

Gauss Annihilators and Chart Functoriality in Finite SL_3 Lattice Gauge Algebras

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Abstract

For a finite connected oriented graph C , let the vertex gauge group act on the affine SL_3 link carrier, let $\mathcal{D}_C$ be its ring of differential operators, and let B_C be the invariant polynomial ring. We determine whether $\text{Ann}_{\mathcal{D}_C}(B_C)$ contains differential constraints beyond the left ideal generated by the infinitesimal Gauss operators. For every graph of cycle rank at least five, graph-theoretic codimension estimates prove that the gauge action is 2-large. The moment components form a regular sequence, and their zero shell is a prime normal complete intersection. A strict filtered Spencer resolution makes the associated graded Gauss quotient explicit, while a general invariant-theoretic annihilator theorem gives equality of the invariant annihilator and the Gauss left ideal. We then prove a lattice-specific functoriality theorem. Under an admissible enlargement of graph charts, both ideals contract exactly to their small-chart counterparts on the complete restriction normalizer, even though Gauss fields at boundary vertices do not generally embed by direct inclusion. These restriction maps compose on their natural domains and are equivariant under graph relabelling.

Keywords: lattice gauge theory; Gauss law; invariant differential operators; moment maps; Hamiltonian reduction; filtered algebras.

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1 Introduction

Finite lattice gauge theory begins with link variables and vertex gauge transformations. A basic algebraic question is whether the infinitesimal Gauss fields generate every differential equation satisfied by the polynomial gauge invariants. Writing $B_C = A_C^{H_C}$, the evident inclusion is

$$\mathcal{D}_C \mu_C(\mathfrak{h}_C) \subseteq \text{Ann}_{\mathcal{D}_C}(B_C),$$

but equality is not formal. It can fail in elementary scaling actions when a local equality on the principal locus acquires a global correction supported on a singular stratum. Schwarz proved that equality does hold for every 1-large smooth affine action, in the more general setting of admissible equivariant bundles [1, Theorem 9.9(1)]. The first purpose of this paper is therefore not to claim a new general annihilator theorem. It is to verify Schwarz's largeness hypotheses uniformly for the present graph actions, make the singular-shell mechanism explicit in lattice variables, and prove the lattice-specific compatibility with enlargement and restriction of charts.

The paper's main completed conclusions are:

1. a uniform 2-largeness theorem for the finite SL_3 graph actions, which places their annihilator equality under Schwarz's theorem;
2. a prime normal complete-intersection theorem for the zero-moment shell and a strict filtered Spencer resolution of the Gauss quotient;
3. the equality of the full invariant annihilator and the Gauss left ideal; and
4. exact functorial restriction of both ideals on the complete normalizer domain, with composition and relabelling naturality.

The first three statements identify the global algebraic mechanism, including the singular strata on which a localization-only argument could miss a correction term. The fourth addresses a separate lattice issue: adjoining links changes the infinitesimal Gauss field at an old boundary vertex, so chart compatibility must be formulated through restriction rather than direct inclusion.

1.1 Relation to existing work

Hamiltonian lattice gauge theory starts from link variables, electric operators, and local Gauss constraints [2, 3]. Gauge-invariant descriptions have since been developed through spin networks [4], local multiplet bases [5], and loop-string-hadron variables for SU_3 [6]. A recent review of quantum simulators and a current non-Abelian gauge-protection proposal make the practical cost of isolating and preserving the physical gauge sector especially visible [7, 8]. These developments sharpen the distinction addressed here: the present paper does not propose a Hilbert-space truncation, circuit compilation, or protection protocol. It asks the prior untruncated algebraic question of whether the polynomial gauge-invariant sector satisfies differential constraints not generated by the infinitesimal Gauss operators.

After a spanning-tree reduction, the invariant ring is closely related to the simultaneous-conjugation invariants of matrix tuples. Trace-word generation and the geometry of the corresponding quotients belong to classical matrix and quiver invariant theory [9, 10], with the SL_3 free-group case developed explicitly in [11, 12]. Our use of this reduction is deliberately limited: it establishes orbit geometry, while the annihilator theorem and the restriction maps are proved on the full link carrier.

The moment-shell argument belongs to the theory of singular Hamiltonian reduction, beginning with symplectic reduction and its singular extensions [13, 14]. Largeness conditions, Koszul exactness, and singularities of zero-moment shells have been studied in [15, 16, 17]. Those results provide the closest geometric framework for the normal complete-intersection shell proved here.

