Intrinsic Non-Locality of Glueball Decay

A Structural Prediction from the Discrete TCH Substrate

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Abstract

The companion paper on glueball masses established the universal formula $m_N^{\text{dressed}} = (2N-1)\Lambda_{\text{QCD}}$ for N-body pure-gauge Fock states on the TCH gauge web, with confirmed predictions for the 0^{++} scalar (1660 MeV) and 2^{++} tensor (2324 MeV) glueballs. The substrate-level Feshbach decay width, however, exhibits a $1/\epsilon$ regulator dependence indicating that the discrete 208-state $\mathcal{Q}$-spectrum is a step-function approximation to a smooth continuum. Direct diagonalisation of the local $\mathcal{Q}$-matrix reveals four exactly-degenerate eigenstates sitting rigidly at $1\Lambda_{\text{QCD}}$ (corresponding to the four vertices of a single C_4 plaquette, each hosting a single-parity-violation transition). Combined with strong-coupling threshold pinning of the dressed pole, this yields the *Threshold Bound State Theorem*: restricted to the local Hilbert space of a single TCH unit cell, the substrate-level glueball tree-level decay width is identically zero. The empirically observed scalar-glueball width $\sim$ 100–300 MeV must therefore arise entirely from non-local macroscopic spatial routing: virtual $q\bar{q}$ pairs propagating across multiple TCH cells before hadronising into asymptotic $\pi\pi/\eta\eta/K\bar{K}$ continua. The physical decay is intrinsically non-local in spacetime, not merely strongly-mixed in Hilbert space. We formalise the macroscopic routing programme as Brillouin-zone integration of the momentum-dependent walk operator $\mathcal{W}_{\mathcal{QQ}}(\mathbf{k})$ over the simple-cubic gauge web, derive phenomenological consequences (scalar-tensor width hierarchy via D-wave angular-momentum suppression, structural explanation of f_0-manifold mixing), and identify two falsifiable signatures distinguishing the framework from standard local-vertex glueball treatments: anomalous lattice-volume scaling of glueball widths, and production-decay correlations in $J/\psi \to \gamma G \to \gamma\pi\pi$.

1 Introduction

The Holographic Circlette (TCH) framework's glueball mass predictions [1] produced two zero-parameter LQCD-matching results — the 0^{++} scalar at 1660 MeV and the 2^{++} tensor at 2324 MeV — via the universal mass formula $m_N^{\text{dressed}} = (2N-1)\Lambda_{\text{QCD}}$ for N-body pure-gauge Fock states. The substrate-level Feshbach machinery underlying that result, however, produced a decay width that scaled strictly as $1/\epsilon$ with the regulator: $\Gamma \cdot \epsilon \approx 2.64 \times 10^6$ MeV^2 across $\epsilon \in [5, 50]$ MeV. The mass was regulator-stable to < 1 MeV; the width was not.

The mass paper explicitly anchored the width-extraction question as a downstream computational target requiring continuum extrapolation of the discrete 208-state $\mathcal{Q}$-spectrum. This paper takes up that target. The headline result is structurally informative rather than purely computational: the local single-cell tree-level decay width is *identically zero*, and the physically observed width must arise entirely from a fundamentally non-local macroscopic process.

We state this as the *Threshold Bound State Theorem* (§4) and develop its consequences. The substrate's discrete spectral structure forces the dressed pole exactly to the kinematic threshold of the virtual-matter continuum; at this position the tree-level phase space $\sqrt{E - E_{\text{th}}}$ identically vanishes; the local glueball cannot decay via any single-cell process. The physical width arises only when the virtual $q\bar{q}$ pair propagates across multiple cells of the macroscopic gauge web before hadronising into asymptotic states. The decay is non-local in spacetime, not merely strongly-mixed in Hilbert space.

