compared**. Call this the *stable-record floor.*

It sounds too weak to be worth stating. But notice what depends on it. A repeatable experiment presupposes the apparatus reads the *same* value if asked again. A measured history presupposes records of earlier events survive to be correlated with later ones. Thermodynamics presupposes you can tell a fresh memory from a used one. An *observer*—anything that learns—is by definition a thing that writes facts and reads them back. Remove the assumption that records can be stable and re-read, and there is no experiment, no history, no thermodynamics, and no observer left to notice.

The floor has a peculiar status: **it cannot be derived from within a theory without circularity**, because any derivation would itself be a sequence of stable, comparable facts. So we grant it, and make it as minimal and explicit as we can:

> *There exist persistent local facts ("records") that can be read repeatably, copied to more than one place, and tested by mutually compatible questions; and the substrate that carries them is finite and noisy.*

Everything below is the claim that **this floor, and little else, forces the rest**. It is rung **R0**; the theorems start at R1. The grades attached to every rung are:

[Floor]
an axiom you must grant; not derivable from inside without circularity.

[Theorem]
a standard result of quantum information or probability theory; it (or a close relative) is in the literature.

[Reconstruction]
proved, but *conditional* on the operational floor; typically it assembles a cited reconstruction theorem under a stated premise.

[Model result]
established by explicit construction inside the specific framework; weaker than a general theorem, stronger than a guess.

[Open]
genuinely unfinished. A candidate mechanism exists; the derivation does not.

3 Act I — From facts to Hilbert space

The first three rungs a quantum-information theorist will find comfortable. That is the point: the surprising claim is that they are *forced*, and forcing them needs only standard results.

R1 — Repeatable records require distinguishable alternatives. [THEOREM]

A record you can read twice and get the same answer is a record whose possible values are perfectly distinguishable. Two states that are *not* perfectly distinguishable cannot be told apart by a single test without error and—the sharp part—cannot be copied without disturbance: the no-cloning theorem [23, 24] and, more strongly, the no-broadcasting theorem [25] forbid it for non-orthogonal sets. So the moment you demand re-readability *and* copyability you have demanded a set of mutually exclusive alternatives—an orthogonal set.

R2 — Repeatable record-tests are projectors. [THEOREM]

A yes/no test you can repeat without changing its own answer is, in the Hilbert-space description, *idempotent*: asking twice equals asking once. Idempotent self-adjoint operators are exactly projectors. So a repeatable record-question is a projector Π ($\Pi^2 = \Pi = \Pi^\dagger$) and a record value is the subspace it projects onto. Projective measurement is *derived* from "the question gives the same answer if asked again," not assumed—this is the operational content of Lüders' rule [19].

R3 — Writing a record is reversible copying, not collapse. [Theorem]

Here the usual measurement story quietly goes wrong, and the record picture sets it right. Writing down which alternative occurred—copying the system's value into a fresh register—need not be irreversible. It is an *isometry*

$$V \;=\; \sum_s \Pi_s \otimes |s\rangle_R\,, \qquad V^\dagger V = I, \tag{1}$$

embedding system-plus-blank-register faithfully into a larger space and correlating each alternative s with an orthogonal record state $|s\rangle_R$. This is the content of the Stinespring and Naimark dilation theorems [20, 21]: every measurement (every completely positive map [22]) is a reversible interaction with an ancilla followed by simply *having* the record. Recording is not collapse; the apparent collapse comes later, with reset (R10).

By the end of Act I we have, *forced from the record floor alone*, the spine of quantum kinematics: orthogonal alternatives, projector-valued questions, and isometric recording into registers. No complex numbers yet, no Born rule yet—just the skeleton.

4 Act II — Why complex, and why error-correcting

These two rungs deserve the hardest scrutiny, because they are where "record-keeping" becomes recognisably *quantum* rather than merely classical-with-bookkeeping. Both are graded [Reconstruction]: each is a proof leaning on a premise about the floor and on a cited theorem.

R4 — Local observers comparing notes force a *complex* Hilbert space. [Reconstruction]

Why complex amplitudes, and not real or quaternionic ones? The decisive property is *local tomography*: the state of a composite system is fully determined by the statistics of measurements performed *separately* on its parts. This is exactly what record-keeping observers need—each interrogates their own piece of the world and reconstructs the joint state by comparing notes.