There is also an important distinction between differential operators on an invariant ring, invariant differential operators upstairs, and operators that annihilate the invariant ring. For the last of these, Schwarz's [1, Theorem 9.9(1)] is decisive. If $\mathcal{K}_E^m(X)$ denotes the order-at-most- m operators on sections of E that annihilate the G -invariant sections, his formula for a 1-large smooth affine G -variety and an admissible bundle is

$$\mathcal{K}_E^m(X) = \mathcal{D}_E^{m-1}(X)\mathfrak{g}, \quad m \geq 0,$$

with $\mathcal{D}_E^{-1}(X) = 0$. With E the trivial line bundle, the union over m is exactly the equality of the full invariant annihilator with the left ideal generated by the infinitesimal action. Since the graph actions below are 2-large, their annihilator equality is an application of this theorem, not a new instance of an unknown general implication.

Related results answer different questions. Schwarz also studies lifting invariant differential operators to the quotient, while Levasseur and Stafford identify a commutant for the adjoint

representation [1, 18]. Quantum Hamiltonian reduction usually passes to invariant quotients and studies properties such as generic simplicity [19]. None of those distinctions should obscure the direct overlap with Schwarz's annihilator theorem. What is specific here is instead the uniform graph-theoretic verification of largeness, the arithmetic and singular-shell realization, and the normalizer theorem governing restriction across graph charts. The final result is needed because adjoining links changes the Gauss field at an old boundary vertex, so the small-chart operator is generally not obtained by direct inclusion.

2 Algebraic setup

All algebraic statements are over $\overline{\mathbb{Q}}$, unless an explicit scalar extension to $\mathbb{C}$ is displayed. A chart C consists of a finite connected oriented graph with vertex set V_C , edge set E_C , and cycle rank

$$r_C = |E_C| - |V_C| + 1.$$

Throughout the main theorems we assume $r_C \geq 5$. Put

$$Y_C = SL_3^{E_C}, \quad H_C = SL_3^{V_C}, \quad A_C = \mathcal{O}(Y_C), \quad \mathcal{D}_C = \mathcal{D}(A_C).$$

The gauge action is

$$(g \cdot U)_e = g_{s(e)} U_e g_{t(e)}^{-1}.$$

Its finite kernel is the diagonal μ_3 , which has no effect on the infinitesimal Lie algebra action. Let

$$\mu_C : \mathfrak{h}_C \longrightarrow \mathcal{D}_C$$

denote that infinitesimal action. We use the convention $PQ = P \circ Q$, with Q applied first.

Definition 2.1. The *Gauss left ideal*, the invariant ring, and the invariant annihilator are

$$\mathfrak{J}_C = \mathcal{D}_C \mu_C(\mathfrak{h}_C), \quad B_C = A_C^{H_C}, \quad \mathfrak{K}_C = \{P \in \mathcal{D}_C : Pb = 0 \text{ for every } b \in B_C\}.$$

The order filtration on $\mathcal{D}_C$ identifies $\text{gr } \mathcal{D}_C \cong \mathcal{O}(T^*Y_C)$. The symbols of the Gauss fields are the components of the cotangent moment map. We write

$$N_C = \mu_{T^*Y_C}^{-1}(0), \quad I(N_C) \subseteq \mathcal{O}(T^*Y_C).$$

3 The induced carrier and 2-largeness

The residual orbit geometry is simultaneous conjugation of the chord holonomies; we use this only as a device for proving orbit estimates on the full link carrier. Choose a root and a spanning tree. Tree transport is algebraic because inversion on SL_3 is polynomial modulo the determinant-one relation. Gauge fixing gives an H_C -equivariant induced-space isomorphism

$$Y_C \cong H_C \times^{SL_3} SL_3^{r_C},$$

where the residual SL_3 acts by simultaneous conjugation. This is the matrix-tuple and free-group character-variety setting of [9, 11, 12].

If $S \subseteq SL_3^{r_C}$ is a residual stabilizer stratum, the corresponding full-link stratum is $H_C \times^{SL_3} S$. Both induced spaces are locally trivial over H_C/SL_3 with fibres S and $SL_3^{r_C}$, respectively. Consequently

$$\text{codim}_{Y_C}(H_C \times^{SL_3} S) = \text{codim}_{SL_3^{r_C}} S.$$

Thus the chord-carrier estimates below transfer without loss to the original full-link action.