This is a substantive physical claim that extends beyond the framework's internal logic. Standard QCD treatments of glueball decay assume an effectively local coupling $\mathcal{L}_{\text{eff}} \sim g_{GG\pi\pi} G\pi\pi$; the framework predicts that this local coupling vanishes identically. The observed broad scalar-glueball width and the empirical impossibility of identifying a single clean f_0 as the glueball state become *structural consequences* of non-locality rather than independent phenomenological puzzles. We identify falsifiable signatures that distinguish the framework from standard local-vertex glueball treatments (§8).

2 Spectral Anatomy of the Local $\mathcal{Q}$-Subspace

Direct diagonalisation of the local $\mathcal{W}_{\mathcal{Q}\mathcal{Q}}$ matrix, restricted to states accessible from a single C_4 plaquette via single-parity-violation transitions, reveals an unexpectedly simple structure:

Eigenstate	Energy	Spectral weight $\lvert V_q\rvert^2$
1	$1\Lambda_{\text{QCD}} = 332.0$ MeV	$\sim 1.1 \times 10^5$ MeV^2
2	$1\Lambda_{\text{QCD}} = 332.0$ MeV	$\sim 1.1 \times 10^5$ MeV^2
3	$1\Lambda_{\text{QCD}} = 332.0$ MeV	$\sim 1.1 \times 10^5$ MeV^2
4	$1\Lambda_{\text{QCD}} = 332.0$ MeV	$\sim 1.1 \times 10^5$ MeV^2

Four exactly-degenerate eigenstates, all at exactly $1\Lambda_{\text{QCD}}$, all with identical spectral weight. This is not a numerical coincidence: a C_4 plaquette has four vertices, and the substrate's single-parity-violation transitions accessible from a closed plaquette break exactly one constraint per vertex, yielding four independent transition pathways. Each pathway costs exactly one $\mathbb{F}_2$ penalty Λ_{QCD}. The local $\mathcal{Q}$-spectrum is therefore a 4-fold degenerate spike at $1\Lambda_{\text{QCD}}$ rather than a smooth distribution.

The Feshbach resolvent

$$G_{\mathcal{Q}}(E) \;=\; (E - \mathcal{W}_{\mathcal{Q}\mathcal{Q}} + i\epsilon)^{-1} \tag{1}$$

evaluated at the threshold-bound pole $E = 1\Lambda_{\text{QCD}}$ reduces to

$$G_{\mathcal{Q}}(1\Lambda) \;\to\; \frac{-i}{\epsilon} \cdot \mathbb{P}_{\text{4-fold deg.}}, \tag{2}$$

where $\mathbb{P}_{\text{4-fold deg.}}$ is the projector onto the degenerate eigenspace. The width's $1/\epsilon$ scaling observed in the mass paper is now structurally explained: the resolvent reads the height of the regulator rather than a smooth density-of-states.

3 Threshold-Pinning Analysis: The Strong-Coupling Limit

The dressed pole's exact pinning at the kinematic threshold is not a generic feature of Feshbach systems but a consequence of the framework-specific strong-coupling regime. Quantifying the coupling magnitude:

For the 0^{++} scalar 3-body Fock state, the Feshbach self-energy carries an effective coupling

$$|W|^2 \;\approx\; N \cdot |\langle \text{plaquette}|\mathcal{W}|q_i\rangle|^2 \;\sim\; 12\,\Lambda_{\text{QCD}}^2, \tag{3}$$

combining the 3-body combinatorial factor (three plaquettes that can each undergo virtual nucleation) and the 4-fold degeneracy (four vertices per plaquette). The continuum gap relative to the Fock-state bare mass is

$$\Delta_{\text{gap}} \;=\; E_{\text{bare}} - E_{\text{th}} \;=\; 6\Lambda_{\text{QCD}} - 5\Lambda_{\text{QCD}} \;=\; 1\,\Lambda_{\text{QCD}}. \tag{4}$$

The relevant dimensionless strong-coupling parameter is the ratio

$$\xi \;\equiv\; \frac{|W|^2}{\Delta_{\rm gap}^2} \;\sim\; \frac{12\Lambda_{\rm QCD}^2}{\Lambda_{\rm QCD}^2} \;=\; 12. \tag{5}$$

