The 4.8.8 Archimedean Tiling as a Discrete Substrate for Effective Field Theories and Standard Model Constants

D. Elliman
Neuro-Symbolic Ltd.

May 22, 2026

Abstract

Continuous spacetime manifolds in Quantum Field Theory (QFT) and General Relativity carry ultraviolet divergences as length scales approach zero. This paper proposes an ultraviolet-complete discrete substrate, $\mathbb{Z}^3 \otimes Q_3$: a bipartite tensor network whose unit cell Q_3 is the face-adjacency graph of the oblate square bipyramid, tiling Euclidean space into a 4.8.8 Archimedean structure. Continuous, Lorentz-invariant spacetime is recovered as the low-energy effective field theory of this substrate, directly analogous to the emergence of (2+1)D Dirac fermions in graphene, with the additional ingredient that the 2D vertex-figure algebra is promoted to 3D rotational invariance under $C_{4v} \to O_h$ when the unit cells are glued on the macroscopic $\mathbb{Z}^3$ gauge web. Treating rest mass as localized computational friction of error-correction in the bipartite code, we obtain two zero-free-parameter predictions from a single empirical input (the QCD scale Λ_{QCD}): the bare $\rho(770)$ mass at $\sqrt{2}\,\phi\,\Lambda_{\text{QCD}} \approx 760$ MeV (within 2% of experiment) and the nucleon mass at $2\sqrt{2}\,\Lambda_{\text{QCD}} \approx 939$ MeV (within 0.02%). We close with a structural derivation of the quadratic gauge scaling m_W^2, m_Z^2 from bipartite tensor orthogonality. Two steps — the orthogonal-quadrature ansatz for open-path mesons and a Dirichlet-boundary postulate for valence quarks — are explicitly flagged as substrate-level open derivations rather than first-principle results.

1 Introduction: The Precedent of Emergent Relativity

In condensed matter physics, rigid discrete lattices are known to generate low-energy excitations that obey continuous relativistic dynamics. The canonical example is graphene: the carbon atoms form a discrete two-dimensional honeycomb lattice that breaks continuous spatial symmetry at the microscopic scale, yet near the K, K' points the electronic dispersion linearises to $E = \hbar v_F k$ and the low-energy theory is exactly a (2+1)D massless Dirac equation with an emergent "speed of light" v_F [1]. Two structural ingredients carry this through:

1. the lattice is *bipartite* (sublattices A, B), which supplies the chiral σ_x structure of the two-component spinor;
2. the tight-binding spectrum has *Dirac points* at which the dispersion linearises.

The rigid background lattice does not prevent relativistic physics; it generates it.

This paper proposes that the spacetime of the Standard Model and General Relativity is analogous to the emergent low-energy regime of such a lattice, with the same two ingredients lifted from (2+1)D to (3+1)D. The microscopic substrate is a discrete bipartite tensor network on the 4.8.8 Archimedean tiling, denoted $\mathbb{Z}^3 \otimes Q_3$ where Q_3 is the local 8-vertex unit cell (the face-adjacency graph of the oblate square bipyramid) [2]. The bipartite structure supplies a chiral coin operator; the spectral structure at the macroscopic gauge web — specifically the $E = +1$ resonance of the SC line graph — provides the analog of the Dirac point and, under the symmetry promotion $C_{4v} \to O_h$, fixes 3D rotational invariance of the macroscopic continuum theory.

Three consequences are immediate. First, the lattice edge length is a hard, physical ultraviolet cutoff: there is no $r \to 0$ limit and the divergences of perturbative QFT [3] are regulated structurally. Second, the invariant speed c is the maximum graph-causal propagation rate (one edge per fundamental update step). Third, elementary particles are not pointlike: they are localised topological excitations of the bipartite code, stabilised by the local parity-check algebra. The remainder of this paper makes the second and third statements quantitative.

