Dark Matter, MOND, and K04 Debris in the Finite-QEC Substrate

David Elliman
Neuro-symbolic Ltd
`dave@neusym.ai`

Revised canon snapshot: 2026-06-25

Abstract

This paper states the current dark-sector canon of the finite-QEC substrate program. The main cleanup is categorical. The dark sector is not a single hidden particle gas, not a single entropy pressure, and not a single defect network. It now contains three separated carriers. K04 crystallisation leaves gauge-blind kinetic wall fossils whose gravitating ledger is the recorded boundary shadow; those fossils are massive and durable but pinned, so they are not the dominant mobile halo medium. R4/MOND is a late line-current response: the actual R4 Kraus/Stinespring repair channel, read as a repeated service-history ledger, gives a count-valued nonexclusive Poisson line count and $\chi_{R4} = 1$ under the shared P1 scheduler premise. The CMB/mobile-halo slot can be carried phenomenologically by a pressureless R4 zero-mode reservoir plus the 17.7 keV sterile-ν_R branch: under the $\alpha_0/208$ boot source law and the 4:1 directed-R4 incidence split, it gives $\Omega_{\nu_R}h^2 = 0.02418$, $\Omega_{\text{zero}}h^2 = 0.09671$, $\Omega_{\text{dark}}h^2 = 0.12089$, and $z_{\text{eq}} = 3430$. The K04 toy configuration model is superseded because it admits nonphysical $K_{3,3}$ order; the embedded $\mathbb{Z}^3$ bond-subset ensemble is the physical substrate. The old boundary-local rescue island-floor abundance is now archival: the durable K04 relic is a kinetically frozen wall network with $n_{\text{wall}} \propto \xi^{-1}$, not a stable rare-island abundance. Remaining open pieces are now sharper: $w_6 \leftrightarrow \Lambda_{\text{QCD}}$, the boot cooling/KZ correlation length, the α_0-billed sterile source event, the zero-mode conserved-dust/Boltzmann/halo branch, the $R4$ core boundary, and the external horizon-class a_0 and G scales. The kill conditions are explicit: K04-as-halo is killed by pinning; MOND/BTFR is killed if P1 or the core rule fails; and the CMB completion route fails if the zero-mode reservoir cannot be justified and implemented as pressureless geodesic dust without double-counting active R4/MOND.

Contents

1 Role of This Paper

This is the dark-sector companion to the overview, foundations, matter, and cosmology papers. It covers the material that should not be merged into late dark energy or inflation:

1. K04 crystallisation and debris, a primordial kinetic fossil mechanism frozen in at the boot, the substrate's initial crystallisation epoch — K04 being the canon's fourth foundational kernel, the topological crystallisation rule that favours 4- and 6-cycles while penalising degree defects;
2. R4 line-current MOND, an ongoing halo-sector service-current mechanism — R4 is the substrate's fourth repair class, the same service channel whose Landauer exhaust sources late dark energy in the cosmology companion;
3. the pressureless R4 zero-mode reservoir and sterile-ν_R source law that now carry the CMB and mobile halo budget, if the remaining zero-mode conserved-dust, $\alpha_0/208$ billing, and Boltzmann gates hold.

The claim discipline is especially important here. Earlier dark-sector claims in the canon often had the right qualitative picture but the wrong ledger: entropy was counted without an event unit, Landauer language supplied coefficients it had not derived, and negative pressure was asked to do jobs that failed explicit Jeans tests. This draft uses the corrected protocol. Every dark-sector claim must specify the carrier, the scheduler, the event count, and the observable map.

The external physics context is standard: defect production by rapid symmetry-breaking or ordering transitions follows the Kibble–Zurek logic [6, 10, 22], extended defects and domain walls have familiar cosmological constraints [20, 21], the particle dark-matter paradigm whose budget this sector re-partitions is reviewed in Bertone et al. [2], and MOND/AQUAL phenomenology is compared with the Milgrom and Bekenstein–Milgrom construction [1, 9, 14]. The reproducibility repository is [8]; `https://neusym.ai` is the public project pointer.

2 Three Dark-Sector Carriers

The current canon separates the dark sector into distinct carriers. This is not just terminology; each carrier dilutes, couples, and gravitates differently.

Carrier	Mechanism	Status	Main remaining gate
K04 debris	Boot-frozen domain-wall shadow from misordered crystallisation	COMPUTED	Pinned fossil bound; absolute w_6 scale and KZ length.
R4 exhaust / MOND	Scheduler-clocked line-current halo records	CONDITIONAL	P1 scheduler premise, external a_0, and cored-profile rule.
R4 zero-mode reservoir	Effective Brown–Kuchar dust count paired to active R4 exchange	CONDITIONAL / imported dust premise	Conserved massive dust charge, $\alpha_0/208$ source billing, and galaxy branch.
Sterile ν_R cores	17.7 keV particle-like share [7]	CONDITIONAL	Same source map and X-ray/structure tests.

The K04 mechanism is a relic abundance problem. It asks how much misordered substrate survives the boot crystallisation and how much of its boundary strain is recorded by the surrounding instrumented crystal. The R4 mechanism is a line-current problem. It asks whether a halo sector of service records has matched creation and erasure rates, nonexclusive occupancy, and one-dimensional support. The zero-mode mechanism is a conserved-charge problem: it asks whether an inactive R4 reservoir can be admitted as rest-count dust even though the documented active R4 Lindbladian has no conserved number charge. These mechanisms can coexist, but a successful calculation for one must not be used as a coefficient theorem for the other.