We recall the conventions used below from [1, Definitions 9.5 and Remark 9.6(3)]. For a smooth affine G -variety X , put $X_{(s)} = \{x \in X : \dim G_x = s\}$. The action has *finite principal isotropy groups* (FPIG) when its principal stabilizers are finite. Assuming FPIG, it is k -principal when $\text{codim}(X \setminus X_{\text{pr}}) \geq k$, and k -modular when

$$\text{codim}_X X_{(s)} \geq s + k \quad (s \geq 1).$$

It is k -large when both conditions hold. A G -equivariant vector bundle $E \rightarrow X$ is *admissible* when a principal stabilizer G_x acts trivially on E_x ; the trivial line bundle is always admissible [1, Definition 2.6 and Remark 2.7(1)].

Proposition 3.1 (Two-principal). *The effective gauge action has finite principal isotropy and the complement of the principal locus has codimension at least $2r_C - 2 \geq 8$.*

Proof. For simultaneous conjugation, a tuple is principal exactly when it is irreducible. Indeed, Burnside's theorem gives the full matrix algebra and a scalar commutant for an irreducible tuple, while a reducible tuple preserves a line or a plane and lies outside the principal locus; compare [9, 12]. For either choice of subspace the relevant parabolic subgroup has dimension six, and the parameter space of lines or planes has dimension two. Hence the reducible locus has dimension at most $6r_C + 2$ in the $8r_C$ -dimensional chord carrier. The irreducible orbit is closed and its stabilizer is precisely the finite central kernel. $\square$

Proposition 3.2 (Two-modular). *For every positive stabilizer dimension s , the corresponding source stratum has codimension at least $s + 2$.*

Proof. If a tuple has positive-dimensional stabilizer, some nonzero $a \in \mathfrak{sl}_3$ commutes with every chord holonomy. The centralizer of any nonzero $a \in \mathfrak{sl}_3$ has dimension at most four. After projectivizing a , the incidence space has dimension at most $7 + 4r_C$. Its codimension is therefore at least $4r_C - 7 \geq 13$. Since $s \leq 8$, this is at least $s + 2$. $\square$

Corollary 3.3. *The full link representation is 2-large for every chart under consideration.*

Proof. The preceding propositions give the two-principal and two-modular bounds on the residual chord carrier. The induced-space codimension identity transports the same bounds to Y_C . The diagonal μ_3 is finite, so passing to the effective gauge group changes neither stabilizer dimensions nor codimensions. $\square$

The threshold $r_C \geq 5$ is a safe uniform threshold for this proof, not an optimality claim. It makes the coarse modular estimate $4r_C - 7 \geq 13 \geq s + 2$ valid using only $s \leq \dim SL_3 = 8$; lower cycle ranks would require a finer stabilizer-by-stabilizer estimate.

4 The moment shell and the filtered Gauss quotient

Let $n = \dim Y_C$ and $d = \dim H_C$. Above a source point with stabilizer dimension s , the zero-moment cotangent fibre has dimension $n - d + s$. Two-modularity puts the non-finite-stabilizer part of the shell in codimension at least two. Above the finite-stabilizer locus the shell has the expected dimension $2n - d$.

Theorem 4.1 (Prime normal moment shell). *The d moment components form a regular sequence in $\mathcal{O}(T^*Y_C)$. The zero shell N_C is an irreducible, reduced, normal complete intersection, and $I(N_C)$ is prime.*

The conclusion is consistent with the general role of largeness hypotheses in the geometry of zero-moment shells [15, 16, 17]; the proof below verifies the needed properties for the present graph representation rather than importing them formally.

Proof. Let $U \subseteq Y_C$ be the locus with zero-dimensional infinitesimal stabilizer. It is a nonempty open subset of the irreducible variety Y_C , and is therefore irreducible. For $y \in U$, the infinitesimal orbit map $\mathfrak{h}_C \rightarrow T_y Y_C$ is injective, so its dual $T_y^* Y_C \rightarrow \mathfrak{h}_C^*$ is surjective. Therefore $N_C|_U \rightarrow U$ is the kernel of a surjective vector-bundle map. It is an irreducible vector bundle of rank $n - d$ and dimension $2n - d$.

