A Holographic Error-Correcting Code in de Sitter Space: How a Finite Quantum-Error-Correction Substrate Relates to AdS/CFT

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Abstract

The most striking idea in modern quantum gravity is that a gravitational theory in a region of space can be *equivalent* to a non-gravitational theory living on that region's boundary — the holographic principle [21, 23], made precise by the AdS/CFT correspondence [16] and sharpened into the statement that the bulk is literally a *quantum error-correcting code* written on the boundary [1, 18, 19]. This note is written for a graduate reader who knows quantum mechanics and a little general relativity but is still climbing into holography. We first explain, plainly, what AdS/CFT and holographic error correction actually say. We then introduce a very different-looking research programme — a finite quantum-error-correction (QEC) "substrate" in which space is the ordered phase of a discrete, self-correcting register network — and ask, honestly, how the two relate. The answer is a clean three-part verdict. They share a deep *principle* (bulk-in-boundary error correction) and a deep *law* (entanglement entropy = a geometric area). They differ in two concrete, checkable ways: the substrate's elementary code is *not* a perfect tensor of the Pastawski–Yoshida–Harlow–Preskill (HaPPY) type, and its spacetime is *not* anti-de Sitter — it is flat with a de Sitter cosmology, and its "boundary" is a causal/cosmological horizon, not a conformal boundary at infinity. The consequence is that the natural bridge is to *de Sitter / cosmological-horizon* holography — the open, under-served side of the subject — where the substrate offers something the field largely lacks: a concrete microscopic holographic code with explicit boundary dynamics. We make precise which AdS/CFT tools transfer and why, verify the Ryu–Takayanagi area law directly on the lattice, and close by relating the picture to the programmes of Susskind (the holographic principle, complexity, ER=EPR) and Baez (the discrete-versus-continuum question, and physics as computation), whose work this both draws on and, we hope, complements. A 2026-06-20 refresh adds the main downstream consequence now in canon: RT/min-cut structure plus an explicit service-current stress tensor supports the linearized Einstein source form at continuum-Jacobson grade, while the observed Planck hierarchy remains a separate nonlocal service-span problem.

Contents

1 The holographic principle, in one paragraph

Black holes told us something strange about information. A black hole's entropy is not proportional to its volume, as for an ordinary gas, but to the *area* of its horizon: $S = A/4G$ in natural units [4, 13]. Since entropy counts microstates, this means the number of degrees of freedom inside a region is bounded by its boundary area, not its volume. 't Hooft and Susskind drew the radical conclusion: a theory with gravity in D dimensions must be equivalent to a theory *without* gravity in $D - 1$ dimensions, living on the boundary — the *holographic principle* [5, 21, 23]. Everything below is about how that principle is realised, first in the textbook case and then in the substrate programme.

2 What AdS/CFT actually says

The duality. Anti-de Sitter space (AdS) is a spacetime of constant *negative* curvature; think of it as a box whose spatial boundary sits at infinity but which light can reach and return from in finite time. Maldacena's correspondence [16] states that a quantum gravity theory in the $(d+1)$-dimensional AdS "bulk" is *exactly the same theory* as a conformal field theory (CFT) — an ordinary, gravity-free quantum field theory with scale invariance — living on its d-dimensional boundary. Two descriptions, one physics: this is a *duality.*

Entanglement is geometry (Ryu–Takayanagi). Pick a region A of the boundary. Its entanglement entropy $S(A)$ — how entangled A is with the rest of the boundary — equals the area of the smallest bulk surface γ_A that hangs from the edge of A:

$$S(A) = \frac{\text{Area}(\gamma_A)}{4G_N}. \tag{1}$$

This is the Ryu–Takayanagi (RT) formula [19]. It says boundary entanglement is *built out of* bulk geometry: areas *are* entropies.

The bulk is an error-correcting code (Almheiri–Dong–Harlow). How is a point deep in the bulk encoded on the boundary? Not in any single boundary region: you can delete a chunk of the boundary and still reconstruct the bulk point, provided the chunk is not too big. That is exactly the defining property of a *quantum error-correcting code* — logical information protected against erasure of part of the physical system. Almheiri, Dong and Harlow [1] showed bulk locality *is* a QEC code: a bulk operator in the "entanglement wedge" of a boundary region A (the bulk region bounded by γ_A) can be reconstructed from A alone. This is *subregion duality.*

An explicit toy: the HaPPY code. Pastawski, Yoshida, Harlow and Preskill [18] built tensor-network toy models that realise all of this exactly. Their tiles are *perfect tensors*: a tensor on $2k$ legs whose state is *absolutely maximally entangled* (AME), meaning maximally entangled across *every* way of splitting the legs in half. Tile a hyperbolic disk with perfect tensors and you get a code in which RT holds exactly (the minimal surface becomes a minimal cut through the network) and bulk operators enjoy clean erasure protection. "Perfect tensor / AME" is the special ingredient that makes HaPPY so clean.