3 K04 Substrate

The K04 substrate is a bond-subset model on the embedded cubic lattice. The physical state space is not the unconstrained configuration model. It is the set of bond subsets of periodic $\mathbb{Z}^3$ in which each site carries exactly three of its six nearest-neighbour bonds. A perfect registered cell is a cube-like eight-vertex component carrying the substrate's syndrome registers — the instrumented degrees of freedom the error-correction layer reads — and the local energy used in the crystallisation scripts is schematically

$$E = -w_4 C_4 - w_6 C_6,$$

where C_4 and C_6 count the square and hexagonal cycle content of the bond graph. The precise implementation uses local plaquette swaps that preserve the degree-three constraint. The signs and relative K04 weights are now better grounded than in the first draft: the Q3 loop-orbit edge-ledger audit conditionally gives

$$\frac{w_4}{w_6} = 2.$$

The absolute unit $w_6/\Lambda_{\mathrm{QCD}}$ is still a separate scale bridge.

3.1 Why the Toy Ensemble Was Superseded

The original configuration-model toy ensemble was useful as a negative artifact. Its apparent large debris plateau was not measuring the physical substrate. The audit found that the small cubic species had the wrong competitor:

$$\frac{E(K_{3,3})}{v} = -(1.5w_4 + w_6), \qquad \frac{E(Q_3)}{v} = -(0.75w_4 + 2w_6).$$

Thus $K_{3,3}$ beats the cube whenever

$$\frac{w_4}{w_6} > \frac{4}{3}.$$

The swept range $w_4/w_6 = 1.5$–2.5, and the paper's earlier $(w_4, w_6) = (2, 1)$ point, lie above that crossover. The toy plateau was therefore contaminated by nonphysical $K_{3,3}$-ordered bulk.

The embedded $\mathbb{Z}^3$ substrate removes this artifact structurally. It is bipartite, so it forbids K_4. No two $\mathbb{Z}^3$ vertices share more than two common neighbours, so it also forbids $K_{3,3}$ as a bond pattern. The physical ensemble is therefore the embedded bond-subset ensemble, not the configuration model.

3.2 What Crystallises

In the embedded ensemble the cube tiling is a true crystal branch. Cold starts remain locked at low temperature, and hot starts nucleate registered cells under the same plaquette dynamics. Equilibrium runs near the ordering window show that the substrate crystallises almost perfectly: the equilibrium deficit just below the transition is at the one-to-two percent level for small lattices and decreases with size. Those equilibrium defects have short healing spectra and are therefore not dark matter.

The original dark-matter candidate was kinetic debris: defects trapped by rapid cooling before the crystal can coarsen. The current canon is narrower. The trapped objects are real, massive, and durable, but the mobility and equation-of-state audits classify them as pinned fossils rather than the dominant mobile halo medium. This remains a Kibble–Zurek-style relic, but in the consolidated budget it is an upper-bound, substrate-static fossil branch unless a new exact zero-barrier translation primitive is derived.

4 Kinetic Debris from Kibble–Zurek Ramps

A cooling ramp with rate parameter R gives a trapped vertex deficit

$$d_{\text{trapped}}(R, L) = 1 - \frac{\text{registered crystal vertices}}{\text{total vertices}}.$$

The K04 ramp sweeps found monotone Kibble–Zurek curves: fast ramps trap large deficits, slow ramps crystallise out. The measured slopes are finite-size and toy-lattice slopes, not continuum exponents; this paper uses them only as a reproducible surface, not as a universal critical exponent.

Two gates changed the status of the debris proposal:

1. **Mass.** Surviving trapped debris carries positive excess energy, with a stable scale near $2.17\,w_6$ per debris vertex in the measured windows.
2. **Durability.** Healing spectra are dominated by extensive alternating trails in the boot-window runs, and low-temperature aging reduces d only slowly. The defects are locality-protected domain walls, not one-move plaquette artifacts.

The percolation gate is equally important. If the ramp is too fast, the visible registered crystal fails to span. Earlier versions therefore treated "viable debris dark matter" as the range where the visible sector percolates and the wall debris remains durable. The later mobility and EoS audits change the interpretation. The trapped debris is pinned to the substrate; extended frozen strings/walls scale as $w = -p/3$, and a pinned fossil cannot follow galaxies through mergers. Thus the debris surface below is retained as a fossil-fraction diagnostic and a consistency bound, not as the cold, pressureless clustering component required by the CMB.

The later defect-network audit also changes which K04 abundance statistic is load-bearing. The canonical quench conserves the $\mathbb{Z}_2^3$ homology class, so strictly topological winding relics are negligible in the thermodynamic limit. The durable relic is kinetic: a frozen, percolating wall network whose wall density follows the codimension-one KZ form

$$n_{\rm wall}(\xi) \simeq \frac{2.63}{\xi}$$

in lattice units, with line tension

$$\mu = w_4 + 4w_6.$$

Thus the live K04 abundance shape is a KZ correlation-length surface, not a rare-island count. The absolute normalisation is now closed by the cooling law below: finite-size scaling pins the trapping exponent $\beta = 0.16 \pm 0.02$ on a glassy plateau $d \simeq 0.77$, so ξ stays at the lattice scale and the bare wall fossil overcloses by $\sim 10^{31}$ — a pinned, shadow-suppressed relic, not a halo.

5 Why Bare Debris Energy Is Not the Gravitating Ledger

The key dark-sector correction is the shadow corollary. A wall interior has no registers. It therefore cannot be read as a fully instrumented local energy density. In the same sense that the cosmological-constant residual is the uncorrected part of a monitored QEC ledger, debris gravitates through recorded boundary strain in the surrounding registered crystal. The gravitating object is not the bare $2.1\,w_6$ per wall vertex. It is a depth-truncated correction shadow.

This resolves an apparent overclosure. Charging all unrecorded wall energy directly to gravity produces a miniature cosmological-constant problem. The wall-shadow reading instead makes the answer depend on how the concatenated QEC hierarchy reads the boundary:

$$\rho_{\rm DM} \propto \sum_{\text{wall-adjacent cells}} r_{\ell(x)},$$

where r_ℓ is the uncorrected strain residual surviving at concatenation depth ℓ of the QEC read hierarchy, so deeper readout leaves a smaller shadow. Shallow wall readout gives too much dark matter; fully adaptive deep readout gives too little or erases the shadow. The archived island-floor branch made this a pairing/orphan test; the current K04 branch instead uses the wall-network density and treats the shadow depth as an upper-bound consistency condition for a pinned fossil component.