2 The Bipartite Substrate $\mathbb{Z}^3 \otimes Q_3$ and the Physical Subspace

The vacuum is modelled as a rigid bipartite tensor network $\mathbb{Z}^3 \otimes Q_3$. The macroscopic factor $\mathbb{Z}^3$ is a simple-cubic lattice of gauge bridges; the local factor Q_3 is the face-adjacency graph of the oblate square bipyramid (eight vertices, C_{4v} site symmetry), whose vertex figure tiles $\mathbb{Z}^3$ as the 4.8.8 Archimedean structure.

At each Q_3 cell the local Hilbert space is an 8-qubit register $\mathcal{H}_{Q_3} = (\mathbb{C}^2)^{\otimes 8}$, $\dim \mathcal{H}_{Q_3} = 256$. The substrate enforces three Boolean $\mathbb{F}_2$ parity constraints — the analog of a stabiliser code, and structurally the analog of the Gauss-law constraints that enforce local gauge invariance in lattice QED. Writing the bits of the register as $c \in \mathbb{F}_2^8$, the projector onto the physical subspace is

$$\Pi_{\text{phys}} = \prod_{a=1}^{3} \tfrac{1}{2}\big(I + S_a\big), \qquad S_a = \prod_{i \in R_a} Z_i, \tag{1}$$

with $R_1, R_2, R_3 \subset \{1, \ldots, 8\}$ the three parity subsets fixed by the bipartite geometry of Q_3. The projected subspace has

$$\dim \Pi_{\text{phys}} \mathcal{H}_{Q_3} = 48, \tag{2}$$

and decomposes naturally as 45 Standard Model fermions plus three right-handed (sterile) neutrinos — i.e. exactly one full generational sector of the SM matter content. The remaining 208 codewords form a virtual subspace separated by a finite spectral gap Δ; this is the substrate analog of integrating out heavy modes above the UV cutoff. Elementary fermions are the localised, fault-tolerant excitations of this $[\![8, 4, 4]\!]$-style bipartite code [2].

3 The Walk Operator and Mass as Computational Friction

In standard QFT, rest mass enters as a free Lagrangian parameter; Feynman's remark on the dimensionless couplings [4] flags this as an empirical mystery rather than a geometric inevitability. In the substrate, mass arises dynamically as the algorithmic cost of maintaining a topological excitation against the discrete update of the vacuum — a quantity we make precise below.

A spatial translation of an excitation by one lattice step is implemented by a Walk Operator

$$\mathcal{W} = \mathcal{S}\,\mathcal{C}, \tag{3}$$

the product of a shift $\mathcal{S}$ (a permutation on the $\mathbb{Z}^3$ factor) and a coin $\mathcal{C}$ acting on $\mathcal{H}_{Q_3}$. The coin is the substrate analog of the Dirac σ_x chirality operator: the bipartite vertex structure of Q_3 forces $\mathcal{C}$ to be a zero-controlled NOT on the chirality bit, with the deterministic Boolean form at the UV scale relaxing into a continuous $U(m)$ family on coarse-graining — recovering the Dirac mass term in the $\lambda \gg a_0$ continuum limit.

To each codeword $c \in \Pi_{\text{phys}} \mathcal{H}_{Q_3}$ we associate a *frustration count*

$$F(c) = \sum_{(i,j) \in E(Q_3)} (c_i \oplus c_j), \tag{4}$$

the Hamming defect of c summed over the edge set of the Q_3 unit cell. F measures how many bit-flips a spatial hop must temporarily execute against the parity-check algebra. Bare informational mass follows from the Boltzmann weight on code survival under the walk:

$$M(c) \;=\; \exp\!\left(\frac{\varphi\, F(c)}{2}\right), \qquad \varphi = \frac{\sqrt{5}-1}{2} \approx 0.618. \tag{5}$$

Equivalently, the topological dwell time is $\tau(c) \propto M(c)$ and the bare propagation probability is $P(c) \propto 1/M(c)$. The appearance of the reciprocal golden ratio φ is not a tuning: it is fixed by the spectral structure of the Q_3 unit cell (cf. the P_4 characteristic polynomial $\lambda^4 - 3\lambda^2 + 1 = 0$ which reappears in §4).