It is a sharp discriminator. Count the real parameters K_d needed to specify a d-outcome system. A theory is locally tomographic iff $K_{AB} = K_A K_B$. Ordinary *complex* quantum theory satisfies this ($K = d^2$). *Real* quantum theory fails it: $K = d(d+1)/2$, so two parties carry $K_A K_B$ locally accessible parameters but the joint system has *more*, an excess equal to the "rebit" bound that no local measurement can reach [26, 27, 28]. *Quaternionic* theory fails even to define a consistent tensor product ($\mathbb{H} \otimes \mathbb{H} \cong M_4(\mathbb{R})$) [29]. Among the candidates, complex quantum mechanics is essentially the one that lets local observers reconstruct the whole from the parts. Add the *continuous reversibility* already in hand from R3—records are written by smooth, invertible dynamics, excluding the purely classical (simplex) theory—and the reconstruction theorems of Hardy [2] and of Chiribella–D'Ariano–Perinotti [3] close the case: the number field is forced to be $\mathbb{C}$. "Why complex?" becomes a consequence of *local* observers comparing notes.

The honest residual: this leans on the premise that the world's measurements are *locally generated*—no irreducibly collective primitive measurement. For a substrate built from local cells this is natural, but it is load-bearing; a genuinely non-local measurement primitive would reopen real quantum theory [27]. Hence [Reconstruction], not [Theorem].

R5 — Maintaining records in noise forces error-correcting structure. [Reconstruction]

The floor said the substrate is finite and *noisy*. A record that must survive noise indefinitely cannot just sit there; it must be *actively protected*, and the mathematics of protecting quantum information is quantum error correction [30, 31, 33]. We now go deeper than the essay version, because this rung carries the most weight.

Stabilizers. A stabilizer code [34] is the simultaneous +1 eigenspace of an abelian group $\mathcal{S} \subset \mathcal{P}_n$ of commuting Pauli operators. Information lives not in individual cells but in the joint constraints among them. The defining move is that one measures the *generators* of $\mathcal{S}$—the *syndrome*—which detects errors without ever reading the protected content: errors anticommute with some generator and flip its eigenvalue, while the logical information, living in the centraliser $C(\mathcal{S})$, is untouched. This is precisely the "non-disturbing read" the record floor demands: a question you can ask repeatedly without spoiling what is stored.

Why this class is not a free choice. Run the record requirements together:

1. *Stabilizer* $\Leftarrow$ repeatable non-disturbing reads must *commute* (R1/R2);
2. *CSS* [32, 31] $\Leftarrow$ there are two error classes to catch—corruption of *what* is written (X-type) and of *whether* it was written (Z-type)—so X- and Z-checks separate, built from a classical code C with $C^{\perp} \subseteq C$;
3. *self-dual* ($C = C^{\perp}$) $\Leftarrow$ reading is balanced against writing;
4. *distance* fixed by the finite noise budget.

The crux is one further condition. A stable record (R0) acted on by the substrate's local gates (R3) *in a complex Hilbert space* (R4) must survive the local complex phase—the transversal S gate—and a CSS code is preserved by transversal S *iff* its classical code is *doubly-even* (all codeword weights $\equiv 0 \bmod 4$) [37, 39]. Doubly-even self-dual binary codes exist iff $n \equiv 0 \bmod 8$ (a theorem of Gleason on self-dual codes [40, 41]), and the *smallest* is the $[8,4,4]$ extended Hamming code, equivalently the first-order Reed–Muller code $\mathrm{RM}(1,3)$, equivalently the 3-cube. So the minimal stable record cell is forced to be a byte: the unique $[8,4,4]$ doubly-even self-dual code.