6 Pairing, Orphans, and the Archived Island Floor

The hierarchy pairing rule was the first K04 abundance calculation that probed fine QEC geometry not visible in homogeneous ρ_Λ. It is now retained as an archived diagnostic rather than the live abundance formula. The measured growth-pairing audits found three policies:

Policy	Verdict	Reason
Strict same-level pairs	Excluded as canonical	Late wall cells stall at shallow depth and overshoot the observed $\rho_{\mathrm{DM}}/\rho_{\mathrm{B}}$ by hundreds.
Fully adaptive rescue	Excluded by locality	It reads through no-register wall interiors and removes the boundary-shadow observable.
Boundary-local rescue	Archived partial branch	A stranded block can be padded into an adjacent registered host; disconnected islands remain as a noisy floor.

The quantitative object in this branch was the island-floor surface:

$$\frac{\rho_{\mathrm{DM}}}{\rho_{\mathrm{B}}} = F_{\mathrm{island}}(d, L; R, \text{rescue policy}).$$

The target value used in the audits is

$$\left(\frac{\rho_{\mathrm{DM}}}{\rho_{\mathrm{B}}}\right)_{\mathrm{obs}} \simeq \frac{0.1200}{0.02237} \simeq 5.36,$$

using the Planck-like physical densities as the reference [16].

6.1 Current Surface Snapshot

The first larger boundary-local-rescue surface is noisy and zero-inflated, which is itself now part of the measurement. Representative four-replica rows from the current $L = 18, 20$ sweep are:

L	R	n	d_{mean}	$\rho_{\mathrm{DM}}/\rho_{\mathrm{B}}$	x_{obs}	island floor
18	1800	4	0.479	16.1	3.01	0.002
18	2200	4	0.476	31.5	5.87	0.003
18	2600	4	0.449	2.28	0.43	0.002
18	3200	4	0.433	0.764	0.14	0.001
20	1800	4	0.497	23.5	4.38	0.003
20	2200	4	0.474	17.2	3.21	0.006
20	2600	4	0.456	44.8	8.35	0.004
20	3200	4	0.436	0.0146	0.00	0.000

Here x_{obs} is the measured ratio divided by 5.36. The larger $L = 18, 20, 22$ surface confirmed the diagnostic rather than reviving debris as halo dark matter. Boundary-local rescue reduces the strict-pair load by roughly 10^3, but the result is zero-inflated and dominated by rare depth-one orphan islands. The archived summary has mean $\rho_{\mathrm{DM}}/\rho_{\mathrm{B}} = 13.9$, median 3.04, and a best median crossing near $(L, R) = (22, 2600)$; rows with detached depth-one islands jump to tens of times the observed load, while no-island rows sit near a few times observed. This is not a stable smooth prediction. It supports the later conclusion: K04 debris is a pinned fossil network whose live abundance variable is the KZ wall correlation length, while mobile halo dark matter must come from a different carrier.

7 The w_6 Anchor and Cooling Driver

The K04 numerical residuals are now split. The relative weight $w_4/w_6 = 2$ is conditionally derived from the Q3 loop-orbit edge ledger. The absolute unit and the cosmological cooling law remain separate bridges.

7.1 The $w_6 \leftrightarrow \Lambda_{\rm QCD}$ Bridge

K04 energies are measured in w_6. There is no K04-internal absolute scale: the Metropolis dynamics is exactly scale-invariant, so no simulation of the K04 move set can by itself determine $w_6/\Lambda_{\rm QCD}$. The anchor epoch is nevertheless no longer an arbitrary choice once the bridge is stated as a service-tension question. The Q3 edge-ledger normal ordering gives $w_4/w_6 = 2$, hence the native K04 line-service tension is

$$\mu = w_4 + 4w_6 = 6w_6.$$

The simulation ramp starts at the same K04 unit, $T_{\rm start} = 6w_6$. Thus ramp start is not merely a convenient high-temperature endpoint; it is the native line-service quantum. If the strong-sector anchor prices that service quantum, $\mu = \Lambda_{\rm QCD}$, then

$$T_{\rm start} = 6w_6 = \Lambda_{\rm QCD}, \qquad w_6 = \frac{\Lambda_{\rm QCD}}{6} \simeq 0.167\,\Lambda_{\rm QCD}.$$

With $e_D \simeq 2.17w_6$, this gives a debris excess scale

$$e_D \simeq 0.36\,\Lambda_{\rm QCD}.$$

Anchoring instead at the transition/arrest epoch gives $w_6/\Lambda_{\rm QCD} \simeq 0.33$–$0.36$. Equivalently, it would make the native line-service tension $\mu \simeq 1.9$–$2.1\,\Lambda_{\rm QCD}$, inserting a new dimensionless offset not supplied by the K04 service ledger. The transition anchor is therefore rejected as a derivation in the present canon; it remains only a named alternative postulate for a future primitive in which the heat-capacity response, rather than the line-service tension, is the object billed by $\Lambda_{\rm QCD}$.

7.2 The Cooling Law

The K04 crystallisation protocol contains a strong candidate cooling driver:

$$\gamma = 0.995, \qquad R = \frac{\ln(0.5/6)}{\ln(0.995)} \simeq 495.7.$$

This is canon-pinned as a protocol parameter. It is not yet derived as a physical boot law. The attempted entropy-spike derivation of $\gamma = \exp(-\alpha_0 \ln 2)$ missed its pre-registered target by a large margin, so the finite-run γ is now treated as a simulation proxy. The live physical statement is boundary-printer dilution:

$$\dot{F} = -nHF, \qquad \tau_Q = \frac{1}{nH_c},$$

so the KZ correlation length is controlled by the crystallisation epoch H_c and the KZ exponent. Both are now derived rather than assumed.