Complexity is action. A last tool we will need: the boundary state's quantum computational *complexity* (how many elementary gates to build it) is conjectured to equal a bulk gravitational quantity — the action of a certain spacetime region ("complexity = action") [6, 22]. Geometry is computation.

3 What the finite-QEC substrate says

The substrate programme [8, 10] starts from a different intuition: that space *is* a quantum computer's memory. Concretely, space is the ordered (crystallised) phase of a discrete, self-correcting register network on $\mathbb{Z}^3$, with an internal 8-bit register Q_3 at each cell. Three features matter here.

The cell is a code. Each cell's register is the self-dual $[8, 4, 4]$ Reed–Muller code — eight physical bits encoding logical (matter) content, with code distance 4. Distance 4 means the logical information survives the erasure of up to three of the eight legs: *the cell is already a small quantum error-correcting code with genuine erasure protection.*

Expansion is boundary printing (HBC). The substrate cannot expand by stretching — stretching the lattice would change the speed of light. Instead, new cells are *printed at the boundary of the causal patch* as the error-correcting machinery radiates waste heat outward and the patch approaches its Bekenstein bound. This "Holographic Boundary Crystallisation" (HBC) makes the cosmological horizon a dynamical *holographic screen*: the universe does not stretch, it prints. This note uses HBC in that architectural sense. It does not, by itself, claim every later inflationary HBC coefficient: the scalar amplitude and stop-rule statements are separate cosmology-sector theorems with their own locality and saturation premises.

Entropy is recorded syndrome; horizons carry area-law entropy. Entropy in the substrate is the number of recorded error-syndrome bits, each costing one Landauer unit [15]. The horizon carries Bekenstein entropy $S = A/4$, with a per-cell record coefficient that the programme fixes at 55/8. So, just as in holography, horizon *area* counts *information*.

4 Where they agree

Two correspondences are genuine and, we claim, rigorous.

(1) Bulk-in-boundary error correction (the ADH principle). The substrate cell does exactly what ADH says holography must: encode bulk (logical) content in boundary (physical) legs with erasure protection. Distance 4 protects against erasure of up to three of eight legs. This is the same *principle* as subregion duality, in a concrete finite code.

(2) Entanglement entropy is a geometric area (RT = Bekenstein = min-cut). The RT law (1) and the substrate's Bekenstein law $S = A/4$ are the same statement: entanglement entropy is a minimal area. We checked this is not merely verbal. In the link-state (bit-thread) limit, where each bulk bond carries one unit of entanglement, the entanglement entropy of a boundary region on a substrate lattice equals the minimal cut separating it from the rest — the max-flow/min-cut theorem, which is precisely how RT works in a tensor network. On a model lattice this reproduces the expected Page curve (entropy rises with region size then falls), an area law (entropy scales with the cut, not the volume), and a well-defined entanglement wedge for every region.[1] So the substrate carries Ryu–Takayanagi structure.

These two legs are the bridge. They also place the substrate squarely in the lineage of the holographic principle [5, 21, 23].

5 Where they differ

Honesty about the differences is what makes the bridge usable rather than a slogan. There are two, both concrete.

(A) The cell is not a perfect tensor. HaPPY's tiles are AME (perfect) tensors: maximally entangled across every balanced cut. The substrate's $[8, 4, 4]$ cell is *not*. Its $[[8, 0, 4]]$ stabiliser state has balanced-cut entanglement that is maximal (4 bits) for most cuts but drops to 2 bits for the cuts that a weight-4 stabiliser lies inside — so it is not AME. (A perfect tensor on 8 legs would need every balanced cut maximal; the distance-4 codewords forbid that.) The substrate therefore belongs to the *general* ADH holographic-code family — codes with erasure protection and RT-like behaviour — but *not* to the perfect-tensor (HaPPY) subclass. RT then holds as a leading-order/area-law statement with sub-leading corrections, rather than as the exact min-cut equality HaPPY enjoys.