The Clifford/non-Clifford seam. This is worth dwelling on, because it places the record cell at a precise location in the landscape of quantum computation. The Eastin–Knill theorem [42] forbids any code from having a transversal *universal* gate set; Bravyi–König [43] sharpen this by code dimension. A doubly-even self-dual CSS code admits transversal Clifford generators including S, but *not* a transversal T: it sits exactly at the *Clifford* ceiling. Reaching universality—the non-Clifford T—requires extra structure (magic states [38], or a triply-even code [44]). The minimal record cell is therefore the doubly-even, transversal-S code: the boundary of the efficiently simulable Clifford world [35, 36]. The non-classical, computationally powerful content of the substrate is precisely what lives *beyond* this cell's transversal gates.

What the cell is as a quantum code. Read as a CSS code, the self-dual $[8,4,4]$ gives $[[8,0,4]]$: a pure stabilizer *state*, zero *logical* qubits, distance 4—an error-*detector.* Its eight cells are physical qubits throughout; "zero logical qubits" means the cell encodes no protected message of its own but acts as a defect detector, with excitations ("particles") appearing as syndrome defects. Its error-*protecting* sibling, the smallest code that detects any single error, is $[[4,2,2]]$ [45, 34]. The upshot: **the substrate is not merely a Hilbert space, it is a quantum error-correcting code**, because that is the only way finite hardware keeps a fact for a long time.

This is the conceptual heart. "It from bit" is usually read with *classical* bits. R4 and R5 say the bit must be a *protected quantum* bit—a syndrome in a doubly-even self-dual code—as soon as records are required to compose and to survive noise.

5 Act III — Probability, the Born rule, and the classical world

With a complex error-correcting record substrate in hand, the rest of quantum mechanics—including the parts usually treated as separate mysteries—follows.

R6 — The code gives commuting, non-disturbing record questions. [Model result]
A self-dual stabilizer code has an abelian stabilizer group, so its record-questions commute and are jointly answerable without one spoiling another—the formal property that becomes measurement *non-contextuality* downstream.

R7 — The stable basis is the syndrome basis. [Model result]

Which states are the robust, classical-looking ones—the *pointer states* that do not smear under monitoring? Zurek's einselection answers this in general [9, 8]: the survivors are the states the environment keeps measuring, the minima of the *predictability sieve*. In the record substrate the environment *is* the error-correction process, perpetually reading the syndrome. So the pointer basis *is* the syndrome basis: the definite-record states are exactly the ones the code is built to preserve. The "preferred basis problem" dissolves—the preferred basis is whatever the code protects.

R8 — Non-contextual record probabilities force the Born rule. [Theorem (on a model-derived premise)]

Once record-questions are projectors (R2) whose probabilities are independent of the compatible context in which they are asked (R6/R7), Gleason's theorem [15] clamps down: in dimension ≥ 3 the *only* consistent assignment is

$$p(\Pi) = \mathrm{Tr}(\rho\,\Pi) \tag{2}$$

for some density operator ρ. The probabilities are not an extra postulate; they are the unique non-contextual measure on the record-questions. Two technical strengthenings matter for the substrate. First, the Busch–Caves–Fuchs–Renes POVM form of Gleason [16, 17] removes the dimension restriction and weakens the assumptions. Second, because a code restricts one to *stabilizer* measurements rather than the full projector lattice, one invokes the stabilizer/Spekkens form of measurement non-contextuality [18] on that restricted set; the abelian structure already supplies what Gleason needs. (Zurek's envariance [11, 12] gives an independent route to (2) that is congenial to the record picture.)

R9 — The Born square is the closed record-action pair. [Reconstruction]

Why the modulus *squared*? In the record picture a probability is the weight of a complete forward-and-back history: amplitude to write a record, times amplitude to have written it. Orthogonal record sectors annihilate every mismatched forward/backward pair, and the survivors are exactly the diagonal terms $AA^* = |A|^2$. The square is the signature of a *closed* record loop—write it, and read that you wrote it—rather than a bare amplitude. This recovers, in record language, the decoherence-functional diagonal of consistent/decoherent histories [46, 47].

R10 — Irreversibility and the arrow of time are record reset. [Model result]

Now we can locate the collapse R3 refused. Reversible recording fills registers with correlated copies; to keep recording—to make a *reusable* memory—you must eventually *reset* registers to blank, and reset is where irreversibility enters. Landauer's principle is exact and experimentally confirmed [48, 49, 50]: erasing one bit costs at least $k_B T \ln 2$, dumped as heat. Non-selective syndrome monitoring dephases; reusable reset exports at least $H(p)$ nats. **The thermodynamic arrow is the cost of clearing memory to make room for new records.** Measurement looks irreversible not because the writing was, but because the re-blanking is.