The line-tension/ramp-start anchor prices the boot bath at the chiral scale, $T_{\rm boot} = 6w_6 = \Lambda_{\rm QCD}$, so crystallisation is a QCD-epoch event and the radiation-era rate is

$$H_c = 1.66\sqrt{g_*}\,\frac{T_{\rm boot}^2}{M_{\rm Pl}} \simeq 1.2 \times 10^{-19}\,{\rm GeV} \simeq 1.8 \times 10^5\,{\rm s}^{-1}$$

($g_* \simeq 62$; Hubble time $\sim 6\,\mu$s), with substrate tick $\tau_0 = 1/\Lambda_{\rm QCD}$ and $\tau_Q/\tau_0 \sim 3 \times 10^{18}$ at $n = 1$. Finite-size scaling of the ramp sweeps ($L = 6$–16, many replicas) pins the codimension-one KZ trapping exponent for the first time,

$$d_{\rm final} \propto R^{-\beta}, \qquad \beta = 0.16 \pm 0.02 \;\; (L \geq 6),$$

so $\xi(R) \propto R^{\beta}$. The exponent is real but *weak*, and the boot-window deficit saturates at an L-independent glassy plateau $d \simeq 0.77$: the K04 quench is a constraint glass, not Allen–Cahn coarsening, and does not coarsen. Within the physical boot window ($R \lesssim 204$) the correlation length therefore never leaves the lattice scale, $\xi \simeq 1\text{–}2\, a_0$; the relic is a *dense, frozen* wall network, not a long-wavelength KZ texture.

7.3 The Fossil Abundance Bound

With ξ pinned at the lattice scale and the pinned-wall equation of state $w = -p/3 = -2/3$, the bare frozen network barely redshifts. Using the sim-measured excess $e_D \simeq 2.17 w_6$ per debris vertex, the wall energy at freeze-out is $\sim 1.4\%$ of the radiation density; evolved as $\rho_{\text{wall}} \propto a^{-1}$ to the present this gives

$$\Omega^{\text{bare}}_{\text{K04}} h^2 \simeq 10^{31},$$

the Zel'dovich domain-wall catastrophe — here *worsened* by the non-coarsening glass, since even an idealised one-wall-per-horizon scaling solution ($\sigma \sim \mu \Lambda^2_{\text{QCD}}$) already overcloses by $\sim 10^3$. K04 is therefore viable only if the wall-shadow truncation of the preceding section suppresses the gravitating fraction by $\sim 10^{-32}$: a near-total erasure. This is the quantitative content of the pinning verdict. The KZ normalisation does not close into a halo abundance; it closes into an overclosure bound that forces the K04 wall fossil to be gravitationally negligible, leaving the cold clustering budget on the $\nu_R + R4$ zero-mode branch.

8 $R4$ Line Currents and MOND

The MOND branch is a separate mechanism. The corrected form is not "negative pressure" and not a finite-register saturation story. It is a Poisson service-history theorem. Here a_0 is the MOND acceleration scale, empirically $a_0 \simeq 1.2 \times 10^{-10}\,\text{m}\,\text{s}^{-2}$ [9, 14]. One exposure should be named up front: a_0 enters this paper as a measured input, not a derived scale. Its proximity to the cosmological acceleration — $a_0 \sim cH_0/2\pi$ to within about fifteen percent — is the coincidence already noted by Milgrom [14]; in this framework it points at the horizon sector, and the closure-gate script in the appendix keeps the MOND mechanism's bookkeeping explicitly separate from the horizon-class a_0 and G questions rather than silently borrowing either.

The finite R4 repair graph has three sterile source corners and six legal repair records:

$$\nu_R \to e_R, \qquad \nu_R \to \nu_L$$

in each of the three generation sectors, with labels I_3 and locked χ/W. One tick is still exclusive: a fresh ancilla records either vacuum/no-op or one repair. The halo observable is not that finite flag. It is the repeated Stinespring service history. Fresh ancillas tensor over ticks, and after order is forgotten they form a count-valued Fock/event ledger. Under the shared P1 active-address scheduler premise, birth and death are the same one-record service clock.

Let

$$x = \frac{|\mathbf{g}|}{a_0}.$$

The scheduler supplies

$$n \to n+1 \quad \text{at} \quad \Gamma_0 x, \qquad n \to n-1 \quad \text{at} \quad \Gamma_0 n,$$

then the stationary distribution is the Poisson law of an $M/M/\infty$ service queue,

$$P(n) = e^{-x} \frac{x^n}{n!}, \qquad \mathbb{E}[n] = \text{Var}(n) = x.$$

The R4 incidence factor is 2/3, because there are two legal R4 repair edges per three generation sectors. Thus

$$\lambda_{R4} = \frac{2}{3}\frac{|\mathbf{g}|}{a_0}$$

when the susceptibility is unity. More generally,

$$\lambda_{R4} = \frac{2}{3}\chi_{R4}\frac{|\mathbf{g}|}{a_0}.$$

The Stinespring-history plus same-service-class result is precisely $\chi_{R4} = 1$. This is stronger than the old conditional Poisson ansatz, but it is not an unconditional derivation of a_0 or of the galaxy core rule.

8.1 AQUAL Action and BTFR

With one-dimensional support and the standard quadratic Newtonian edge stiffness, the effective action is

$$S_{R4} = \int \frac{|\mathbf{g}|^3}{12\pi G a_0}\, d^3x.$$

Variation gives the deep-MOND/AQUAL equation

$$\nabla \cdot \left(\frac{|\mathbf{g}|}{a_0}\mathbf{g}\right) = 4\pi G\rho_b.$$

For a spherical baryonic mass M_b, this gives

$$|\mathbf{g}| = \frac{\sqrt{GM_b a_0}}{r}, \qquad v_\infty^4 = GM_b a_0.$$

With susceptibility retained,

$$v_\infty^4 = \frac{a_0}{\chi_{R4}} GM_b.$$

Thus $\chi_{R4} = 1$ is not cosmetic. It is the BTFR normalization. The empirical surface it must reproduce is sharp: $v_\infty^4 \propto GM_b$ holds over roughly five decades of baryonic mass with normalisation set by a_0 [12, 19], and its local generalisation, the radial acceleration relation, ties the observed acceleration to the baryonic one galaxy by galaxy [13].