This is the deep-strong-coupling regime. In standard Feshbach resonance theory, when $\xi \gg 1$, the physical pole is violently driven toward the kinematic threshold of the virtual continuum. The pole-to-threshold binding energy satisfies

$$\epsilon_B \;\sim\; |W|^2/E_{\rm bare} \;\sim\; \frac{12\Lambda_{\rm QCD}^2}{6\Lambda_{\rm QCD}} \;=\; 2\,\Lambda_{\rm QCD} \;\approx\; 664\text{ MeV} \tag{6}$$

in the standard estimate. In practice this exceeds the gap itself, indicating that perturbative Feshbach corrections break down: the pole is rigidly pinned at threshold to within regulator-dependent microscopic corrections. The discrete-substrate calculation makes this transparent; the continuum-limit calculation would place the pole microscopically below threshold with an exponentially-small binding energy $\epsilon_B \sim \exp(-c/g)$ characteristic of strong-coupling Efimov-like binding.

The threshold-pinning is therefore not a generic accident: it is the strong-coupling asymptote of a Feshbach system with $\xi \sim O(10)$. The framework's specific coupling structure (combinatorial 3-body $\times$ 4-fold vertex degeneracy) places it firmly in this regime, and the discrete spectrum makes the pinning rigorous to within ϵ.

4 The Threshold Bound State Theorem

We can now state the central structural result.

The local Hilbert space. For the rigorous formulation, "local" means: the substrate-level Hilbert space restricted to single-TCH-unit-cell internal degrees of freedom (the 256-dimensional 8-qubit register of Q_3), with the $\mathcal{P} \cup \mathcal{Q}$ decomposition into 48 physical + 208 invalid codewords (ANCHOR [4, §2.6]), with couplings $\mathcal{W}_{PQ}$ restricted to single-parity-violation transitions from a fixed closed gauge cycle (Hamming-distance-1 in $\mathcal{Q}$-space). This explicitly excludes:

- Multi-cell propagation of the virtual $q\bar{q}$ pair across the macroscopic gauge web;
- Coupling to states with two or more simultaneous parity violations (multi-loop excursions);
- Continuous-momentum routing on the macroscopic $L(\mathbb{Z}^3)$ lattice.

Theorem (Threshold Bound State). *For an N-body pure-gauge Fock state on the TCH gauge web, restricted to the local Hilbert space defined above, the substrate-level Feshbach calculation yields a dressed pole exactly at the kinematic threshold $E_{th} = (2N-1)\Lambda_{QCD}$. At this pole position, the tree-level decay phase space $\sqrt{E_{pole} - E_{th}}$ vanishes identically, and the local tree-level decay width is*

$$\Gamma^{(N\text{-}body)}_{tree\text{-}local} \;=\; 0 \tag{7}$$

to all orders in the local coupling.

Proof sketch. The kinematic threshold for single-cycle virtual $q\bar{q}$ nucleation in the N-body Fock state is $E_{\rm th} = (N-1)\cdot 2\Lambda_{\rm QCD}$ (spectator energy of $N-1$ unperturbed cycles) plus $1\Lambda_{\rm QCD}$ (energetic cost of one $\mathbb{F}_2$ parity violation), giving $E_{\rm th} = (2N-1)\Lambda_{\rm QCD}$. The strong-coupling Feshbach machinery of §3 drives the dressed pole exactly to this position (rigorously, to within microscopic regulator corrections). The standard tree-level decay-width formula

$$\Gamma_{\rm tree} \;\propto\; \int d\Pi_n \, |\mathcal{M}|^2 \, \delta(E_{\rm pole} - E_{\rm cont}) \cdot \rho(E_{\rm cont}) \tag{8}$$

vanishes when $E_{\text{pole}} = E_{\text{th}}$ because the density of states $\rho(E_{\text{cont}})$ at threshold has no states below it: the integral collapses to evaluation at $\sqrt{E_{\text{pole}} - E_{\text{th}}} = 0$. □

Physical interpretation. The pure-gauge glueball cannot shed its energy via any single-cell internal-degree-of-freedom process. To decay, the virtual $q\bar{q}$ pair nucleated locally must propagate spatially — traversing gauge bridges to neighbouring cells, eventually reaching asymptotic infinity to form physical $\pi\pi$, $\eta\eta$, $K\bar{K}$, or other meson final states. This propagation is excluded from the local Hilbert space by construction.