(B) It is not anti-de Sitter. This is the decisive difference. AdS/CFT, the HaPPY construction, and complexity = action all live in *negative*-curvature anti-de Sitter space with a *conformal boundary at spatial infinity*. The substrate is a *flat* $\mathbb{Z}^3$ lattice carrying a *de Sitter* cosmology (positive cosmological constant, accelerating expansion), and its "boundary" is the *causal/cosmological horizon* — the HBC printing front — not a conformal boundary at infinity. The two settings are,

[1] Verified in `substrate_rt_wedge.py`; the broader correspondence map and the entanglement computations of §5 are in `substrate_holographic_bridge.py` [11].

in a real sense, opposite (negative vs positive curvature; timelike conformal boundary vs causal horizon).

6 What transfers, and why it is useful

The differences do not break the bridge; they relocate it. The ADH *principle* and the RT *law* are statements about "holography + quantum error correction" in general, not about anti-de Sitter space specifically. So the tools built on *those* transfer, provided one translates "conformal boundary" to "cosmological horizon":

AdS/CFT tool	Status in the substrate
Subregion duality / entanglement wedges	**Transfers.** Follows from the QEC structure; the wedge of a horizon region is the bulk inside its RT (min-cut) surface.
Ryu–Takayanagi / bit threads	**Transfers (leading order).** Verified as min-cut on the lattice; corrections from the non-perfect cell.
Complexity = action/volume	**Suggestive.** The substrate's Landauer/record cost is a microscopic complexity; the conjecture becomes a question about counting recorded operations along horizon histories.
ER=EPR (entanglement = geometry)	**Natural.** In the substrate, geometry is built from the entanglement of records — the same slogan, microscopically.
Entanglement first law / Einstein form	**Transfers at leading order.** RT/min-cut first variations plus the service-current stress tensor give the linearized Einstein source form at continuum-Jacobson grade; the observed Planck hierarchy is still a nonlocal span theorem.
Perfect-tensor (HaPPY) constructions	**Does not transfer verbatim** — the cell is not AME; one needs general (non-perfect) holographic-code methods.

The use is concrete. A graduate student wanting to study entanglement wedges, subregion recovery, or holographic complexity in a *de Sitter* universe has, in AdS/CFT, a magnificent toolkit but the wrong spacetime; in the substrate, the right spacetime (a positive-Λ cosmology with an explicit horizon) but, until now, no dictionary. The bridge lets the former be carried to the latter.

7 A canon-refresh consequence: the Einstein source form

The RT leg is no longer only a structural analogy. The current canon combines three ingredients: the lattice RT/min-cut theorem above, the entanglement first-law route from area variations to linearized gravity [12, 14], and an explicit service-current stress tensor

$$T^{\mu\nu}_{\rm svc} = \frac{1}{V}\sum_e r_e \frac{p_e^\mu p_e^\nu}{p_e^0}.$$

Here e labels local service events, r_e is the record weight, and p_e^μ is the event momentum. The tensor passes the finite-ledger checks needed for a source: symmetry, positivity, Kirchhoff conservation, a radiation control with $P/\rho = 1/3$, and a dust control. With that source in hand, the all-region entanglement first-law constraint has the right localizing rank to give the linearized Einstein/source form,

$$\delta S = \delta\langle K\rangle \quad \Rightarrow \quad \delta G_{\mu\nu} = 8\pi G_{\rm eff}\,\delta T_{\mu\nu},$$

at leading-order continuum grade. This is a genuine new use of the bridge: the framework no longer has to import a generic matter stress tensor for the linearized source-form gate.

The boundary is equally important. This argument gives the form of the source equation and the local/bare coupling scale. It does not generate the observed M_P hierarchy, which remains tied to the separate horizon-scale service-span theorem in the gravity paper. In short: the bridge now reaches the Einstein equation's linearized form, but not yet the full observed gravitational normalization.

8 A second canon-refresh consequence: a hyperbolic emergent bulk, a thermal horizon, and $\gamma_{\mathrm{TEE}} = 0$

The RT leg of §5 is a statement about *how much* entanglement a boundary region carries (an area). A direct lattice measurement (refresh 2026-06-23) lets us ask the sharper question underneath holography: *what geometry does the entanglement itself encode?* The method is elementary. Between any two regions i, j of the lattice the mutual information $I(i:j)$ measures how strongly they are entangled; turn it into a *distance*, $d(i, j) = -\log I(i:j)$, so that strongly-entangled regions sit "close." Collect the full table of pairwise distances and feed it to multidimensional scaling — the standard "recover the shape from its distance matrix" procedure — but allow the target to be a curved model space of constant curvature κ, and read off the κ that fits best. This turns "geometry from entanglement" from a slogan into a number.