8.2 What Was Retired

Two older obstructions or stories are now retired:

- **Zero-bias thermal obstruction.** Finite R4 repair moves have raw strain changes

  $$\Delta F \in \{-4, -1, +1\}.$$

  A KMS/Boltzmann reading therefore cannot produce matched rates. The current canon says this was the wrong ensemble: repair rates are scheduler-clocked service records, not thermal weights on raw strain-energy deltas.

- **Negative pressure / constant-tension Jeans support.** A literal *constant*-tension Jeans model does not produce the required halo structure. The surviving support is the action and line-current route, equivalently a *variable* anisotropic log-tension $\tau(r) \to v_\infty^2 \ln(r/r_c)$ (see Cored Profiles and Jeans Targets, below); only the constant-tension equation of state is retired.

This is progress, not full closure. The same scheduler-clock premise now appears in the cosmological-constant chain, which is a useful consistency check. The count-valued nonexclusive ledger is no longer an extra many-microedge assumption once the Stinespring history is the observable. The remaining halo-sector gates have now split. The scheduler part of P1 is no longer a broad rate-matching assumption: under the same monitored service-class reading, creation and recycling are one Γ_0-clocked R4 event class, so the common clock cancels and the virial ledger is Poisson. The residual is the separate one-a_0 phase-return latch, plus the inherited Newtonian/G stiffness normalisation and external galaxy phenomenology.

9 Cored Profiles and Jeans Targets

The line-current action gives the asymptotic MOND law. A finite galaxy still needs a core regulator. The minimal profile currently carried in canon is

$$\rho_{\text{line}}(r) = \frac{A}{r^2 + r_c^2}.$$

It is the minimal [0/1] Pade regularisation of a finite-center, $1/r^2$-tail halo with no extra shape parameter — in fact the classical pseudo-isothermal halo of rotation-curve phenomenology. The empirical preference of dwarf and low-surface-brightness rotation curves for such cored profiles over the cuspy Navarro–Frenk–White form of collisionless cold-dark-matter simulations [5, 15] is what makes the core regulator a live structural target rather than a cosmetic choice. It is not the unique consequence of finite center plus $1/r^2$ tail alone; higher rational families exist.

The minimal/no-extra-shape premise is supplied by the same single-service structure. The matched-rate Poisson line ledger is a *single*-rate process — one service rate Γ_0, one boundary-QEC event per tick, not parallel service — so it carries a single scale and a single Poisson pole. The two load-bearing conditions, "no shape parameter beyond r_c" and "minimal Padé degree", are therefore not extra assumptions but the single-rate, single-event structure; a higher rational regulator would need a second independent service rate (parallel service), which the same-service theorem forbids. Thus the scheduler leg forces the [0/1] cell $\rho = A/(r^2 + r_c^2)$. The core radius then uses a different residual: the one-a_0 phase-return latch. If R4 commits on the primitive KMS phase-return projector, hidden integer cadences are excluded by repeatability and minimality, and the central constant-density cell reaches one a_0 quantum at $g_{\text{in}}(r) = \frac{4\pi}{3} G\rho(0)\, r = a_0$ when $r = r_c$, giving $r_c = r_{\text{M}}/3$. This is the cell-local reading of the one-a_0 rule; the global enclosed-field value derived next is its continuum-integral counterpart, and the two bracket the actual MOND phantom ($\simeq 0.29\, r_{\text{M}}$).

Two core-radius rules have circulated, and they are now reconciled: they are the *same* boundary condition $g(r_c) = a_0$ read at two different radii. With the exact halo field $g_h(r) = (v_\infty^2/r)\,[1 - (r_c/r)\arctan(r/r_c)]$,

$$g_h(r_c) = \frac{v_\infty^2}{r_c}\Big(1 - \tfrac{\pi}{4}\Big) = a_0 \quad\Longrightarrow\quad \frac{r_c}{r_{\text{M}}} = 1 - \frac{\pi}{4} \simeq 0.2146, \qquad r_{\text{M}} = \sqrt{\tfrac{GM_b}{a_0}}.$$

The earlier $r_c = r_{\text{M}}/3$ is this *same* condition in the $r \ll r_c$ limit, where $g_h \to v_\infty^2 r/(3r_c^2)$: the 1/3 is the leading small-radius (central-cell) coefficient, and it overshoots the exact value by $(1/3)/(1 - \pi/4) \simeq 1.55$. So the two are not rival fits — $1 - \pi/4$ is the self-consistent *global* enclosed-field core, and 1/3 is its *local central-cell* reading. A direct check against the actual MOND phantom density of a point mass ($\rho_{\text{ph}} = \frac{1}{4\pi G}\nabla\cdot\mathbf{g} - \rho_b$) puts the half-asymptote core at $r_c \simeq 0.29\, r_{\text{M}}$ for the simple interpolation and $\simeq 0.75\, r_{\text{M}}$ for the standard one — bracketing $1 - \pi/4$ and confirming that the pseudo-isothermal r_c is an interpolation-dependent *regulator* with the global $1 - \pi/4$ as its principled reference, not a parameter-free constant.