Relation to the mass paper. The mass result $m_N^{\text{dressed}} = (2N-1)\Lambda_{\text{QCD}}$ [1] is robust against the Threshold Bound State theorem: the mass is regulator-stable and emerges from the principal value of the resolvent, which is finite and well-defined even when the pole sits exactly at the threshold cliff. The mass result and the zero-tree-level-width result are consistent and complementary statements about the same dressed pole.

5 The Continuum Limit: Brillouin-Zone Formalism

The local-Hilbert-space restriction is what kills the tree-level width. The physical width requires lifting this restriction to the continuum. We formalise this via momentum-dependent promotion of the $\mathcal{Q}$-subspace operator.

From local matrix to Bloch Hamiltonian

The local 208×208 matrix $\mathcal{W}_{\mathcal{Q}\mathcal{Q}}$ describes the internal dynamics of single-cell virtual $q\bar{q}$ excitations. On the macroscopic gauge web $L(\mathbb{Z}^3)$, each cell carries the same internal structure, and the macroscopic translation symmetry of $\mathbb{Z}^3$ admits Bloch decomposition. Promoting the local matrix to a momentum-dependent Bloch Hamiltonian:

$$\mathcal{W}_{\mathcal{Q}\mathcal{Q}} \;\longrightarrow\; \mathcal{W}_{\mathcal{Q}\mathcal{Q}}(\mathbf{k}) \;=\; \mathcal{W}_{\mathcal{Q}\mathcal{Q}}^{\text{local}} + \sum_{\hat{n}} \mathcal{T}_{\hat{n}}\, e^{i\mathbf{k}\cdot\hat{n}a_0}, \tag{9}$$

where $\mathcal{T}_{\hat{n}}$ is the inter-cell hopping operator in direction $\hat{n}$ (taking values along the six nearest-neighbour directions of $\mathbb{Z}^3$), $a_0 = 0.594$ fm is the canonical chiral-scale lattice spacing (ANCHOR [4, §1.4]), and $\mathbf{k}$ runs over the macroscopic Brillouin zone of $\mathbb{Z}^3$.

The resulting Bloch Hamiltonian $\mathcal{W}_{\mathcal{Q}\mathcal{Q}}(\mathbf{k})$ is a 208-band matrix-valued function over the BZ. Its eigenspectrum at each $\mathbf{k}$ defines a 208-band structure $\omega_n(\mathbf{k})$ for $n = 1, \ldots, 208$. The local 4-fold degeneracy at $1\Lambda_{\text{QCD}}$ splits into momentum-dependent bands; for the lowest bands describing single-$q\bar{q}$-pair excitations, the dispersion near band minimum takes the standard effective-mass form

$$\omega_{\min}(\mathbf{k}) \;\approx\; 1\,\Lambda_{\text{QCD}} + \frac{(\hbar k)^2}{2m_*} + O(k^4), \tag{10}$$

where m_* is the effective mass of the virtually-nucleated $q\bar{q}$ pair on the discrete substrate. Crucially, m_* is not a free parameter: in standard discrete tight-binding theory, the effective mass at the band minimum is rigidly fixed by the hopping amplitude t (the dominant matrix element of $\mathcal{T}_{\hat{n}}$) and the lattice spacing a_0 via

$$m_* \;=\; \frac{\hbar^2}{2\,t\,a_0^2}, \tag{11}$$

with $a_0 = 0.594$ fm the canonical chiral-scale lattice spacing (ANCHOR [4, §1.4]) and t the framework-derived inter-cell hopping amplitude of the walk operator $\mathcal{W} = \mathcal{S}\mathcal{C}$. The continuum momentum dispersion is therefore entirely constrained by the discrete Boolean topology of the substrate, with no free continuous parameters: m_* is a substrate-derived quantity computable from the explicit construction of $\mathcal{T}_{\hat{n}}$ in step (1) of the macroscopic-routing programme below. This continuum band structure replaces the 4-fold-degenerate spike of the local calculation.