The generation hierarchy is a *two-tier* phenomenon. Local F-averages over the 36 valid quark codewords give $\bar{F}_{\text{Gen 1}} = 5.67$, $\bar{F}_{\text{Gen 2}} = 6.33$, $\bar{F}_{\text{Gen 3}} = 5.33$ — strikingly, Gen 3 is *less* locally frustrated than Gen 2. Local frustration alone therefore cannot reproduce the empirical hierarchy. The macroscopic shell impedance of the $\mathbb{Z}^3$ gauge web — areal progressions $R^2 \in \{1, 4, 9\}$ — supplies the load-bearing $\mathbb{Z}_2$-routing obstruction that lifts the Gen 3 mass above Gen 2. We flag this explicitly: the mass hierarchy is a global routing phenomenon, not a local micro-cell property.

4 Zero-Parameter Prediction I: the $\rho(770)$ Bare Mass

In standard QCD the mass of the $\rho(770)$ is non-perturbative and currently accessible only via numerical lattice simulation. In the substrate it follows analytically from the spectral algebra of the gauge flux tube. The derivation rests on three statements; the first is rigorously graph-theoretic, the second imports a continuum-string postulate, the third is a phenomenological ansatz. We separate them.

Line Graph Theorem (graph-theoretic)

A meson flux tube spanning n edges of a Q_3 octagon traces the vertex path P_{n+1}. Gauge fields live on edges, matter on vertices, so the gauge-field dynamics of the flux tube are governed by the line graph $L(P_{n+1}) = P_n$. The adjacency matrix of P_n has the exact discrete spectrum

$$\lambda_k^{(P_n)} = 2\cos\!\left(\frac{k\pi}{n+1}\right), \qquad k = 1, \ldots, n. \tag{6}$$

Antinode Theorem (graph-theoretic up to a Dirichlet postulate)

The ρ is the $J^{PC} = 1^{--}$ ($k = 1$ dipole) harmonic of the D_8 octagonal symmetry in the E_{1u} irrep, with spatial amplitude $\psi_n^{(k=1)} = \cos(\pi n/4)$. *If* the valence quarks act as Dirichlet sources/sinks for the chromoelectric flux tube (a string/bag postulate imported from the continuum), maximal $k = 1$ coupling forces antinode placement at $\psi_0 = +1$, $\psi_4 = -1$, giving antinode separation $n = 4$. The Z_2 reflection symmetry of the lattice then independently selects $d = 4$ over $d = 3, 5$ on parity grounds alone. The Dirichlet step is a continuum postulate, not a substrate first-principle; we flag it.

For $n = 4$ the line graph is P_4, with adjacency matrix

$$A_{P_4} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \qquad p(\lambda) = \lambda^4 - 3\lambda^2 + 1, \tag{7}$$

and eigenvalue set $\{+\phi, +\phi^{-1}, -\phi^{-1}, -\phi\}$ where $\phi = (1+\sqrt{5})/2 \approx 1.618$ is the full golden ratio (and $\phi = 1/\varphi$, with φ as defined in §3). The dominant mode of the flux tube is $\lambda_1 = \phi$.

Orthogonal Quadrature (phenomenological ansatz)

The two half-octagon paths between the antinodes carry the dominant mode independently. *Treating them as orthogonal transverse channels* gives a quadrature sum:

$$m_\rho^{\text{bare}} \;=\; \sqrt{(\phi\Lambda_{\text{QCD}})^2 + (\phi\Lambda_{\text{QCD}})^2} \;=\; \sqrt{2}\,\phi\,\Lambda_{\text{QCD}}. \tag{8}$$

This is an ansatz, not a forced consequence of the C_8 spectrum. If the same modes were instead treated as normal modes of a single Hamiltonian and superposed linearly (as we do for the nucleon, §5), the result would be $2\phi\Lambda_{\text{QCD}}$. The orthogonal-quadrature postulate is required to recover the empirical ρ mass; a substrate-first-principle derivation forcing the orthogonal framing over the normal-mode framing is an open structural problem.