Jeans support now closes. Reading the cored profile as an anisotropic static stress ($p_r = -\tau\rho$, $p_t = 0$) in the halo field, the equilibrium $\tau' + [2r_c^2/(r(r^2 + r_c^2))]\,\tau = g_h$ has the regular solution

$$\frac{\tau(r)}{v_\infty^2} = \frac{r^2 + r_c^2}{r^2}\int_0^{r/r_c} \frac{u}{1+u^2}\Big[1 - \tfrac{\arctan u}{u}\Big]du \xrightarrow{r \gg r_c} \ln\frac{r}{r_c},$$

with $\tau(0) = 0$ ($\tau \sim r^2/12$ near the centre) and $\tau > 0$ throughout. The support is therefore a *variable* log-tension, not a constant one: the retired constant-tension Jeans model failed only because it assumed the wrong equation of state. The core boundary and the support equation are now both in closed form — a sharp structural target for the MOND paper, not a loose phenomenological fit.

10 $R4$ Zero-Mode Reservoir and CMB Completion

The active $R4$ Stinespring line ledger does not by itself complete the CMB. In a homogeneous recombination background the halo acceleration field is zero, so the active Poisson line count has zero homogeneous density. Nor does a cumulative service archive behave as conserved dust: it is a time-integrated event history, not a local number density with $\rho \propto a^{-3}$. Frozen R4 lines have $w = -1/3$, and K04 walls are even more dark-energy-like. The CMB third peak therefore requires a separate pressureless slot.

The live phenomenological slot is the R4 zero-mode reservoir. It is not derived from the active R4 immigration–death ledger: that documented Lindbladian has only the identity as a conserved observable and relaxes to the vacuum at homogeneous recombination. The zero-mode therefore requires a separate non-dissipative conserved massive record number. If that premise is admitted, the finite operator inventory contains a rest-count Hamiltonian and local active exchange only. Gradient stiffness, reservoir hopping, phase stiffness, and elastic shear would all be new operators. Therefore the minimal Hamiltonian has

$$w = 0, \qquad c_s^2 = 0, \qquad \rho_{\text{zero}} \propto a^{-3}.$$

This is the finite record-count analogue of Brown–Kuchar dust [3]: a constrained rest-density variable rather than a propagating scalar with gradient pressure.

The current conditional source law is

$$\frac{n_{\nu_R}}{n_\gamma} = \frac{\alpha_0}{208},$$

where $208 = 256 - 48$ is the service-complement alphabet, not the old "$208 - 3$" gravity count. The $R4$ source algebra is generation-blind: the three sterile source corners couple through the unique S_3-invariant bright port rather than through three independent generation ports. Three ports would overproduce the dark budget. With

$$m_{\nu_R} = \alpha_0^2 \Lambda_{\text{QCD}} \simeq 17.7\,\text{keV},$$

and the directed R4 service-edge incidence giving a 4:1 zero-mode-to-sterile split, the density ledger is

$$\Omega_{\nu_R} h^2 = 0.02418, \qquad \Omega_{\text{zero}} h^2 = 0.09671, \qquad \Omega_{\text{dark}} h^2 = 0.12089,$$

with $z_{\text{eq}} = 3430$. A diagnostic CAMB sweep [11] confirms the shape of the requirement: the sterile share alone leaves equality near recombination and underproduces the relative third peak, while adding $\Omega_x h^2 \simeq 0.096$ of pressureless low-sound-speed matter restores equality and the third-to-second peak scale.

This is a conditional effective completion, not a free fit and not a fully native R4 theorem. The source-map clause is sharp at the bare-α_0 tier, but it only applies once the conserved dust carrier itself is admitted. The release power p in $n_{\nu_R}/n_\gamma = \alpha_0^p/208$ is *predicted* to be $p = 1$ by the non-unitary billing rule — one α_0 per non-unitary syndrome firing, the same statement ($\alpha_0 = \mathrm{Tr}_{\text{non-unit}}$, item 79) that fixes the fine-structure constant — and is *independently data-selected*: $p = 0$ (free release) overproduces by $1/\alpha_0 \simeq 137$ and $p = 2$ (two firings) underproduces by the same factor, so the CMB third peak fixes $p = 1$ to one part in 137, robustly to m_{ν_R}. The dark-source rate is therefore not a new constant: it is the same α_0 that bills electromagnetism, charged once through the single S_3-singlet sterile port (derived) and addressed uniformly over the 208-label service complement (derived). What remains is not the billing but the absolute mass $m_{\nu_R} = \alpha_0^2 \Lambda_{\mathrm{QCD}}$ (item 118), and the Boltzmann/halo implementation. A halo-model depletion audit now narrows this fork: if the zero-mode is treated as CDM-like dust, standard halo profiles do not remove it from galaxies. Keeping active R4/MOND as an independent galaxy force law would therefore require a new depletion/screening theorem, globally at the $> 95\%$ level and locally at the 90–97% level in realistic outer-disk halo brackets. A fair-sample zero-mode halo plus an independent active-MOND force double-counts the same galaxy mass budget. Relativistic MOND/AeST constructions [18] are useful comparison points because they show what a pressureless additional field must do, but this paper treats the zero-mode reservoir as the framework-native candidate.

11 Debris, MOND, and Observations

The two dark mechanisms make different predictions.

K04 debris is gauge-blind because the wall interior has no registers. Its effective gravitational mass is localized through the boundary shadow, but the later depinning audit shows that the canonical local plaquette dynamics does not translate these walls: the substrate drive is tens of orders of magnitude below the transport barrier, and thermal creep is negligible. K04 is therefore substrate-static fossil structure, not ordinary collisionless halo matter. Bullet-cluster style lensing [4] is still a genuine discriminant, but the sign is now an upper bound: any mass component that tracks galaxies through a merger is charged to R4/MOND, sterile-ν_R, or another mobile completion, not to K04. The same merging systems bound dark-sector self-interaction near $\sigma/m \lesssim 1\,\mathrm{cm}^2\,\mathrm{g}^{-1}$ [17]; this is background context for mobile dark matter, while the canonical K04 branch is pinned rather than scattering.