The macroscopic-routing decay width

The Bloch-Hamiltonian resolvent

$$G_{\mathcal{Q}}(E,\mathbf{k}) \;=\; (E - \mathcal{W}_{\mathcal{Q}\mathcal{Q}}(\mathbf{k}) + i0^+)^{-1} \tag{12}$$

has poles at the band energies $\omega_n(\mathbf{k})$ for each $\mathbf{k}$. The Feshbach self-energy for the dressed glueball state becomes

$$\Sigma(E) \;=\; \int_{\mathrm{BZ}} \frac{d^3k}{(2\pi)^3}\, \mathcal{W}_{P\mathcal{Q}}(\mathbf{k})\, G_{\mathcal{Q}}(E,\mathbf{k})\, \mathcal{W}_{\mathcal{Q}P}(\mathbf{k}), \tag{13}$$

where $\mathcal{W}_{P\mathcal{Q}}(\mathbf{k})$ is the momentum-dependent transition amplitude from the local Fock state to the band-n momentum-$\mathbf{k}$ excitation. The decay width follows from the imaginary part:

$$\Gamma \;=\; -2\,\mathrm{Im}\,\Sigma(E_{\mathrm{pole}}) \;=\; 2\pi \int_{\mathrm{BZ}} \frac{d^3k}{(2\pi)^3} \sum_n |\mathcal{W}_{P\mathcal{Q}}(\mathbf{k})|_n^2\, \delta(E_{\mathrm{pole}} - \omega_n(\mathbf{k})). \tag{14}$$

Near the band minimum, the delta-function constraint and 3D phase-space measure $d^3k = k^2\,dk\,d\Omega$ combine to give the standard $\sqrt{E - E_{\mathrm{th}}}$ behaviour for S-wave decay. For the glueball at $E_{\mathrm{pole}} = (2N-1)\Lambda$ sitting right at threshold, the integrand has support only for $\mathbf{k}$ strictly satisfying $\omega_n(\mathbf{k}) = E_{\mathrm{pole}}$; if the pole is microscopically below the continuum band minimum, the width vanishes; if the pole is at or above the band minimum, the width is positive and computable.

This is exactly the structure that produces the empirical scalar-glueball width $\sim$ 100–300 MeV in continuum lattice QCD: the integral (14) is the framework's substrate-derived analogue of the standard lattice-QCD glueball width calculation.

6 The Macroscopic Routing Programme

The Brillouin-zone formalism converts the decay-width problem into a finite (in principle) computational programme:

1. **Inter-cell hopping operator.** Construct $\mathcal{T}_{\hat{n}}$ explicitly: enumerate gauge-bridge transitions connecting a single-parity-violation state in one cell to the equivalent state in an adjacent cell. For the standard walk operator $\mathcal{W} = \mathcal{S}\mathcal{C}$ on $\mathbb{Z}^3 \otimes Q_3$ (ANCHOR [4, §3.1]), this is a structurally explicit combinatorial calculation.

2. **Bloch Hamiltonian band structure.** Diagonalise $\mathcal{W}_{\mathcal{Q}\mathcal{Q}}(\mathbf{k})$ at each $\mathbf{k}$ in the BZ. The lowest band defines the $q\bar{q}$ continuum threshold as a function of momentum; higher bands correspond to multi-pair and higher-frustration excitations.

3. **Asymptotic state projection.** Project the band excitations onto asymptotic physical meson states ($\pi\pi$, $\eta\eta$, $K\bar{K}$). The framework's existing pion, η, η' derivations (ANCHOR [4, §9.9]) supply the matter-side meson sector; the projection onto these states gives the channel-resolved width contributions.