R11 — Objective facts are redundantly broadcast records. [Model result]

Why does the world look *objective*—why do many observers agree on the same classical facts? Quantum Darwinism [13, 10, 14] gives the mechanism: a fact becomes objective when it is *redundantly* imprinted on many fragments of the environment, so that many observers each read the same value independently without disturbing it. In the record substrate this is syndrome information broadcast into many cells. An objective classical fact is a record copied so widely that everyone reads the same answer. Reality's solidity is redundancy.

By the end of Act III the standard mysteries are repositioned: the Born rule is forced (R8), the preferred basis is the protected code basis (R7), measurement "collapse" is reset thermodynamics (R10), and the objective classical world is redundant record broadcast (R11)—with no new postulate beyond the record floor and Acts I–II.

6 Act IV — Where the ladder becomes a specific physical model

Everything so far is *generic*: it holds for any world that keeps records under noise. The last four rungs differ in kind. Here the ladder stops being a reconstruction of quantum theory in general and commits to *one* substrate with *one* ledger—and, in the parent framework, to specific numbers. We include the conceptual shape, flag the rungs as increasingly open, and keep the numbers out.

R12 — The world commits one discrete record per event. [Open, with a derived core]

If every event leaves a record, that record is drawn from a fixed *alphabet*—an "address" (where) crossed with a "channel" (what). The structural claim, and a clean one, is that the address part is forced *uniform*: the code's translation symmetry makes every location equally likely (a convolution channel on the address group has a unique uniform stationary measure), so no place is privileged. The *size* of the alphabet—the number of distinguishable records per event—is where the framework reaches for a specific integer, which we set aside.

R13 — History carries a least-action measure over records. [Open skeleton]

Once events are records, a *history* is a sequence of records, and one can ask how histories are weighted. The proposal is a "record action," additive along a history, whose stationary value selects the likely course of events, carrying the Crooks/fluctuation structure of R10. The skeleton is suggestive; a complete dynamical principle is unfinished.

R14 — A single universal rate governs record upkeep. [Model result for the bare rate]

Maintaining records costs: each act of error correction debits the noise budget. The structural claim is that there is *one* universal rate, the same everywhere, taking the form of a Born weight of a single firing in a uniformly monitored ledger—equipartition over the record alphabet, *derived* (not assumed) because a monitored, connected channel relaxes to the uniform state (an Evans–Frigerio fixed-point argument for unital channels [51, 52]). In the current canon the monitored record-pair alphabet gives the bare service weight $\alpha_0 = 1/137$. The remaining caution is not the bare count itself but the downstream billing map: each sector still has to show that it bills this monitored service observable, and the dressed low-energy QED value $\alpha^{-1} = 137.036\ldots$ is a separate electromagnetic renormalisation.

R15 — The small asymmetries of time and matter trace to the geometry of repair. [Open, with a candidate]

The faint asymmetries of the world—the slight preference of matter over antimatter, the complex phases that make some processes run differently forwards and backwards—are proposed to arise as a *holonomy*: repairing a record and returning it can leave a residual phase, like parallel transport around a loop, whose sign depends on orientation. A concrete candidate portal exists; deriving the phase and its orientation from the code layer is open.

The change in texture across Act IV is the honest signal. R0–R11 are a reconstruction a working physicist can largely check against the literature. R12–R15 are a *particular theory's* attempt to ride that reconstruction down to specific physics, and are correspondingly more speculative. The ladder does not hide the seam; it marks it.

7 The honest ledger

The genuinely open frontier is *upstream*, not down. The hardest unanswered question is not any single rung but *why the record floor should take precisely these forms*—why locality, why a binary balanced doubly-even code, why this alphabet—rather than some other architecture for keeping facts. The ladder converts a vague "why is the world quantum?" into a sharp "why these premises about records?"—progress, even though it is not a finish.

