# A Frozen Defect Network in the Dark Sector: A Consolidated, Self-Contained Account of K04 Crystallisation Debris in the Finite-QEC Substrate

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**Abstract**

This note collects, in one place and in plain language, a chain of small but exact results about the K04 crystallisation-debris component of the finite quantum-error-correction (QEC) substrate programme. The programme models space as a discrete, self-repairing lattice whose ordered ground state tiles space with eight-element register cells — geometrically oblate square bipyramids, whose face-adjacency graph is the combinatorial three-cube $Q_3$. The K04 relic is not a new particle but *crystallisation debris*: defects left behind when the lattice ordered too quickly to heal. The June 2026 canon update narrows the interpretation. K04 debris is a pinned, substrate-static dark-sector fossil, not the dominant mobile, pressureless halo component and not the carrier that completes the CMB third peak. We answer four questions that a graduate scientist can follow without prior exposure to the programme. (1) **Taxonomy:** which defects are permanent? A defect is permanently locked only if it winds the system, so the strictly topological relic is an extended string, not a point particle; finite local defects heal. The actual K04 quench conserves the homology class, so its durable abundance is kinetic rather than topological. (2) **Mobility:** the relic network is pinned by enormous Peierls barriers, which rules it out as a free collisionless halo gas. (3) **Energy scale:** a string costs $w_4+4w_6$ per lattice step and a frustrated domain wall costs $2w_4+12w_6$ per cut cell, while many domain walls are exactly free because the cell crystal is highly degenerate. (4) **Abundance shape:** the live abundance is a Kibble–Zurek wall-network law, $n_{\text{wall}} \simeq 2.63/\xi(R)$, not the older zero-inflated island count. The remaining unknowns are the boot cooling law, the KZ correlation length, and the shadow truncation of the gravitating fossil. The lattice unit $w_6$ is now tied to the QCD anchor at derived-conditional tier by the native line-service tension, $w_4 + 4w_6 = 6w_6 = \Lambda_{\text{QCD}}$. The pressureless mobile dark budget is discussed only as a separate, conditional R4-zero-mode/sterile-neutrino branch. Every numerical claim is reproduced by a short self-checking program; the programs are listed in an appendix.

## Contents

## 1 Why this note exists

The identity of dark matter is the largest unanswered question in physical cosmology. Roughly five-sixths of the gravitating matter in the Universe is invisible, and the default hypothesis for forty years has been that it is a gas of as-yet-undiscovered elementary particles [1]; its cosmic density is measured very precisely, $\Omega_{\rm DM}h^2 \approx 0.12$ [18], but no particle has been found. It is therefore worth taking seriously alternatives in which some dark-sector structure is not a particle gas at all, but *frozen structure* — a relic pattern locked into the fabric of space. The present note is about that K04 fossil component. It no longer claims that the fossil network is the whole dark matter, because later mobility and equation-of-state audits show that it is pinned and not pressureless dust.

This note is about one such component, developed inside the finite-QEC substrate programme [8, 10]. The reader does not need to accept (or even know) the rest of that programme. The point of the note is narrow and self-contained: *given* a particular discrete model of space, work out what its frozen defects are, how stable they are, how much they cost, and how many of them a fast cooling would leave behind. The answers turn out to be unusually clean, and several are exact integers or simple fractions rather than fitted numbers. Their cosmological interpretation is narrower than the title of the first draft: K04 is a pinned dark-sector fossil and a useful falsification target, while the mobile pressureless halo/CMB role belongs, in the current canon, to a separate R4-zero-mode/sterile-neutrino branch if a conserved massive dust charge and source theorem are admitted.

The four ingredients — a defect *taxonomy*, a *mobility* verdict, an energy *tension*, and an *abundance* law — were established one at a time. Here they are assembled into a single picture, with the technical apparatus explained from scratch.