4. **Phase-space integration.** Evaluate (14) numerically or analytically, with appropriate restriction to the kinematically accessible momenta $\omega_n(\mathbf{k}) = E_{\mathrm{pole}}$.

Steps (1)–(2) are framework-internal combinatorial calculations of comparable difficulty to the mass-result derivations of [1]. Steps (3)–(4) require coupling to the matter sector and standard phase-space integration, both well-developed in continuum lattice gauge theory.

The expected output is a channel-resolved decay width $\Gamma = \sum_{\mathrm{channels}} \Gamma_c$ with absolute magnitude comparable to LQCD predictions ($\sim$ 100–300 MeV for the scalar). Successful execution would lift the framework's glueball description from "parameter-free masses + zero local tree-level width" to "parameter-free masses + parameter-free macroscopic-routing widths" — a complete substrate-level result for the pure-gauge sector.

7 Phenomenological Consequences

The intrinsic non-locality of glueball decay carries specific phenomenological signatures, several of which match long-standing empirical puzzles structurally.

Scalar-tensor width hierarchy via angular momentum

The scalar 0^{++} glueball decays to $\pi\pi$ in S-wave (angular momentum conservation: $J = 0 \to L = 0$ with two spinless mesons), while the tensor 2^{++} glueball decays in D-wave ($J = 2 \to L = 2$). The phase-space measures near threshold differ dramatically:

$$\Gamma^{(\text{scalar})}_{S\text{-wave}} \propto \sqrt{E - E_{\text{th}}}, \qquad \Gamma^{(\text{tensor})}_{D\text{-wave}} \propto (E - E_{\text{th}})^{5/2}. \tag{15}$$

The $(E-E_{\text{th}})^2$ extra centrifugal barrier suppresses the tensor decay rate substantially compared to the scalar near threshold.

The framework's role in this hierarchy is structural: the cycle-topology distinction (3 plaquettes $\to A_{1g} \to J = 0$ for the scalar; 4 Petrie hexagons $\to T_{2g} \to J = 2$ for the tensor) fixes the angular momentum of the decay channel, and the angular-momentum content determines the centrifugal suppression. The empirical reading that tensor glueballs are cleaner, narrower resonances than scalar glueballs (consistent across LQCD and phenomenological analyses) is therefore a structural prediction of the framework's cycle-topology mass-ladder.

Empirical mixing of the f_0 manifold

The scalar glueball candidate has resisted decades of identification efforts: the $f_0(1370)$, $f_0(1500)$, $f_0(1710)$, and $f_0(2020)$ states are all empirically mixed, with no single state cleanly identifiable as "the" glueball. Standard analyses treat this as a strong-coupled-channel mixing problem requiring complex coupled-channel formulations.

The framework provides a structural explanation: *the scalar glueball has no local decay vertex to identify.* The decay process is intrinsically spatially-distributed across the macroscopic gauge web, with the asymptotic //KK final states emerging from the integrated macroscopic-routing integral (14). There is no point in spacetime where the glueball "decays" into a single meson pair; rather, the decay is smeared across the entire integration domain.

This naturally produces a phenomenological signature: the physical resonance structure is intrinsically broad and mixed. Different experimental probes (different production mechanisms, different decay channels) sample different parts of the macroscopic-routing integral, giving the appearance of multiple "glueball candidates" in the empirical scalar sector. The framework predicts that no single f_0 state will ever be cleanly identifiable as the pure glueball, because no such single state exists — the glueball is the full spatially-distributed structure.

Lattice-QCD volume dependence

Standard lattice-QCD calculations of glueball widths exhibit notable sensitivity to lattice volume and operator basis, beyond what is expected for local-vertex resonances. Local-vertex decays have width converging exponentially in lattice volume (finite-volume corrections $\sim e^{-mL}$); non-local decays should have width with stronger volume dependence reflecting the spatial extent of the routing integral.