The emergent bulk is hyperbolic. Applied to the physical gauge sector — the gapless simple-cubic Maxwell photon, kernel $\mathcal{K}(\mathbf{k}) = 6 - 2\sum_i \cos k_i$ — the best fit is a space of constant *negative* curvature, $\kappa \approx -1.5$, embedding the entanglement to about 5% distance error, against 48% for the best flat (Euclidean) fit. Two independent diagnostics agree: a small Gromov δ (a parameter-free measure of how "tree-like," hence hyperbolic, a metric is) and an *exponential* growth of the number of regions within a given entanglement-distance — the defining hallmark of hyperbolic space, where volume grows exponentially with radius. Crucially, repeating the measurement on a *gapped* kernel returns flat, so the hyperbolicity is the fingerprint of the gapless (critical) photon, not an artefact of the procedure. The picture is then the AdS/CFT arrangement made concrete: the *physical* lattice is flat $\mathbb{Z}^3$, but the geometry encoded in its *entanglement* is a negatively-curved (hyperbolic) bulk emergent from it.

Two honest bounds. A careful reader should keep two limits in view. First, this emergent geometry is the holographic/renormalisation-group bulk reconstructed from entanglement; it is *not* the physical spacetime, which remains flat with a de Sitter cosmology (§6). The two coexist for exactly the reason they do in AdS/CFT, where a flat-space conformal field theory lives on the boundary of a curved AdS interior. Second, the result is *kinematic*: a hyperbolic mutual-information geometry is the universal "MERA skeleton" that *any* scale-invariant theory possesses; it is necessary for a genuine AdS dual but not sufficient, and a free field in particular has no Einstein-gravity dual. We have measured the right *pattern*, not established dynamical gravity.

No hidden topological order: $\gamma_{\mathrm{TEE}} = 0$. A separate question is whether the vacuum hides the kind of nonlocal entanglement that protects a topological code. The standard diagnostic is the topological entanglement entropy γ, extracted from a Kitaev–Preskill combination of three regions arranged so the area-law piece cancels and only the universal constant survives. For the photon we find $\gamma_{\mathrm{TEE}} \approx 0$ ($|\gamma| \ll \log 2$). This confirms from entanglement what the programme holds on physical grounds: the vacuum is the deconfined *Coulomb* phase — long-range correlated but with *no intrinsic topological order* — which is precisely what a massless emergent photon demands. (A gapped $\mathbb{Z}_2$ string-net would instead give $\gamma = \log 2$ but no photon; the microscopic line-graph Chern number $C_{\mathcal{S}_7} = -1$ of §7.1 is a separate band-topology feature, and reconciling the two phases is left open.)

The cosmological horizon is a thermal screen. The checks so far used a *spatial* region. The object proper to de Sitter is the cosmological *horizon* as the screen, so we take a causal-patch ball whose boundary *is* the horizon. Two things hold. Its entanglement obeys a horizon *area* law, $S \propto$ Area — the Gibbons–Hawking–Bekenstein $S \propto A/4$. And — the signature specific to a horizon — its modular ("entanglement Hamiltonian") spectrum turns *thermal*: the modular gap, the energy of the most strongly entangled mode, *closes* as the patch grows (from ≈ 1.83 down to ≈ 1.11 over the radii probed), trending towards the gapless, continuous spectrum of a thermal Gibbons–Hawking state, whereas a gapped, horizonless field has a modular gap that *saturates* (at ≈ 3.9, some $3.5\times$ larger). So the cosmological horizon behaves as a *thermal holographic screen* — an area law plus a modular gap that closes with size. This is the static seed of the genuinely dS step "RT on the horizon, not on a spatial disk"; the full dynamical horizon, with holographic-boundary crystallisation, remains open.

Both sectors carry the pattern. Finally, the same two signatures appear in the *matter* sector. The single-particle walk $W = S \cdot C$ of §3 is a gapless Dirac band; fed through the identical measurement it returns a hyperbolic emergent geometry and the same closing (thermal) horizon modular gap as the photon. The AdS-pattern — a hyperbolic emergent bulk over flat physical space, with a thermal cosmological horizon — is therefore carried by *both* the gauge and matter sectors of the substrate, the common cause being simply that both are gapless/critical.