Numerical evaluation

Given the empirical scale $\Lambda_{\text{QCD}} = 332$ MeV [5] — the single empirical input — geometric evaluation yields

$$m_\rho^{\text{bare}} \;\approx\; \sqrt{2} \times 1.618 \times 332 \text{ MeV} \;\approx\; 760 \text{ MeV}, \tag{9}$$

within 2% of the experimental peak of 775 MeV [5]. The framework natively dictates a $\sqrt{2}$ rather than $\sqrt{3}$ scaling factor, distinguishing it from continuum string models; lattice Euclidean-time plateau analyses are consistent with $\sqrt{2}$. The residual 15 MeV gap is consistent in both sign and magnitude with the standard Kramers–Kronig dispersive shift of the unstable $\rho \to \pi\pi$ resonance pole; the sign of that shift is fixed structurally by the p-wave centrifugal barrier.

5 Zero-Parameter Prediction II: the Nucleon Mass

The same Λ_{QCD}, the same P_4 algebra, applied to a closed C_8 octagon cycle rather than an open path: this is the cross-check that distinguishes structural emergence from one-off tuning. A baryon is a closed three-quark cycle on the C_8 matter octagon; its mass is set by the $k = 1$ and $k = 7$ modes of the closed cycle, with eigenvalues $a_1 = a_7 = \sqrt{2}$. Because both modes belong to a single effective Hamiltonian on the closed cycle, they superpose linearly rather than in quadrature:

$$M_N \;=\; a_1 + a_7 \;=\; 2\sqrt{2}\,\Lambda_{\text{QCD}} \;\approx\; 939.04 \text{ MeV}, \tag{10}$$

within 0.02% of the experimental isospin-averaged nucleon mass. The contrast between this calculation and the meson one is structurally informative: open-path mesons require the orthogonal-quadrature ansatz (flagged in §4), closed-cycle baryons take the linear-superposition route forced by the closed-cycle normal-mode structure. The cross-validating ratio (conditional on this ansatz pair) is parameter-free:

$$\frac{m_\rho^{\text{bare}}}{M_N} \;=\; \frac{\sqrt{2}\phi\Lambda_{\text{QCD}}}{2\sqrt{2}\,\Lambda_{\text{QCD}}} \;=\; \frac{\phi}{2} \;\approx\; 0.8090, \tag{11}$$

against the experimental 0.8257 ± 0.0003: a 2% deficit consistent with the same dispersive shift that closes the absolute ρ value. *Two zero-free-parameter mass predictions from a single empirical scale* — the empirical content per result is one bit (the choice of cycle topology, open vs closed), and the algebra is the same in both cases.

6 The Scalar Sector and Quadratic Gauge Scaling

A separate structural question: why do fermions couple linearly to the scalar sector ($\mathcal{M}_f \propto m_f$) while massive vector bosons couple quadratically ($\mathcal{M}_V \propto m_V^2$)? In the Standard Model this is a feature of the Higgs mechanism's representation theory; in the substrate it follows directly from bipartite tensor orthogonality.

The Higgs vacuum expectation value is identified with the physical crystallisation of the local chirality constraint, and the scalar resonance with a localised transient relaxation of that constraint. For a single-node fermion excitation, the transition amplitude under the walk operator is linearly proportional to its discrete query rate $N_{\text{query}} \equiv m_f$:

$$\mathcal{M}_f \;\propto\; m_f. \tag{12}$$

Vector bosons $W^{\pm}$, Z^0, by contrast, are *bilocal* chiral excitations spanning a gauge bridge between two distinct Q_3 nodes A and B. The bipartite structure makes nodes A and B belong to opposite sublattices, so a Taylor expansion of $\mathcal{W}$ on the bridge has its first-order matrix element *vanish* by sublattice orthogonality — the substrate analog of the vanishing of nearest-neighbour same-sublattice hopping in graphene. The leading non-zero contribution is the second-order cross-derivative:

$$\mathcal{M}_V \;\propto\; m_A m_B \;\equiv\; m_V^2. \tag{13}$$

The m_V^2 gauge scaling is therefore a structural consequence of the bipartite geometry, not an ad-hoc parameterisation. The same mechanism explains why the substrate's emergent vector-boson sector resists a perturbative single-node description and requires the macroscopic line-graph $L(\mathbb{Z}^3)$ gauge web.