## 2 The model: a self-repairing lattice that crystallises into register cells

Picture ordinary three-dimensional space replaced by the cubic lattice $\mathbb{Z}^3$: a regular grid of points, each joined to its six nearest neighbours by potential "bonds". The substrate programme treats this lattice as the memory of a quantum error-correcting code — a system that constantly measures and repairs itself, in the spirit of topological quantum memories [7, 17]. For the present purpose only one structural rule matters:

**Degree-three rule.** Every lattice site keeps exactly three of its six bonds.

A configuration obeying this rule is, in graph-theory language, a *3-factor* of the lattice. The programme assigns each configuration an energy that rewards short closed loops of bonds:

$$E = -w_4\, C_4 - w_6\, C_6, \tag{1}$$

where $C_4$ and $C_6$ count the four-bond and six-bond loops, and $w_4, w_6 > 0$ are fixed positive weights. (Numerically we use the programme's central values $w_4 = 1.7$, $w_6 = 1$ for worked examples; the algebra keeps them symbolic.)

**The ground state: a honeycomb of bipyramidal register cells.** What does the lowest-energy configuration look like? The lattice self-assembles into disjoint eight-element *register cells*. Geometrically each cell is an *oblate square bipyramid* — two right square pyramids joined base-to-base and flattened just enough to tile space — and its eight triangular faces carry the cell's eight-bit register [8]. These bipyramidal cells pack space, interleaved with truncated-cube gauge cells, into the *truncated cubic honeycomb* (the structure from which the wider programme, and its code repository, take their name).

The *combinatorial* structure of a cell — how its eight faces, equivalently their eight centre points, are wired together as nearest neighbours — is the three-cube graph $Q_3$: eight elements addressed by three bits, each joined to the three others that differ in a single bit. This $Q_3$ is a *graph*, not a shape. Following the foundational description we call it a *combinatorial cube*, and for the rest of this note "cube" means *only* this register graph, never a geometric cube. (Calling the cell a cube would mislead twice over: the cells are bipyramids, and even regular octahedra cannot tile space without tetrahedral fillers.)

For the crystallisation and defect analysis we adopt the programme's embedded model: a bond subset of the periodic lattice $\mathbb{Z}^3$ in which every site keeps exactly three of its six bonds, and whose ordered phase groups the sites into these eight-vertex $Q_3$ cells. One such cell is rich in short loops — six four-loops and sixteen six-loops — which is exactly what the energy (1) rewards. Concretely, in the "phase-zero" ordering a bond along axis $a$ is present precisely when the site's $a$-coordinate is even, giving every site one bond per axis and grouping the sites into eight-vertex cells spanning coordinates $\{0, 1\}, \{2, 3\}, \ldots$ in each direction.

**Eight crystal orientations.** There is a small but important freedom. In each of the three axes the cells can start at even coordinates or at odd coordinates — a binary choice, like deciding whether to lay bricks starting flush with the wall or offset by half a brick. Three independent binary choices give $2^3 = 8$ distinct crystal "phases". We label a phase by a triple $\phi = (\phi_x, \phi_y, \phi_z)$ with each $\phi_a \in \mathbb{Z}_2 = \{0, 1\}$. This phase is the *order parameter* of the crystal: a quantity that is uniform inside an ordered region and that can differ between regions. As we will see, defects are precisely the places where $\phi$ cannot be made uniform.

## 3 Defects, and how they heal

A *defect* is any local deviation from the perfect crystal of $Q_3$ cells — a patch where the bonds are rearranged so that the clean cells are spoiled. The energy (1) makes defects cost energy, because spoiling a cell destroys some of its short loops.

**The repair move.** The substrate heals defects by a local, reversible rewiring that preserves the degree-three rule. The elementary version acts on a single lattice square (a "plaquette"): if two opposite edges of the square are present and the other two absent, the move swaps them — removing the first pair, adding the second. Every site on the square loses one bond and gains one, so all four sites keep degree three. We call this a *Kempe move*, by analogy with the chain-swapping moves of graph colouring. It is the same kind of local update that connects configurations in dimer and tiling models [23], and it is the microscopic step of the substrate's annealing dynamics.

**The simplest defect heals immediately.** The smallest inter-cell defect is what we call a *peanut*: take two neighbouring cells, break one bond on each of their facing faces, and re-join them with two bonds across the gap, so the cells are pinched together at a waist. This is a valid degree-three configuration, and its energy is higher than two clean cells by exactly

$$\Delta E_{\text{peanut}} = 4w_4 + 16w_6 \ (> 0). \tag{2}$$

But the peanut is literally *one Kempe move away* from two clean cells: a single plaquette swap undoes the waist and the defect vanishes, running downhill in energy. The simplest defect self-heals. This raises the real question: are there defects that *cannot* be healed, no matter how many local moves you apply? Those would be the candidate dark-matter relics, because only an unhealable defect survives.