R4/MOND line current is tied to baryonic gravitational strain. Its clearest prediction is the BTFR normalization and the deep-MOND interpolation regime. Its cored-profile regulator must reproduce galaxy cores without resurrecting the refuted constant-pressure Jeans mechanism. The line-current side is sharper after the Stinespring-history audit. The one-tick finite R4 Kraus map is still exclusive, but the repeated service history uses fresh environment records. After virial coarse-graining the same coarse line therefore carries a count-valued event ledger, $N = 0, 1, 2, \ldots$, rather than a single saturated syndrome flag. Under the P1 scheduler-clock reading, creation and erasure are one-record events on the same service clock, so the stationary immigration–death law is Poisson with mean $|g|/a_0$. The finite R4 incidence then gives

$$\lambda_{R4} = \frac{2}{3}\frac{|g|}{a_0}, \qquad \chi_{R4} = 1.$$

This dissolves the old rate-matching and finite-flag saturation obstruction, but only at P1 theorem grade. It does not by itself derive the absolute a_0 phase latch or the minimal cored-profile regulator.

For the core, the newer ledger-selection result separates two questions that were previously conflated. The minimal no-extra-shape regulator $\rho = A/(r^2 + r_c^2)$ is selected only after imposing a minimal finite-centre, $1/r^2$-tail rational profile. The boundary readout, however, belongs to the local R4 service-current ledger rather than the post-summed enclosed gravitational field. That local central-cell readout gives

$$\frac{r_c}{r_M} = \frac{1}{3}$$

once one local a_0 service quantum is granted. The exact enclosed-field condition would give $1 - \pi/4$, but that is a wrong-ledger observable for the R4 service-current core rule. The remaining microscopic lemma is therefore the one-a_0 phase-latch/service-quantum rule, not an enclosed-field boundary.

The eventual mobile-halo budget reconciliation is not charged to K04 unless a new mobility primitive is derived. The archived island-floor surface and the KZ wall-network scaling instead set fossil upper bounds and bookkeeping checks on any smooth substrate-static component. The CMB third peak requires a cold, pressureless clustering component already present at recombination; K04 walls do not supply it. The live budget therefore sits on the $\nu_R + R4$-zero-mode branch for the acoustic-peak role. Galaxy halos then have a narrowed fork: use the zero-mode as the CDM-like mobile halo component, with active R4/MOND demoted to a possible subleading polarisation response, or derive a new depletion/screening operator before retaining active R4/MOND as an independent baryonic response. The current halo-model audit finds no natural $> 95\%$ depletion in the zero-mode-as-CDM dynamics. A sharper non-double-counted target is now available: N_{active} may be the baryon-coupled response $\delta N_{\text{active}}[N_{\text{zero}}, \rho_b]$ of the same zero-mode reservoir. In that reading the active R4 field arranges the zero-mode mass, and the cored profile fixes a finite edge

$$\frac{M_z(R_t)}{M_b} = \frac{\Omega_z}{\Omega_b f_{\text{ret}}}, \qquad \frac{M_z(y r_M)}{M_b} = y - \frac{1}{3}\arctan(3y).$$

The full-retention lower edge is $R_t = 4.8179\, r_M$; for a Milky-Way-like $6 \times 10^{10} M_\odot$ baryonic galaxy with $f_{\text{ret}} = 0.25$, the same ledger gives $R_t \simeq 159\,\text{kpc}$. The constitutive formation law has now been reduced to a local shell-capture rule. The R4 $p = 3$ susceptibility fixes the asymptotic shell demand $dM_z/dr = 4\pi A$. Regular geodesic zero-mode capture must vanish as r^2 at the origin, while the deep-MOND tail must saturate to that demand. With no extra shape parameter the minimal even latch is

$$F(x) = \frac{x^2}{1 + x^2}, \qquad x = r/r_c,$$

so $dM_z/dr = 4\pi A r^2/(r^2 + r_c^2)$ and $\rho_z = A/(r^2 + r_c^2)$. The one-a_0 central-cell phase latch then gives $r_c = r_M/3$. The local shell-capture latch is now derived relative to P1: the bounded record has two states, U and L, with matched service rates $U \to L : \Gamma_0 x^2$ and $L \to U : \Gamma_0$, so stationarity gives $F = x^2/(1 + x^2)$. The scheduler leg is now a same-service-class theorem; the remaining local coefficient is the primitive phase-return operator that licenses one a_0 per central-cell service record. Accepting that latch leaves the full galaxy formation and fitting problem, not a free halo-shape parameter. A first nonlinear bracket gives $R_t \simeq 159\,\text{kpc}$ and an outer-disk residual of 0.047 dex for a Milky-Way-like $6 \times 10^{10} M_\odot$ baryonic galaxy with $f_{\text{ret}} = 0.25$.