7 Scope and Open Structural Problems

For a physicist-reader, the following statements should be made explicit:

- **Empirical input.** The QCD scale $\Lambda_{\text{QCD}} = 332$ MeV is taken as an empirical input. "Zero free parameters" in §§4–5 means *given this single scale*, no further phenomenological inputs enter; it does not mean Λ_{QCD} is itself derived in this paper. (A first-principle derivation of Λ_{QCD} from substrate lattice scales is an open framework target.)

- **Two flagged ansatzes.** The Dirichlet-boundary postulate for valence quarks (§4) imports a continuum string/bag assumption rather than deriving it. The orthogonal-quadrature ansatz for open-path mesons (§4) is required to reach the empirical ρ mass; the alternative normal-mode framing — the one used for the nucleon in §5 — would give $2\phi\Lambda_{\text{QCD}}$ instead of $\sqrt{2}\phi\Lambda_{\text{QCD}}$. Both are anchored as open derivations in the full framework, not glossed.

- **2D-vertex-figure scope.** The ρ and nucleon derivations operate on the 2D 4.8.8 vertex figure; their 3D-bulk projection under $C_{4v} \to O_h$ inherits the algebra but the explicit reduction has not been verified at the bulk level.

- **What this paper is not.** This is a reframing of the substrate's structural mechanics in physicist-native language; it is not a finished UV-complete theory. The aim is to show that two highly non-trivial hadronic masses emerge from a single discrete spectral algebra with one empirical scale, and to make the open structural problems precise enough to be attacked.

8 Conclusion

Taking a discrete bipartite tensor network $\mathbb{Z}^3 \otimes Q_3$ on the 4.8.8 Archimedean tiling as the substrate of the vacuum, and treating continuous spacetime as its macroscopic effective field theory in direct analogy to the graphene → (2+1)D Dirac limit, we have obtained: (i) a $[\![8,4,4]\!]$-style codeword count matching one full SM matter generation plus three sterile neutrinos; (ii) the bare $\rho(770)$ mass at $\sqrt{2}\phi\Lambda_{\text{QCD}} \approx 760$ MeV (2% of experiment); (iii) the nucleon mass at $2\sqrt{2}\Lambda_{\text{QCD}} \approx 939$ MeV (0.02% of experiment); (iv) a structural derivation of the quadratic gauge scaling $\mathcal{M}_V \propto m_V^2$ from bipartite sublattice orthogonality. All four results follow from one empirical input — the QCD scale — and a single discrete spectral algebra. Two ansatzes (Dirichlet boundaries, orthogonal quadrature) remain open structural targets. The aim of this paper has been not to settle the framework, but to express it in the language of the readership best placed to test it.

References

[1] Castro Neto, A. H., Guinea, F., Peres, N. M. R., Novoselov, K. S., & Geim, A. K. *The electronic properties of graphene.* Reviews of Modern Physics, 81(1), 109 (2009).

[2] Elliman, D. G. *A Model for John Wheeler's It from Bit: An Eight-Bit Discrete Origin for the Laws of Nature.* Neuro-Symbolic Ltd. Amazon Publishing (2026). ISBN 978-1919558875

[3] Wilson, K. G. *Confinement of quarks.* Physical Review D, 10(8), 2445 (1974).

[4] Feynman, R. P. *QED: The Strange Theory of Light and Matter.* Princeton University Press (1985).

[5] Workman, R. L., et al. (Particle Data Group). *Review of Particle Physics.* Progress of Theoretical and Experimental Physics, 2022(8), 083C01 (2022).