## 4 Taxonomy: only winding defects are permanent

The decisive tool is topological, and it is worth stating carefully because it does all the work.

**The difference of two configurations is a set of loops.** Compare any configuration $C$ with the reference crystal $C_0$. Form their *symmetric difference* $D = C \triangle C_0$: the set of bonds present in one but not the other. Because both $C$ and $C_0$ obey the degree-three rule, at every site the number of $D$-bonds is even (three plus three, minus twice the shared bonds). A set of bonds with an even number at every site is, by a classical fact of graph theory, a union of closed loops. So $D$ is always a collection of loops.

**Local moves cannot change how loops wrap.** A Kempe move changes $C$ by a single four-bond loop (the plaquette). Hence it changes $D$ by that small loop too. Now small loops are *contractible*: they can be shrunk to a point. On a space that wraps around on itself (a torus, the natural setting for a periodic lattice), loops fall into classes according to how many times they wind around each of the three directions before closing — a contractible loop winds zero times. The technical name for this bookkeeping is the *first homology group*; for our purposes it is just three yes/no winding counts, one per axis, i.e. an element of $\mathbb{Z}_2^3$ [11]. Adding a contractible loop never changes the winding counts. Therefore:

> **Conservation law.** The winding class $[D] \in \mathbb{Z}_2^3$ is unchanged by every local Kempe move. (And not only by plaquette moves — by *any* local rewiring, since any local move changes $D$ by a contractible loop.)

The crystal itself has $[D] = 0$. A configuration can therefore be healed back to the crystal by local moves only if it already has $[D] = 0$. Anything with $[D] \neq 0$ is **permanently locked**: no sequence of local repairs can remove it.

**Locked means winding, and winding means extended.** A non-zero winding class requires $D$ to contain a loop that genuinely wraps the system — a defect that threads all the way around. A purely local, finite defect (like the peanut) is built from contractible loops, so it has $[D] = 0$ and is *not* topologically locked. The smallest locked object is therefore a single *winding string*: flip the bonds along one straight line that wraps the lattice. One checks directly that this keeps every site at degree three, carries winding class $(1, 0, 0)$, and costs

$$\mu_{\text{string}} = w_4 + 4w_6 \quad \text{per lattice step} \tag{3}$$

(equal to $5.7\,w_6$ at the example weights); physically it is the three-dimensional cousin of the soliton that separates the two dimerisations of a one-dimensional chain [22]. This is the central taxonomy result:

> **There is no permanently-locked point particle.** The minimal indestructible defect is one-dimensional — a frozen string — and, more generally, two-dimensional sheets (domain walls) that wrap the system. On infinite space a locked defect must be infinite in extent.

This is exactly the logic by which ordered media classify their defects — vortices, disclinations, domain walls — through the homotopy of the order parameter [15], and the same logic that makes cosmic strings and domain walls stable in field theory [24, 25].

**Finite defects really do heal.** The conservation law says finite defects are *not protected*; it does not by itself prove they actually unwind. We checked this by brute force: enumerate every degree-three configuration of small closed blocks of the crystal and follow the Kempe moves. For blocks of 16, 32, and 32 sites (a $2 \times 2 \times 4$, a $2 \times 4 \times 4$ and a $4 \times 4 \times 2$ region) *every* configuration — 9, then 8467, then 8467 of them — lies in a single connected web of moves containing the crystal. None is stranded. Finite defects heal in practice, not just in principle, at every size we can enumerate. (This is the discrete analogue of the well-known fact that local moves explore a whole topological sector of a tiling model [23].)

## 5 Mobility: the locked debris is frozen, not collisionless

A separate calculation in the programme asked whether these defects can be *pushed* — whether a gravitational pull could drag a wall or string through the lattice. The answer is an emphatic no: moving a defect by one lattice step requires climbing an energy barrier of order $w_6$, while the energy a realistic astrophysical acceleration supplies over one lattice spacing is about 42 orders of magnitude smaller. The defects are pinned in place.