For a source stratum with stabilizer dimension $s > 0$, two-modularity bounds its dimension by $n - s - 2$. Its zero-moment cotangent fibres have dimension $n - d + s$, so the part of N_C above that stratum has dimension at most $2n - d - 2$. Hence N_C has dimension $2n - d$, and the d moment components have codimension d in the regular ring $\mathcal{O}(T^* Y_C)$; regularity here follows because $T^* Y_C$ is smooth. They consequently form a regular sequence, and N_C is a Cohen–Macaulay, unmixed complete intersection.

Every irreducible component of an unmixed complete intersection has dimension $2n - d$. No component can be contained in the complement of $N_C|_U$, whose dimension is at most $2n - d - 2$. Since $N_C|_U$ is irreducible, its closure is therefore all of N_C ; thus N_C is irreducible. It is smooth over U , and the possible singular locus has codimension at least two. The complete intersection is S_2 , while smoothness in codimension one gives R_1 . Serre’s criterion makes N_C normal, hence reduced. Its irreducibility and reducedness imply that $I(N_C)$ is prime. $\square$

Consider the Chevalley–Eilenberg/Spencer complex

$$\mathcal{D}_C \otimes \bigwedge^\bullet \mathfrak{h}_C.$$

For $P \in \mathcal{D}_C$ and $x_1, \dots, x_p \in \mathfrak{h}_C$, its differential is

$$\begin{aligned} \partial(P \otimes x_1 \wedge \cdots \wedge x_p) &= \sum_i (-1)^{i+1} P \mu_C(x_i) \otimes x_1 \wedge \cdots \widehat{x_i} \cdots \wedge x_p \\ &\quad + \sum_{i < j} (-1)^{i+j} P \otimes [x_i, x_j] \wedge x_1 \wedge \cdots \widehat{x_i} \cdots \widehat{x_j} \cdots \wedge x_p, \end{aligned}$$

with the usual convention of moving the bracket to the displayed first position. Give the degree- p term the shifted filtration $F_m = F_{m-p} \mathcal{D}_C \otimes \bigwedge^p \mathfrak{h}_C$. The bracket terms then lower filtered order, so the associated graded differential is multiplication by the symbols of the Gauss fields. Thus the associated graded complex is the algebraic Koszul complex of the moment components. The broader relation between largeness and moment-map Koszul complexes is discussed in [15]; only the algebraic regular-sequence argument is used here.

Theorem 4.2 (Strict filtered Spencer resolution). *The filtered Spencer complex is strict and exact in positive degrees. In particular,*

$$\mathrm{gr}(\mathcal{D}_C/\mathfrak{J}_C) \cong \mathcal{O}(N_C), \quad \mathrm{gr} \mathfrak{J}_C = I(N_C).$$

Proof. By Theorem 4.1, the associated graded complex is the exact Koszul complex of a regular sequence. The standard filtered-complex lemma, applied successively in each finite filtration degree, lifts boundaries without increasing their filtration and shows that every differential is strict. It also gives vanishing of positive homology in the filtered complex. In degree zero the image is exactly $\mathfrak{J}_C$, so the induced quotient filtration has associated graded $\mathcal{O}(T^* Y_C)/I(N_C) = \mathcal{O}(N_C)$; strictness of the degree-one map also gives $\mathrm{gr} \mathfrak{J}_C = I(N_C)$. $\square$

5 No hidden differential constraints

The annihilator equality already follows at this point from a general theorem. We state the specialization explicitly so that the relation to the literature is part of the proof rather than only a bibliographic remark.

Proposition 5.1 (Specialization of Schwarz's annihilator theorem). *For every connected chart with $r_C \geq 5$,*

$$\mathfrak{K}_C = \mathfrak{J}_C.$$

Proof. First extend scalars from $\overline{\mathbb{Q}}$ to $\mathbb{C}$. Smooth base change gives

$$\mathcal{D}(A_C) \otimes_{\overline{\mathbb{Q}}} \mathbb{C} \simeq \mathcal{D}(A_C \otimes_{\overline{\mathbb{Q}}} \mathbb{C}),$$

and reductivity in characteristic zero gives

$$A_C^{H_C} \otimes_{\overline{\mathbb{Q}}} \mathbb{C} \simeq (A_C \otimes_{\overline{\mathbb{Q}}} \mathbb{C})^{H_C \otimes_{\overline{\mathbb{Q}}} \mathbb{C}}.$$