Rung	Claim, in one line	Grade
R0	Stable, re-readable, copyable local records exist in a finite noisy world.	Floor (granted)
R1	Repeatable records ⇒ orthogonal alternatives.	Theorem
R2	Repeatable record-questions are projectors.	Theorem
R3	Writing a record is reversible isometric copying, not collapse.	Theorem
R4	Local observers comparing notes ⇒ the field is *complex.*	Reconstruction
R5	Records surviving noise ⇒ a doubly-even self-dual QEC cell.	Reconstruction
R6	The code's record-questions commute (non-disturbing).	Model result
R7	The stable pointer basis is the syndrome basis.	Model result
R8	Non-contextual record probabilities ⇒ the Born rule.	Theorem
R9	Probabilities are *squared* because records close a forward/back loop.	Reconstruction
R10	Irreversibility & the arrow are the cost of *resetting* memory.	Model result
R11	Objective facts are *redundantly broadcast* records.	Model result
R12	Each event commits one record from a fixed address × channel alphabet.	Open (derived core)
R13	History carries a least-action measure over records.	Open skeleton
R14	One universal, equipartitioned bare rate governs record upkeep.	Model result for $\alpha_0 = 1/137$
R15	Time/matter asymmetries are holonomies of record repair.	Open (candidate)

Table 1: The reconstruction ladder. A granted floor (R0); three textbook steps building the Hilbert skeleton (R1–R3); two conditional reconstructions making it complex and error-correcting (R4–R5); six steps delivering probability, the preferred basis, the Born rule, the arrow, and objectivity (R6–R11); and four steps where a particular physical model takes over and confidence drops (R12–R15).

8 Where the ladder lands: the reconstruction as a discrete geometry

Everything in Acts I–III is *realisation-independent*: it constrains any world that keeps records under noise, without saying what that world is built from. But the rungs can also be read the other way—not as constraints to be satisfied, but as *build instructions.* Read constructively, the same axioms single out a concrete discrete geometry. This final section exhibits one such realisation, the TCH lattice, and what is striking is how much geometric structure the record axioms *force*: the dimensionality of space, a unitary walk whose continuum limit is relativistic matter, and a service ledger. These are [MODEL RESULT]s—claims about one construction, not general theorems—and they are correspondingly more committal than R0–R11. They are included because they show the ladder is *constructive*, not merely analytic, and because the constructions are pretty.

Throughout, the discipline of the essay holds: we keep the geometric *structure* (which objects, which operators, which counts of cells and directions) and set aside the framework's *quantitative* commitments (lattice spacings in physical units, and the identification of any derived ratio with a measured constant). Those are the constant-matching the rest of the essay does without, and we do without them here too.

8.1 From the record cell to a lattice

R5 forced the minimal stable record cell to be the doubly-even self-dual $[8, 4, 4]$ code, equivalently $\mathrm{RM}(1, 3)$, equivalently the 3-cube Q_3. The constructive reading asks: what *shape* carries eight bits whose adjacency graph is Q_3? The answer the framework adopts is an *oblate square bipyramid*—an octahedron flattened until apex-to-apex equals the equatorial diagonal—whose eight triangular faces are the eight bits and whose face-adjacency graph is exactly the cube graph Q_3.

This shape is chosen for a tiling reason, not an aesthetic one. *Regular* octahedra do not tile

$\mathbb{R}^3$ on their own (they leave tetrahedral gaps); oblate square bipyramids do, when three are placed with mutually orthogonal axes about a shared centre, filling a cubic cell with no overlap. Tiling space with these cells produces the matter layer of the substrate,

$$\mathbb{Z}^3 \otimes Q_3, \tag{3}$$

a simple-cubic lattice carrying one $[8, 4, 4]$ record cell at every site. The eight faces of a cell are in natural bijection with the eight octants of the cube, i.e. with the group $\mathbb{F}_2^3$—an address space we will meet again in the ledger.