This sharpens the picture rather than spoiling it. A pinned object cannot behave like a free, collisionless particle: in a cluster collision it would stay put rather than sailing through with the galaxies, which is the opposite of what gravitational lensing of merging clusters shows for dark matter [4], and any mobile self-interacting dark component is in any case tightly bounded [19]. So the debris is *not* a viable free particle gas. That does not erase the defect result. It fixes its role. The proposal that survives is frozen structure — a relic pattern that gravitates but does not flow — and for that, extreme pinning is a feature. The taxonomy and the mobility verdict agree: the durable K04 component is an *extended, frozen defect network*, not a gas of mobile lumps and not the dominant halo component.

## 6 Energy scale: wall tension, and a large hidden degeneracy

To turn "a network of frozen defects" into a mass density we need the energy per unit size of the defects — their *tension*. The string tension $\mu_{\text{string}} = w_4 + 4w_6$ is above. For the two-dimensional walls there is a surprise.

**Most walls cost nothing.** Recall that the crystal has eight phases. A *domain wall* is an interface between two regions in different phases. One might expect every wall to cost energy. It does not. Because the perfect crystal is built from *isolated* cells — cells that carry no bonds to their neighbours — the total energy does not care how neighbouring cells are oriented relative to one another. Sliding a whole slab of cells by one step, or flipping its phase in a direction transverse to the interface, produces a different but *equally low-energy* configuration. We verified that such a wall has $\Delta E = 0$ exactly: it is a genuine alternative ground state.

The consequence is that the cell crystal is *massively degenerate*: there are exponentially many distinct lowest-energy tilings, not just eight, and a huge class of domain walls between them is free. This is the same phenomenon as the residual entropy of geometrically frustrated magnets such as spin ice, where many configurations share the ground-state energy [2]. Free walls are configurational entropy — they reshuffle at no cost — and they are *not* durable gravitating relics.

**The walls that do cost.** A wall costs energy only when it is *frustrated*: when the two tilings cannot meet without cutting cells. Geometrically this happens when the interface is forced to pass through the middle of cells rather than along their boundaries. Such a frustrated wall has an exact tension

$$\sigma_{\text{wall}} = 2w_4 + 12w_6 \quad \text{per cut cell} \tag{4}$$

(equal to $15.4\,w_6$ at the example weights): each cut cell loses two four-loops and twelve six-loops. By the cubic ($O_h$) symmetry of the cell the cost of cutting a cell does not depend on which face is opened, so this is the tension of any frustrated wall. Importantly, a frustrated wall has winding class $[D] = 0$: it is an *energetic* defect, not a topological one. It would heal if it could cross its energy barrier — but it cannot, because the barriers are large compared with the present temperature and the defect is pinned (the previous section). It is durable by being *frozen*, not by being topologically protected. The topologically protected objects (the winding strings) are, by contrast, so rare — at most three of them can exist in a periodic box of any size — that they contribute negligibly to the mass budget. The abundance is carried by the frozen, frustrated walls.

## 7 Abundance: how many frustrated walls a fast cooling leaves

The last ingredient is how many costly defects a cooling Universe freezes in. This is the domain of the *Kibble–Zurek mechanism*, one of the most successful ideas in non-equilibrium physics [6, 12, 26]. Its logic is simple: when a system orders quickly, regions that are too far apart to communicate must each pick an ordering *independently*. Where regions with incompatible choices meet, the mismatch freezes in as a defect. The size of the independently-choosing regions is the correlation length $\xi$ at the moment of freeze-out, and $\xi$ is set by how fast the system was cooled (the cooling rate $R$): slower cooling gives larger $\xi$ and fewer defects. The same mechanism makes cosmic strings in the early Universe and has been confirmed in superfluids, liquid crystals and trapped-ion experiments.

**Which junctions are costly?** In our crystal, each independently-ordered domain picks one of the eight phases at random. Where two domains meet across an interface perpendicular to axis $n$, is the junction free or frustrated? The criterion is clean. The cells of a phase have their boundaries at fixed positions along each axis; for the axis $n$, phase $\phi_n = 0$ puts boundaries at the odd coordinates and $\phi_n = 1$ at the even coordinates — two *disjoint* sets. So two domains share a clean cell-aligned interface (a free wall) exactly when they agree on the *normal* component $\phi_n$, and they are forced to cut cells (a frustrated wall) exactly when they disagree on $\phi_n$. Differences in the other two (transverse) components are always free.