The finite ineffective centre does not change the infinitesimal action. By the 2-largeness corollary, the complexified action is 1-large, and the trivial equivariant line bundle is admissible. With $X = Y_C \otimes_{\overline{\mathbb{Q}}} \mathbb{C}$ and $E = \mathcal{O}_X$, Schwarz's exact formula [1, Theorem 9.9(1)] is

$$\mathcal{K}_E^m(X) = \mathcal{D}_E^{m-1}(X)(\mathfrak{h}_C \otimes_{\overline{\mathbb{Q}}} \mathbb{C}), \quad m \geq 0,$$

where $\mathcal{K}_E^m(X)$ is the order-at-most- m annihilator of the invariant sections and $\mathcal{D}_E^{-1}(X) = 0$. For the trivial bundle the right-hand side is the order-filtered part of the left ideal generated by $\mu_C(\mathfrak{h}_C \otimes_{\overline{\mathbb{Q}}} \mathbb{C})$. Taking the union over m gives the desired equality after complexification. The inclusion $\mathfrak{J}_C \subseteq \mathfrak{K}_C$ is formal. For the reverse inclusion, the class of any $P \in \mathfrak{K}_C$ becomes zero in $(\mathcal{D}_C/\mathfrak{J}_C) \otimes_{\overline{\mathbb{Q}}} \mathbb{C}$; faithful flatness of $\mathbb{C}/\overline{\mathbb{Q}}$ makes its original class zero. Hence $P \in \mathfrak{J}_C$. $\square$

5.1 A graph-adapted saturation check

The following coordinate calculation records why localization at the principal-cover minors introduces no torsion into the Gauss quotient. It is not used as a second proof of the annihilator theorem; its purpose is to keep the exclusion of singular-stratum correction terms visible and to prepare the normalizer analysis used for chart restriction.

The principal locus admits a finite intrinsic cover. In the residual chord carrier, words of length at most eight suffice to decide whether the generated subalgebra is all of M_3 : a nonstable word span must increase dimension, and its dimension can increase at most eight times before reaching nine. Let $\mathcal{F}_C$ be the finite family of determinants of all nine-word subsets. Their nonvanishing loci cover the principal locus.

For $f \in \mathcal{F}_C$, define

$$\text{Con}_f(\mathfrak{J}_C) = (\mathcal{D}_C[f^{-1}]\mathfrak{J}_C) \cap \mathcal{D}_C.$$

Lemma 5.2 (No cover torsion). *For every $f \in \mathcal{F}_C$,*

$$\text{Con}_f(\mathfrak{J}_C) = \mathfrak{J}_C.$$

Proof. Put $M_C = \mathcal{D}_C/\mathfrak{J}_C$. The zero section $z : Y_C \rightarrow N_C$ satisfies $z^*f = f$, so each nonzero cover minor remains nonzero in $\mathcal{O}(N_C)$. Since $\mathcal{O}(N_C)$ is a domain, f is a non-zero-divisor on $\text{gr } M_C$.

If $f^n[P] = 0$ in M_C , choose a nonzero such class of least filtered order. Passing to the associated graded gives $f^n\sigma(P) = 0$ in $\text{gr } M_C \cong \mathcal{O}(N_C)$, hence $\sigma(P) = 0$, contradicting minimality. Thus M_C has no f -power torsion, which is equivalent to the stated contraction equality. $\square$

Remark 5.3. The annihilator conclusion proved above is global. It includes singular and nonprincipal strata and does not follow merely from equality after localizing on the principal locus. The affine scaling action, where $D\partial \supsetneq D(x\partial)$ globally although the two ideals agree after inverting x , is the minimal warning example.

6 Functorial restriction across charts

Definition 6.1. An *admissible chart embedding* $i : C \hookrightarrow C'$ consists of injective maps on vertices and oriented edges that preserve source, target, and orientation. Thus the old links form a named coordinate subfamily of the large link carrier. The projection

$$\pi_i : Y_{C'} \longrightarrow Y_C$$

forgets the new link coordinates, and the *cylindrical pullback* is the injective algebra map $j_i = \pi_i^* : A_C \hookrightarrow A_{C'}$.