The framework's substrate-level prediction is testable: anomalous volume dependence of scalar-glueball widths in LQCD calculations (e.g., Athenodorou-Teper extrapolations) is a positive signature of intrinsic non-locality. Existing LQCD literature does report unusual scalar-glueball volume scaling; comparison with framework predictions for the routing-integral cutoff dependence on physical volume would either confirm the non-locality picture or expose a quantitative tension.

8 Falsifiable Signatures

The intrinsic-non-locality claim must be empirically distinguishable from "standard local-vertex with strong mixing." Two observable signatures separate them.

Production-decay angular correlations

A local-vertex glueball has factorised production $\times$ decay: the angular distribution of the decay products is independent of the production kinematics. A spatially-distributed glueball has correlations between production momentum-transfer and decay angular distribution, because the spatial integration over routing channels mixes momentum and angle.

Specifically, in radiative J/ψ decay $J/\psi \to \gamma G$ followed by $G \to \pi\pi$, the photon momentum $\mathbf{q}$ and the pion-pair opening angle $\theta_{\pi\pi}$ should be uncorrelated for a local-vertex glueball but should exhibit measurable correlations for a non-local glueball. The framework would predict a specific correlation pattern from the macroscopic-routing integral evaluated at the experimental kinematic configuration.

This is experimentally challenging but conceptually clean. BES-III and Belle-II data on $J/\psi \to \gamma f_0 \to \gamma\pi\pi$ could in principle be analysed for such correlations; existing analyses have not specifically searched for them.

Lattice volume scaling

A local-vertex glueball has width $\Gamma(L) = \Gamma_\infty + O(e^{-mL})$ in lattice volume L, with exponentially-suppressed finite-volume corrections. A spatially-distributed glueball has $\Gamma(L) = \Gamma_\infty + O(1/L^p)$ for some power p, with power-law volume dependence reflecting the cutoff of the routing integral at the lattice boundary.

The framework's prediction is specific: p depends on the dimensionality of the routing integral. For 3D macroscopic-routing on $\mathbb{Z}^3$, $p = 1$ or $p = 2$ depending on the integration measure. Detailed framework-internal calculation of the BZ-integral asymptotics at finite volume cutoff would yield the specific power.

Comparison against the volume dependence of LQCD scalar-glueball width extractions (where data exists) is the cleanest available falsification test. This work is anchored as the explicit downstream comparison target.

9 Scope and Open Work

What this paper establishes

- The local 4-fold degeneracy of the $\mathcal{Q}$-spectrum at $1\Lambda_{\text{QCD}}$, structurally derived from the four vertices of a single C_4 plaquette and the single-parity-violation transition structure.
- The strong-coupling regime $\xi = |W|^2/\Delta^2 \sim 12$ for the scalar 0^{++} glueball, placing the framework in the deep-threshold-pinning asymptote where the pole rigidly parks at the kinematic threshold.
- The Threshold Bound State Theorem: restricted to the local single-cell Hilbert space, the substrate-level glueball tree-level decay width is identically zero.
- The intrinsic non-locality of physical glueball decay: the empirically observed width arises entirely from macroscopic spatial routing of virtual $q\bar{q}$ pairs across the gauge web.
- The Brillouin-zone formalism for the continuum-limit calculation: $\mathcal{W}_{\mathcal{QQ}} \to \mathcal{W}_{\mathcal{QQ}}(\mathbf{k})$ promotion with explicit decay-width integral (14).

- Phenomenological consequences: scalar-tensor width hierarchy via D-wave angular-momentum suppression, structural explanation of f_0-manifold mixing.
- Falsifiable signatures: production-decay angular correlations in $J/\psi \to \gamma G \to \gamma\pi\pi$, and anomalous lattice-volume scaling of scalar-glueball widths.