**The frustrated fraction is** $4/7$**.** Now count. With the eight phases chosen at random and independently for adjacent domains:

$$P(\text{the junction is a wall at all, i.e. } \phi \neq \phi') = 1 - \tfrac{1}{8} = \tfrac{7}{8}, \tag{5}$$

$$P(\text{the junction is frustrated, i.e. } \phi_n \neq \phi'_n) = \tfrac{1}{2}. \tag{6}$$

The fraction of domain walls that actually cost energy is therefore

$$f_{\text{frust}} = \frac{1/2}{7/8} = \boxed{\frac{4}{7}} \approx 0.571. \tag{7}$$

This is exact (it is a finite count over the $8 \times 8$ pairs of phases) and we confirmed it by direct Monte-Carlo sampling. Four-sevenths of the frozen domain-wall network is durable; the other three-sevenths is free entropic reshuffling.

**Putting it together.** The number of domain walls per unit volume scales as $1/\xi$ (a wall is a two-dimensional sheet, so its area density falls inversely with the domain size — the standard Kibble–Zurek scaling for a codimension-one defect). Multiplying the wall density by the costly fraction $f_{\text{frust}}$ and by the tension $\sigma_{\text{wall}}$ gives the relic mass density:

$$\boxed{\rho_{\text{dark}} \;\sim\; \sigma_{\text{wall}}\, f_{\text{frust}}\, n_{\text{wall}}\big(\xi(R)\big) \;\sim\; \frac{4}{7}\,\sigma_{\text{wall}}\,\frac{1}{\xi(R)}}\,. \tag{8}$$

The shape of this law — inverse in the freeze-out correlation length — is fixed, and two of its three factors ($\sigma_{\text{wall}}$ and $f_{\text{frust}}$) are now exact numbers.

**What this replaces.** An earlier attempt to predict the debris abundance counted isolated "orphan" clumps left by the cooling, but that count was dominated by rare events and never settled to a stable value. Equation (8) replaces a noisy clump-count with a clean scaling law: the abundance is a Kibble–Zurek order parameter — the frozen length of the frustrated defect network — not a head-count of rare islands. Whether the network percolates (forms a single connected frozen web rather than disconnected pieces) is a percolation question [21]; at the densities of a fast quench it does percolate, consistent with a single rigid relic structure.

## 8 What is settled, and what is not

**Settled (exact or computed).** The defect taxonomy (locked ⇔ winding; smallest locked object is a 1D string), the two tensions ($\mu_{\text{string}} = w_4 + 4w_6$, $\sigma_{\text{wall}} = 2w_4 + 12w_6$), the ground-state degeneracy and the free/frustrated wall criterion, and the frustrated fraction $f_{\text{frust}} = 4/7$ are all exact integers or simple fractions, each reproduced by a self-checking program (appendix). The mobility verdict (extreme pinning) is a computed result. Together they fix the *form* of the abundance law (8).

**Open (one external input and one suppression law).** Two ingredients are still needed to turn the proportionality (8) into a surviving gravitating fossil density, and both belong to the wider programme rather than to the defect taxonomy:

1. the **freeze-out length** $\xi(R)$, which requires the substrate's boot cooling law — the schedule by which the early Universe crystallised. An earlier attempt to derive this from an entropy argument failed, so it remains genuinely open.

2. the **shadow truncation** that tells how much of the pinned wall energy is recorded by the boundary ledger and therefore gravitates. The bare wall network overcloses catastrophically, so this is not optional.

The unit conversion is no longer a free convention in the current canon. With the edge-ledger value $w_4/w_6 = 2$, the native line-service tension is $\mu_{\text{string}} = w_4 + 4w_6 = 6w_6$, exactly the ramp-start unit. Pricing this service tension by the strong-sector anchor gives $w_6 = \Lambda_{\text{QCD}}/6$. Anchoring instead at the heat-capacity transition would make $\mu_{\text{string}} \simeq 2\Lambda_{\text{QCD}}$ and is a different response-temperature postulate, not a derivation. With the freeze-out and shadow inputs, (8) would give an absolute K04 fossil density. That density should be compared with observational upper bounds on pinned, substrate-static dark components, not identified automatically with the full $\Omega_{\text{DM}}h^2 \approx 0.12$ budget [18].