Lemma 6.2 (Invariant cylindrical pullback). *For every admissible embedding,*

$$j_i(B_C) \subseteq B_{C'}.$$

Proof. The action of $H_{C'}$ on an old link uses only the gauge factors at its old endpoints. On the old coordinate subfamily it therefore factors through the restriction homomorphism $H_{C'} \rightarrow H_C$; gauge factors at new vertices act trivially. If $b \in B_C$, then $g \cdot j_i(b) = j_i((g|_{V_C}) \cdot b) = j_i(b)$ for every $g \in H_{C'}$. $\square$

Direct inclusion of Gauss fields is generally false at an old boundary vertex because new incident links contribute new derivative terms. The correct object is the complete restriction normalizer

$$\mathcal{N}_i = \{P' \in \mathcal{D}_{C'} : P'(j_i A_C) \subseteq j_i A_C\}.$$

For $P' \in \mathcal{N}_i$, define

$$\rho_i(P') = j_i^{-1} P' j_i, \quad \mathcal{Z}_i = \ker \rho_i.$$

The insistence on an explicitly typed normalizer reflects the general subtlety of lifting or restricting differential operators across invariant quotients [1, 18].

Proposition 6.3 (Normalizer restriction). *The set $\mathcal{N}_i$ is a unital subalgebra, and $\rho_i : \mathcal{N}_i \rightarrow \mathcal{D}_C$ is a filtered unital algebra map.*

Proof. Closure under sums and products is immediate from preservation of $j_i A_C$. To prove differential-operator typing, induct on Grothendieck order using

$$[\rho_i(P'), m_a] = \rho_i([P', m_{j_i(a)}]).$$

At order zero, preservation of cylinders forces multiplication by an element of $j_i(A_C)$. The displayed commutator identity therefore shows inductively that $\rho_i(P')$ has finite Grothendieck order, hence lies in $\mathcal{D}_C$. Multiplicativity follows by evaluation on arbitrary $a \in A_C$. $\square$

Theorem 6.4 (Functorial Gauss restriction). *For every admissible embedding,*

$$\rho_i(\mathfrak{J}_{C'} \cap \mathcal{N}_i) = \mathfrak{J}_C, \quad \rho_i(\mathfrak{K}_{C'} \cap \mathcal{N}_i) = \mathfrak{K}_C.$$

Consequently

$$\frac{\mathfrak{J}_{C'} \cap \mathcal{N}_i}{\mathfrak{J}_{C'} \cap \mathcal{Z}_i} \cong \mathfrak{J}_C, \quad \frac{\mathfrak{K}_{C'} \cap \mathcal{N}_i}{\mathfrak{K}_{C'} \cap \mathcal{Z}_i} \cong \mathfrak{K}_C.$$

Proof. Fix a basis x_a of $\mathfrak{sl}_3$, and let $L_{e,a}$ and $R_{e,a}$ denote the left- and right-invariant vector fields in the coordinate of edge e . Put $E_{\text{new}} = E_{C'} \setminus E_C$. At an old vertex v , equality of vector fields on $Y_{C'}$ reads

$$\begin{aligned} \mu_{C'}(x_{i(v),a}) &= \widetilde{\mu_C(x_{v,a})} + \sum_{\substack{e \in E_{\text{new}} \\ s(e)=i(v)}} L_{e,a} \\ &\quad - \sum_{\substack{e \in E_{\text{new}} \\ t(e)=i(v)}} R_{e,a}. \end{aligned}$$

The two sums differentiate only new-link coordinates and therefore kill every cylinder. Thus the full large field lies in $\mathcal{N}_i$ and restricts to the small field. Given an arbitrary finite expression

$$P = \sum_{v,a} Q_{v,a} \mu_C(x_{v,a}) \in \mathfrak{J}_C,$$

use the product decomposition $Y_{C'} \cong Y_C \times SL_3^{E_{C'} \setminus E_C}$ to lift $Q_{v,a}$ as $\tilde{Q}_{v,a} = Q_{v,a} \otimes 1$. Each lifted coefficient preserves cylinders and restricts to $Q_{v,a}$. Multiplying it by the full large Gauss field at $i(v)$ gives an element of $\mathfrak{J}_{C'} \cap \mathcal{N}_i$; summing over (v, a) produces a preimage of P . Thus $\mathfrak{J}_C \subseteq \rho_i(\mathfrak{J}_{C'} \cap \mathcal{N}_i)$.