9 The horizon's dynamics: adiabatic printing, and the temperature as an open residual

The results so far are *static* — properties of a fixed causal patch. The substrate's expansion, however, is dynamical: Holographic Boundary Crystallisation (HBC) *prints* new cells at the horizon rather than stretching the lattice, so it is fair to ask whether that printing *sustains* the horizon just characterised. Modelling the gauge web as a free Gaussian field on a growing causal-patch ball (a deliberate modelling choice, set out in `python_code/dS_hbc_dynamics_scope.md`), we print boundary shells in their local vacuum and ramp their couplings on over a tunable time. The outcome is clean and carries a physical lesson: *adiabatic* (slow) printing tracks the ground-state area law to a few percent, whereas a *fast* quench leaves the freshly-printed cells under-entangled and the entropy drifts off the area law towards a volume scaling. So HBC printing sustains a de Sitter horizon *only when it is adiabatic* — the expansion rate is bounded above by the substrate's microscopic relaxation, and a universe that printed too fast would carry a torn, under-entangled horizon rather than a smooth one.

An honest open residual: the horizon temperature. One quantity resisted a clean lattice determination — the magnitude of the Gibbons–Hawking temperature $T_{\mathrm{GH}} = H/2\pi \propto 1/R_H$. We attempted it three ways: the entanglement first law $T_{\mathrm{ent}} = \delta E/\delta S$, and the Bisognano–Wichmann/CHM modular weight read off both the π^2 term and the full local energy density T_{00}. The modular-weight method is sound — its self-consistency check holds to $\sim 10^{-9}$, and in 1+1D it reproduces the known CHM parabola $\beta(x) \propto R^2 - x^2$ exactly — and in 3D it confirms the right *structure*: a parabolic modular weight whose centre value rises with R, so the horizon's local temperature *falls* as the patch grows. But the *coefficient* $\beta(\text{centre}) \propto R$ does not emerge at the radii reachable here ($R \leq 8$): a local ultraviolet constant dominates the centre weight by *both* readouts, so the clean $1/R$ law is finite-size-limited. This is a genuine residual, not a failure of principle. Pinning T_{GH} quantitatively needs the continuum regime (radii far above the lattice cutoff — a large computation with uncertain odds) or an analytic CHM calculation, rather than a modest run. The horizon's *thermality* and *modular structure* are not in doubt; only the temperature's numerical coefficient is left open, and is flagged as such.

10 The positioning: holography without anti-de Sitter

AdS/CFT is, by the standards of theoretical physics, the *solved* case of holography — extraordinarily rich, but anchored to a spacetime that is not ours (we live in a flat, positive-Λ universe). Holography for *de Sitter* space, and for cosmological horizons generally, is the open and underserved frontier [5, 20]: there is no boundary at infinity to put a CFT on, the would-be dual is contested, and many of the AdS tools fall silent.

This is exactly where the substrate has something to offer. It is a *concrete microscopic holographic code in the de Sitter regime*: an explicit cell code (distance 4, holographic) with explicit boundary dynamics (HBC printing at the cosmological horizon). Rather than re-deriving AdS/CFT, the programme can serve as a *constructive model* for dS holography — a place to ask, with an actual code in hand, what subregion duality and RT mean when the screen is a cosmological horizon. The honest pre-emption of the obvious referee question — "how does this relate to holographic error-correcting codes?" — is therefore: *the same ADH principle and the same RT law; a different code (not perfect-tensor) and a different spacetime (de Sitter, not anti-de Sitter).*

11 Relation to the Susskind and Baez programmes

We close by placing this beside two bodies of work it draws on and, we hope, complements.

Susskind. The substrate is, in spirit, a microscopic mechanism for several of Susskind's central ideas. HBC is a literal realisation of the *holographic principle* [21]: degrees of freedom are added only at the boundary screen. The substrate's identification of entropy with recorded, Landauer-costed syndromes makes *complexity = action* [6, 22] a concrete counting problem — recorded operations along horizon histories — rather than a conjecture about abstract gate counts. And because the substrate builds geometry from the entanglement of records, it is a microscopic instance of the *ER=EPR* slogan [17] that entanglement and geometric connection are the same thing. Where Susskind's programme is top-down (from gravity and complexity), the substrate is bottom-up (from an explicit code); the overlap is the holographic principle and complexity, and the complement is a microscopic carrier for them in de Sitter space.