**How it could be tested or killed.** Because the durable relic is a frozen, pinned, extended network rather than a particle gas, it makes qualitatively distinctive predictions: it should not behave as a mobile collisionless halo in cluster mergers [4]; it should be bounded as a smooth or substrate-frame-static fossil component; and its abundance follows the single scaling law (8). Any one of these can falsify the K04 fossil reading. The cored-profile and radial-acceleration phenomenology of galaxies [5, 14, 16] is now charged to the separate R4/MOND line-current sector rather than to K04 itself. Likewise, the pressureless mobile mass budget is charged to the R4 zero-mode plus sterile-$\nu_R$ branch in the current canon, not to the frozen network.

## 9 Summary

Starting from a single structural rule — a self-repairing lattice that crystallises into bipyramidal $Q_3$ cells — we obtained a complete, mostly-exact account of its frozen defects. The strictly protected defects are extended, never point particles, by a topological conservation law; the actual KZ relic is kinetic and homology-trivial; it is frozen in place by large energy barriers; its energy costs are exact small integers in the lattice weights; and the costly fraction of a random domain-wall network is exactly 4/7. The resulting K04 fossil abundance has a clean Kibble–Zurek scaling form, $\rho_{\text{K04}} \propto \sigma_{\text{wall}}/\xi(R)$, with the $w_6$ unit fixed conditionally by the line-service anchor and the surviving density awaiting the cooling law and shadow truncation. Whatever its ultimate phenomenological size, the picture is sharply different from the particle paradigm and sharply testable: K04 is the frozen scar tissue of a Universe that crystallised too fast, while the mobile dark-matter role must be supplied elsewhere in the dark sector.

## 10 Note added in revision (2026-06): the equation of state, and the CMB

The results above fix what the frozen defects *are* and how many a fast cooling leaves, but not how their energy density tracks cosmic expansion — the equation of state $w = p_{\rm dark}/\rho_{\rm dark}$ that decides their cosmological role. A short calculation, added after the first version, both sharpens the picture and bounds the title's claim.

**A frozen $p$-dimensional network has $w = -p/3$.** A relic pinned to the expanding substrate is conformally stretched: a connected $p$-dimensional defect keeps a fixed comoving count while its physical extent grows with the scale factor (length $\propto a$), so its energy scales as $a^p$ and its density as $\rho \propto a^{p-3}$, that is $w = -p/3$ — the standard scaling for frozen topological defects [24, 25]. Point defects ($p = 0$) give the $w = 0$ of matter; strings ($p = 1$) give $w = -1/3$; domain walls ($p = 2$) give $w = -2/3$. This is in fact already implicit in the abundance law (8): once the freeze-out length stretches with expansion, $\xi \propto a$, the wall density $\rho_{\rm dark} \sim \sigma_{\rm wall}/\xi$ falls as $a^{-1}$ — precisely $w = -2/3$.

**So the frozen scar is not the clustering cold dark matter.** The dominant relic here is the frustrated *wall* network ($p = 2$), so its equation of state is $w = -2/3$: a negative-pressure, *anti-clustering* component — in cosmological behaviour closer to a species of dark energy than to dark matter. Only a gas of *point* defects would give the $w = 0$ of cold dust, and the taxonomy result above is exactly that no permanently locked point defect exists. The network therefore gravitates and is pinned, as the mobility and Bullet-cluster discussion describes, but it does *not* act as the $w \simeq 0$ matter that collapses into halos and carries the cosmic clustering budget $\Omega_{\rm DM}h^2 \simeq 0.12$. In the substrate programme that clustering role belongs to the other two dark-sector mechanisms named above — the residual sterile neutrino and the MOND-like line-current — not to the frozen scar. The honest reading of the title is therefore narrower than it first sounds: the frozen network is a pinned, gravitating, *negative-pressure* relic, a genuine and distinctive dark component, but not a $w = 0$ particle-replacement for the whole dark-matter budget.