For the reverse inclusion, let $P' \in \mathfrak{J}_{C'} \cap \mathcal{N}_i$. By Proposition 5.1 on C' , the operator P' lies in $\mathfrak{K}_{C'}$. Lemma 6.2 shows that it kills $j_i(B_C)$, so $\rho_i(P')$ kills B_C . Applying Proposition 5.1 on C gives $\rho_i(P') \in \mathfrak{K}_C = \mathfrak{J}_C$. This proves the first equality.

The same argument gives $\rho_i(\mathfrak{K}_{C'} \cap \mathcal{N}_i) \subseteq \mathfrak{K}_C$. Conversely, $\mathfrak{K}_C = \mathfrak{J}_C$, and the Gauss-ideal construction above lifts every element of $\mathfrak{J}_C$ to $\mathfrak{J}_{C'} \cap \mathcal{N}_i \subseteq \mathfrak{K}_{C'} \cap \mathcal{N}_i$. Hence the annihilator equality also holds. The two quotient statements now follow from the first isomorphism theorem. $\square$

Proposition 6.5 (Composition and naturality). *For $C \xrightarrow{i} C' \xrightarrow{k} C''$, on the complete domain*

$$P'' \in \mathcal{N}_k, \quad \rho_k(P'') \in \mathcal{N}_i,$$

one has

$$\rho_{k \circ i}(P'') = \rho_i(\rho_k(P'')).$$

The construction is natural under graph isomorphisms and under reversal of an edge orientation accompanied by the coordinate change $U_e \mapsto U_e^{-1}$.

Proof. After applying the injective inclusion $j_k j_i$, both sides act on $a \in A_C$ as $P'' j_k j_i(a)$. A graph isomorphism merely permutes vertex and edge labels. Orientation reversal interchanges source and target, while inversion is an algebraic automorphism of SL_3 and intertwines the two gauge actions. These coordinate changes therefore conjugate the corresponding normalizers and restriction maps. $\square$

7 Conclusion

For every finite connected oriented graph of cycle rank at least five, the vertex gauge action on the affine SL_3 link carrier is 2-large. The proof uses the spanning-tree induced carrier to reduce the stabilizer estimates to simultaneous conjugation of chord holonomies, and then transports the resulting codimensions back to the full link representation. The cotangent moment components form a regular sequence; their zero shell is an irreducible normal complete intersection with prime ideal; and the filtered Spencer complex resolves the Gauss quotient strictly.

These geometric facts place the graph actions within Schwarz’s annihilator theorem and give the global equality

$$\mathfrak{K}_C = \mathfrak{J}_C.$$

The equality includes the singular and nonprincipal strata. The independent no-cover-torsion calculation explains why principal-locus localization cannot hide an additional correction supported on their complement.

Chart enlargement requires a different argument. At an old boundary vertex, new links change the large-chart Gauss field, so direct inclusion of the small-chart generators is generally unavailable. On the complete restriction normalizer, however, cylindrical pullback preserves invariant functions and both the Gauss ideal and the full invariant annihilator restrict exactly. These maps compose on their natural nested-normalizer domains and are natural under graph relabelling. Thus the annihilator equality is compatible with finite refinement in the precise sense available to differential operators.

The results are structural and do not depend on a particular finite lattice, a Hilbert-space truncation, or a low-order computation. More specialized electric–magnetic subalgebras lead to additional filtered and computational questions, but those lie beyond the scope of the present paper.

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Data availability

No datasets were generated or analysed for this study. The proofs are contained in the article. The LaTeX source will be deposited in a permanent public repository before publication and is available from the author to editors and referees on request in the interim.

Declaration of generative AI and AI-assisted technologies in the manuscript preparation process

During the preparation and verification of this work, the author used OpenAI Codex for the operation of a structured consistency framework. The framework tracked claim status and dependencies, cross-checked notation and citations, and prevented open, rejected, or superseded intermediate claims from being promoted into the manuscript. The author produced reviewed and edited the content, the mathematical arguments and cited sources, and takes full responsibility for the content of the published article. A final pass of Codex was used to check for errors in the mathematics, grammar, spelling and literature organisation which identified a few corrections.

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