Baez. Two strands of Baez's work bear directly on this. First, *Struggles with the Continuum* [2] lays out how badly the continuum behaves in fundamental physics; a discrete, finite substrate is one disciplined response, and the present note shows such a substrate can still carry the continuum holographic structures (RT, entanglement wedges) in an emergent limit — discreteness need not cost holography. Second, the *Rosetta Stone* [3] that aligns physics, topology, logic and computation is, in a sense, the substrate's founding premise made categorical: if physical processes *are* computations, a self-correcting code is a natural candidate for the fabric of space. The overlap is the physics-as-computation viewpoint and the seriousness about discreteness; the complement is a specific, calculable model in which those commitments produce holographic error correction.

12 Status, and an invitation

We are candid about tiers. The two bridge legs — the ADH bulk-in-boundary principle (the cell is a distance-4 holographic code) and the RT = Bekenstein = min-cut law (verified on the lattice) — are rigorous. The two mismatches — not a perfect tensor; not anti-de Sitter — are likewise established, not hand-waved. What remains *open* is the full dictionary: importing each AdS tool (subregion recovery maps, complexity = action, the dS dual) and making it quantitative on the cosmological horizon is genuine work, only begun here. And the substrate programme itself is a research programme, not an established theory; this note claims a structural relationship, not a proof of either side.

With those caveats, the relationship seems worth stating plainly, because it points somewhere useful: a concrete holographic error-correcting code that lives in the de Sitter regime where the AdS machinery is most wanted and least available. We would welcome correspondence with workers in holographic quantum gravity — the overlap with the holographic-principle, complexity and ER=EPR programmes on one side, and with the discreteness and physics-as-computation programmes on the other, is, we think, real and mutually informative.

A Reproducibility

The correspondence map, the cell-code entanglement computation, and the RT/entanglement-wedge check are self-asserting programs in the repository [11] (each exits 0 only if its checks pass): `python_code/substrate_holographic_bridge.py` (the map; the $[[8,0,4]]$ balanced-cut entanglement showing min $2 < 4$, hence not AME) and `python_code/substrate_rt_wedge.py` (RT = min-cut on the lattice; Page curve; area law; entanglement wedge). The refreshed Einstein-source consequence is checked by `python_code/gravity_service_current_stress_tensor_gate.py` (the explicit $T^{\mu\nu}_{\rm svc}$ and its controls) and `python_code/intrinsic_gravity_linearized_einstein_gate.py` (RT/first-law/service-current localization). The 2026-06-23 emergent-geometry refresh is checked by six further self-asserting programs: `python_code/item_geo_a_gaugeweb.py` (area law + $d \sim \log r$ on the canon kernel), `python_code/item_geo_a_curvature.py` (curved-MDS measured $\kappa \approx -1.5$ with Gromov-δ and volume-growth cross-checks; massless-hyperbolic versus massive-flat), `python_code/item_geo_b_maxwell.py` (the faithful vector Maxwell photon: area law, hyperbolic emergent geometry, $\gamma_{\rm TEE} \approx 0$), `python_code/dS_horizon_modular.py` (the causal-patch cosmological horizon: a horizon area law + a modular gap that closes with patch size, the thermal-screen signature), `python_code/walk_band_geometry.py` (the matter walk $W = S \cdot C$: the same hyperbolic emergent geometry and thermal horizon as the photon), and `python_code/candidate_macroscopic_model.py` (one explicit operator set for the macroscopic gate, showing $\gamma_{\rm TEE}$ is a phase choice—a $\mathbb{Z}_2$-topological branch with $\gamma = \log 2$ versus a trivial/Coulomb branch with $\gamma = 0$ that hosts the photon). The dS-dynamics thread adds the build spec `python_code/dS_hbc_dynamics_scope.md` and four self-asserting checks: `python_code/dS_hbc_phase01.py` (adiabatic printing sustains the horizon area law; a fast quench breaks it), and `python_code/dS_hbc_phase2.py`, `python_code/dS_hbc_bw.py`, `python_code/dS_hbc_bw_fullT00.py` (the three Gibbons–Hawking-temperature routes and their documented finite-size limitation). The substrate, its $[8, 4, 4]$ code, the Bekenstein coefficient, and HBC are in the sector papers [7–10].

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