What this paper does NOT yet produce

- Numerical values for the physical glueball decay widths. The Brillouin-zone integral (14) requires explicit construction of the inter-cell hopping operator $\mathcal{T}_{\hat{n}}$ and diagonalisation of $\mathcal{W}_{QQ}(\mathbf{k})$, both deferred to a follow-up computational paper.
- Channel-resolved branching ratios for the scalar glueball into $\pi\pi$, $\eta\eta$, $K\bar{K}$, 4π, and other channels. These require projection onto the framework's matter-sector pseudoscalar spectrum and are also deferred.
- Quantitative predictions for the production-decay correlation strength in $J/\psi \to \gamma G \to \gamma\pi\pi$. The structural existence of correlations is predicted; their explicit magnitude requires the full BZ-integral calculation.

Recommended next steps

1. **Construct the inter-cell hopping operator** $\mathcal{T}_{\hat{n}}$ from the canonical walk operator $\mathcal{W} = \mathcal{SC}$ on $\mathbb{Z}^3 \otimes Q_3$. This is a finite combinatorial calculation enumerating gauge-bridge transitions between adjacent cells.
2. **Diagonalise** $\mathcal{W}_{QQ}(\mathbf{k})$ over a discrete sample of the macroscopic Brillouin zone (e.g., $32 \times 32 \times 32$ grid). Extract the band structure; identify the band minimum and effective mass m_* of the lowest $q\bar{q}$ continuum.
3. **Evaluate** (14) numerically at the threshold-bound pole positions $E = (2N-1)\Lambda_{\mathrm{QCD}}$ for $N = 3$ (scalar) and $N = 4$ (tensor). Compare against LQCD width values.
4. **Compute** the lattice-volume scaling of the routing integral by repeating step (3) at varying lattice cutoffs; extract the power-law exponent p and compare against LQCD volume dependence.
5. **Cross-check** against the framework's mass paper's ϵ-stability: the BZ-integral should produce a width with no ϵ-dependence (the continuum smooths the discrete spectrum), confirming that the $1/\epsilon$ artefact of the mass paper's local calculation arose from the discrete spectrum and is resolved by the continuum promotion.

10 Conclusion

The TCH framework's substrate-level structure forces a sharp prediction for glueball decay: at the local single-cell level, the tree-level decay width is identically zero (Threshold Bound State Theorem); the physically observed width arises entirely from non-local macroscopic spatial routing of virtual $q\bar{q}$ pairs across the extended gauge web.

This is a substantive physics claim that distinguishes the framework from standard local-vertex glueball treatments. It carries falsifiable signatures (production-decay correlations, anomalous lattice volume scaling) and structurally explains several long-standing empirical puzzles in the scalar-glueball sector (the impossibility of identifying a single f_0 as the glueball, the systematic broadness of scalar resonances, the difficulty LQCD has had with width extractions).

The decay width is therefore not a phenomenological accident but a structural observable: it measures the framework's macroscopic-routing integral on $L(\mathbb{Z}^3)$, with the routing distance set by the propagation extent of virtual $q\bar{q}$ pairs from local nucleation to asymptotic hadronisation. The decay is non-local in spacetime, not merely strongly-mixed in Hilbert space — and the difference between these two pictures is empirically distinguishable.

The framework's glueball description, taken together with the mass paper, now stands as:

- *Mass*: universal formula $m_N^{\text{dressed}} = (2N-1)\Lambda_{\text{QCD}}$, confirmed at LQCD-band precision for $N = 3$ and $N = 4$.

- *Width*: zero at local tree-level (Threshold Bound State); finite physical width arising entirely from macroscopic routing (intrinsic non-locality).

- *Phenomenology*: scalar broader than tensor (D-wave centrifugal suppression of tensor decay); f_0 manifold structurally explained as the absence of a single local decay vertex.

- *Falsifiability*: production-decay angular correlations in $J/\psi \to \gamma G$; anomalous lattice volume scaling of scalar widths.

The remaining computational work is the explicit Brillouin-zone integration to produce numerical width predictions and the comparison against LQCD finite-volume data. These are concrete targets following standard lattice field theory machinery applied to the framework's substrate-derived Bloch Hamiltonian.

References

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