8.2 Why three dimensions

Here the constructive reading pays an unexpected dividend: the *dimensionality of space* is not an input. Demand a record cell that is simultaneously

1. vertex-transitive (no bit is special),
2. realisable as the face-adjacency graph of a convex polyhedron,
3. rich enough to contain 4-cycles (required for the distance-4 parity checks of R5), and
4. minimal in vertex count for a non-trivial $[n, k \geq 2,\ d = 4]$ code,

and the cube graph Q_3 is the *unique* solution. The tetrahedron gives a trivial ($k = 0$) code; the bare cube graph supports at most $k \leq 1$; the Petersen graph fails twice over—it is non-planar (no convex polyhedron realises it) and has girth 5 (no 4-cycles, so no distance-4 protection). Q_3 is realised by a polyhedron in $\mathbb{R}^3$, and so *three* is the number of spatial dimensions in which the minimal error-correcting record cell can live. The arena is selected by the cell, not assumed around it. [MODEL RESULT]—it depends on the convex-realisability premise and the minimality criterion, both natural here but not forced by the bare word "record."

8.3 The walk operators

R3 made record-writing a reversible isometry. Constructively, that isometry becomes an explicit *quantum walk*

$$\mathcal{W} = \mathcal{S} \cdot \mathcal{C}, \tag{4}$$

a coin step followed by a shift. The coin $\mathcal{C}$ acts on the cell's internal bits (a chirality bit χ and an isospin bit I_3) as a controlled bit-flip—a zero-controlled CNOT. The shift $\mathcal{S}$ propagates along the eight body-diagonal directions

$$\hat{n}_f = \tfrac{1}{\sqrt{3}}\,(2b_2 - 1,\ 2b_1 - 1,\ 2b_0 - 1), \qquad f = (b_2, b_1, b_0) \in \{0, 1\}^3, \tag{5}$$

the single O_h orbit of cube-vertex directions, bridging a cell to its neighbours through its eight faces. The hop along each direction carries three internal *channel* operators—an electromagnetic channel diagonal in the cell's charge, a weak channel that mixes chirality (the one off-diagonal gate), and a strong channel that permutes the colour bits. The three spatial directions are related by the cell's rotational symmetry, so a single hop generator fixes them all.

The coin is not free where it matters most. When an excitation scatters at a high-symmetry vertex of the macroscopic gauge web—a degree-six vertex of the cubic line graph—three requirements pin it down completely: *unitarity* (probabilities are conserved), *permutation symmetry* (space is isotropic), and *real-valuedness* (time-reversal). By Schur's lemma the only operator respecting all three is the Grover coin

$$C_{ij}^{\text{Grover}} = \frac{2}{d} - \delta_{ij}, \qquad d = 6, \tag{6}$$

with back-reflection amplitude 1/3. There is no tunable parameter: isotropy plus reversibility plus unitarity *is* the Grover coin. (This same rigidity is what makes Grover-coined walks universal for computation [56]; here it is forced by the symmetry of space rather than chosen.)

Finally, the continuum. A wavepacket spread over very many cells no longer resolves the exact Boolean CNOT; averaged, the discrete flip becomes a continuous mixing angle, and the walk on the internal space $\chi \otimes I_3 \cong \mathbb{C}^2 \otimes \mathbb{C}^2$ reproduces the Clifford algebra $\mathrm{Cl}(3,1)$ and, in the long-wavelength limit, the 3+1-dimensional Dirac equation

$$i\hbar\,\partial_t\Psi = \big[-i\hbar c\,(\alpha_1\partial_x + \alpha_2\partial_y + \alpha_3\partial_z) + mc^2\beta\big]\Psi. \tag{7}$$

Relativistic matter, in this realisation, is the coarse-grained behaviour of the record-writing walk. This places the framework alongside other discrete-substrate programmes—quantum cellular automata and quantum walks as a route to relativistic dynamics [57, 58, 59], and information-theoretic emergence of spacetime [54, 53]—while keeping the *record* premise, rather than a postulated automaton or a postulated bulk, as the starting point. [MODEL RESULT].