**A consequence for the microwave background.** This was the sharpest pressure on the dark sector, and the current canon now separates it cleanly from K04. The third acoustic peak of the cosmic microwave background requires a cold, pressureless, clustering component of density $\Omega_c h^2 \simeq 0.12$ present at recombination. The frozen wall network cannot supply it, because its $w = -2/3$ density anti-clusters rather than seeding the required potential wells. A frozen R4 line network would give $w = -1/3$, not dust. The live effective completion is instead a separate R4 zero-mode reservoir: if a conserved massive dust charge is admitted, it has a Brown–Kuchar/Stueckelberg dust count [3] with $w = c_s^2 = 0$, paired to the 17.7 keV sterile-$\nu_R$ source branch. Under the current conditional source law

$$\frac{n_{\nu_R}}{n_\gamma} = \frac{\alpha_0}{208}$$

and the directed-R4 4:1 zero-mode-to-sterile incidence, the ledger gives

$$\Omega_{\nu_R}h^2 = 0.02418, \qquad \Omega_{\rm zero}h^2 = 0.09671, \qquad \Omega_{\rm dark}h^2 = 0.12089, \qquad z_{\rm eq} = 3430.$$

A diagnostic CAMB calculation [13] confirms that a pressureless $\Omega_x h^2 \simeq 0.096$ component is the right shape for the third-peak/equality repair. This is not a K04 success and not a free fit: it is a separate conditional/AeST-class branch that inherits the conserved-dust premise, the $\alpha_0$-billed sterile source theorem, the sterile mass anchor, and a Boltzmann/halo implementation gate. If the zero-mode is used as ordinary CDM-like halo mass, active R4/MOND must not be added as a second independent galaxy force law; if active MOND is retained, the zero-mode needs a galaxy depletion or screening theorem. Relativistic MOND/AeST constructions [20] remain useful comparison models for this bookkeeping. None of this affects the defect results above; it bounds their cosmological interpretation.

# A Reproducibility

Every numerical statement above is produced by a short, self-asserting Python program in the project repository [10] (each prints `exit 0` only if all its internal checks pass):

| Program | What it establishes |
|---|---|
| `python_code/debris_dark_matter_audit.py` | The peanut defect and its one-move self-healing; why the toy ensemble was superseded. |
| `python_code/k04_kempe_locked_defect.py` | The homology conservation law; locked ⇔ winding; finite blocks heal (enumeration); the string tension $w_4 + 4w_6$. |
| `python_code/depinning_mobility_gate.py` | The ∼42-order-of-magnitude pinning of the debris. |
| `python_code/k04_wall_tension.py` | Ground-state degeneracy (free walls); the frustrated-wall tension $2w_4 + 12w_6$ per cut cell. |
| `python_code/k04_frustration_fraction.py` | The free/frustrated criterion and $f_{\text{frust}} = 4/7$ (enumeration + Monte Carlo). |
| `python_code/k04_defect_network_abundance.py` | The quench conserves winding; the abundance scaling $\rho \sim 1/\xi$. |
| `python_code/k04_w6_line_tension_anchor_theorem.py` | The conditional unit bridge $w_6 = \Lambda_{\text{QCD}}/6$ from $\mu_{\text{string}} = w_4 + 4w_6 = 6w_6$. |
| `python_code/item123_k04_glass_dust_test.py` | The frozen-defect equation of state $w = -p/3$; a glass freeze gives $w = -1/3$ (strings) / $-2/3$ (walls), not the $w = 0$ of dust. |
| `python_code/item123_cmb_zero_mode_theorem.py` | The current CMB zero-mode effective-dust gate: $\nu_R$ + $R4$ zero-mode gives a conditional pressureless component with $\Omega_{\text{dark}}h^2 = 0.12089$ if the conserved-dust premise is admitted. |
| `python_code/item123_cmb_boltzmann_sweep.py` | CAMB diagnostic showing that a pressureless $\Omega_x h^2 \simeq 0.096$ component restores equality and the third-peak scale. |

## Acknowledgement of scope

This is an expository consolidation aimed at a general scientific reader, not a primary research paper. The underlying programme, its assumptions, and its other sectors are documented separately [8–10]; the dark-sector companion in particular states the claim tiers and the competing mechanisms in full.

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