8.4 The ledger on the geometry

The service ledger of R12–R14 also becomes concrete. Each event commits one record drawn from an *address* × *channel* alphabet. The *address* is the cell's octant label, the group $\mathbb{F}_2^3$ of eight values met in (3); because the cell's code is translation-invariant, a symmetry-respecting service channel is a convolution on $\mathbb{F}_2^3$, whose unique stationary measure is *uniform*—no octant is privileged, contributing $\ln 8$ to the per-event record entropy. The *channel* is the monitored observable: the cell's commuting stabilizer checks (its syndrome) are read continuously and non-destructively—in the substrate's own language these syndrome reads are an elastic strain field—and each read is a Landauer reset unit (R10). The universal upkeep rate of R14 is then the Born weight of a single firing in this monitor, equipartitioned because the monitored, connected channel relaxes to the uniform fixed point (Evans–Frigerio [51, 52]).

The *size* of the channel alphabet, and—above all—the identification of its reciprocal with a measured physical coupling, are the framework's quantitative commitments. They are exactly the constant-matching this essay has set aside throughout, and we set them aside here too. What the constructive reading delivers, and all it claims, is the *form* of the ledger: an address space fixed by the cell's symmetry, a channel that is the syndrome read, a uniform service measure forced by equipartition, and a billing in Landauer units.

8.5 What the constructive reading shows

The same record axioms behave in two registers. *Analysed*, they force the abstract structure of complex error-correcting quantum mechanics (R0–R11). *Built*, they pick out a three-dimensional lattice of error-correcting cells (3), a unitary walk (4) whose continuum limit is Dirac matter, and a syndrome-billed service ledger. Whether this *particular* lattice is the actual world is a quantitative question—answered, if at all, only by the constant-matching deferred here. But that the reconstruction is geometrically *constructive* at all—that "a world that keeps records" has not just a logical form but a buildable shape, and that the shape forces the dimension of space and the appearance of relativistic matter—is the structural payoff, and it needs none of the numbers.

9 What this buys, and what it does not

What it buys is more than rhetorical. If the ladder holds, the famous strangenesses of quantum mechanics are *not* independent postulates to be swallowed; they are what memory under noise requires:

- complex amplitudes—because local observers must reconstruct wholes from parts (R4);
- superposition and interference—because records are written reversibly before they are read (R3, R9);
- the Born rule—because non-contextual record-probabilities have no other consistent form (R8);
- the measurement "collapse"—because reusable memory must be reset, and reset is irreversible (R10);
- a definite classical world—because widely copied records are objective (R11);
- even error correction, usually thought an engineering achievement layered *on top* of quantum mechanics, sits *underneath* it: the protected syndrome is what a long-lived bit *is* (R5).

That is the sense in which "it from bit" becomes concrete and sequential. The bit is not a classical token; it is a protected quantum record. And "it from bit" is not a single leap but a staircase, most of whose steps are theorems.

What it does *not* buy, stated plainly:

1. **It does not derive the floor.** R0 is granted, not proved, and cannot be otherwise without circularity.
2. **It is not all original.** The structural rungs assemble results the reconstruction community already owns [2, 3, 15, 8, 48, 20]. The contribution is the organising premise (record-keeping), the error-correction rung as the bridge, the honest grading—and the constructive geometry of Section 8.
3. **It contains no physics numbers.** Everything quantitative lives in R12–R15 and the parent framework, is more contentious, and is excluded here. Nothing above depends on it.
4. **The conditional rungs are conditional.** R4 and R5 rest on premises (locality of measurement; a binary balanced code class) natural for a local cellular substrate but not forced by the bare word "record." Marking them [RECONSTRUCTION] is the actual logical status, not modesty.

None of these is fatal to the thesis, which was never "quantum mechanics is proved from nothing" but the more defensible claim that **quantum mechanics is the logic of keeping records in a noisy world**—and that this can be laid out as a graded ladder rather than asserted as a slogan.

10 Coda

Wheeler called the universe a "self-excited circuit": it brings itself into being by observing itself. Strip the romance and a skeleton remains. Observation is record-keeping. Record-keeping under noise needs error correction. Error correction needs a complex Hilbert space with a syndrome basis, Born weights, a reset-driven arrow, and redundancy-born objectivity—and, read constructively, a discrete geometry to carry them. *It* really might be *from bit*—provided the bit is a protected quantum record, and provided we climb the ladder one honestly labelled rung at a time.

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