12 Status Ledger

Claim	Status	Current reading
Toy K04 plateau	Retired	Configuration model admitted $K_{3,3}$ bulk order; not physical debris.
Embedded K04 crystallisation	Computed	$\mathbb{Z}^3$ bond-subset ensemble crystallises strongly; equilibrium defects sparse and healable.
KZ trapped debris	Computed / Conditional	Fast ramps produce positive-mass, durable wall debris; later audits classify it as pinned fossil/subdominant, not mobile halo matter.
Bare debris energy as gravity source	Retired	Wall interiors have no registers; gravity reads recorded boundary shadow.
Boundary-local rescue	Retired / diagnostic	Strict pairing and fully adaptive rescue are excluded; island floor is zero-inflated and archival.
K04 wall-network abundance	Computed	FSS pins $\beta = 0.16 \pm 0.02$ on a glassy plateau $d \simeq 0.77$ ($\xi \sim$lattice); bare network overcloses $\sim 10^{31} \Rightarrow$ shadow-suppressed fossil, not halo.
w_4/w_6	Conditional	$w_4/w_6 = 2$ from Q3 loop-orbit edge-ledger normal ordering.
w_6 anchor	Conditional	Ramp-start $w_6 = \Lambda_{\mathrm{QCD}}/6$ follows from the line-service anchor $\mu = w_4 + 4w_6 = 6w_6 = \Lambda_{\mathrm{QCD}}$; transition anchoring would make $\mu \simeq 2\Lambda_{\mathrm{QCD}}$ and is a new postulate.
$\gamma = 0.995$ cooling	Computed	Proxy retired; physical law $\tau_Q = 1/(nH_c)$ with $H_c \simeq 1.2 \times 10^{-19}$ GeV derived at $T_{\mathrm{boot}} = \Lambda_{\mathrm{QCD}}$.
Kibble–Zurek fossil EoS / mobility	Computed	Pinned extended defects have $w = -p/3$; dominant K04 is not cold dust and cannot follow galaxies without new dynamics.
R4 Poisson line theorem	Conditional theorem	Actual R4 Stinespring service history gives a count-valued nonexclusive virial line ledger; same monitored service class gives Poisson(x), Fano 1, and $\chi_{R4} = 1$ without an η/κ knob. The finite one-tick Kraus flag remains exclusive.
AQUAL/BTFR action	Conditional	Follows from $d = 1$ support, $\chi_{R4} = 1$, and inherited Newtonian edge stiffness; the Poisson/scheduler leg is no longer the rate-matching blocker.
Cored profile	Conditional	Minimal $\rho = A/(r^2 + r_c^2)$ is now the shell-capture law $dM_z/dr = 4\pi A x^2/(1 + x^2)$ with $x = r/r_c$: R4 $p = 3$ fixes A, regular zero-mode capture fixes the central x^2, and the same-service two-state latch gives the denominator. The local central-cell $r_c = r_{\mathrm{M}}/3$ rule still rests on the primitive one-a_0 phase latch; the first nonlinear bracket is viable but not a full galaxy fit.
$\nu_R + R4$ zero-mode CMB slot	Conditional / imported dust premise	If a conserved massive zero-mode dust charge is admitted, Brown–Kuchar form and 4:1 split give $\Omega_{\mathrm{dark}} h^2 = 0.12089$; documented active R4 dynamics do not derive that charge. Residuals: source billing, m_{ν_R}, full likelihood, and halo branch discipline.
Halo branch	Computed / Conditional	Halo-model audit rejects natural $> 95\%$ galaxy depletion for CDM-like zero-mode dust; current conservative branch is zero-mode CDM-like halos plus at most subleading active-R4 polarisation. A stronger P1-conditional branch treats active R4 as the response of the same zero-mode reservoir, with a finite cored-profile edge and no second mass ledger.

13 Open Frontiers

1. **KZ wall-network normalization** (*closed, conditional*). The boot cooling law, $H_c \simeq 1.2 \times 10^{-19}$ GeV (QCD epoch), and the KZ length $\xi(R) \propto R^{0.16\pm0.02}$ are now derived; finite-size scaling ($L = 6$–16) shows a glassy plateau ($d \simeq 0.77$, ξ ~lattice), so the bare wall fossil overcloses by $\sim 10^{31}$ and survives only as a near-totally shadow-truncated pinned relic. What remains open is the *shadow-depth* normalisation fixing the surviving gravitating fraction.

2. w_6 **scale.** The anchor epoch is now selected at derived-conditional tier: $w_4/w_6 = 2$ makes the native K04 line-service tension $\mu = w_4 + 4w_6 = 6w_6$, exactly the ramp-start unit. Pricing μ by Λ_{QCD} gives $w_6 = \Lambda_{\mathrm{QCD}}/6$; the transition anchor survives only as a different primitive that would price an emergent response temperature rather than the service tension.

3. **Zero-mode dust and source billing.** The pressureless slot has the right effective form only if a conserved massive zero-mode charge is admitted; documented active R4 dynamics do not supply it. Conditional on that carrier, the S_3-singlet port, uniform 208-addressing, and 4:1 incidence give the current $\nu_R + N_{\mathrm{zero}}$ budget. The remaining load-bearing questions are the sector billing/source map, m_{ν_R}, and the halo non-double-counting clause below.

4. **Boltzmann/halo branch.** Implement the zero-mode as a Brown–Kuchar dust species in a perturbation/structure calculation. The current halo-model audit says CDM-like zero-mode dust does not naturally deplete from galaxies; retaining galaxy-scale active R4/MOND as an independent response therefore requires a new $> 95\%$ depletion/screening theorem. The cleaner alternative is the polarised-zero-mode theorem target: active R4 is not an independent mass ledger, but the baryon-coupled response that arranges the same zero-mode reservoir into the cored halo profile. The profile derivation now closes relative to the same-service shell-capture latch $F = x^2/(1+x^2)$, not to a free halo shape; the remaining local step is the one-a_0 phase-return operator, followed by full galaxy/formation phenomenology.

5. **P1 and MOND core.** The Stinespring-history line ledger plus same monitored service class closes the nonexclusive virial count and retires the finite-flag saturation and matched-rate objections. The minimal cored-profile law is now the same-service shell-capture latch; the remaining local core target is the primitive one-a_0 central-cell phase-latch rule. The enclosed-field $1 - \pi/4$ boundary is now a rejected wrong-ledger readout for the R4 service-current core.

6. **External scales.** The MOND law still imports the horizon acceleration a_0, with $a_0 = cH_0/(2\pi)$, together with G and the Newtonian edge stiffness from the shared horizon/gravity sector.

7. **New wall transport primitive.** K04 is pinned under the canonical local move class. Reopening K04-as-halo requires a new exact zero-barrier translation primitive, not a parameter adjustment.

A Reproducibility Starter Table

Script	Sector	Purpose
`python_code/debris_dark_matter_audit.py`	K04 debris	Shows the toy $K_{3,3}$ artifact, embedded-substrate rescue, and five-property debris ledger.
`python_code/k04_embedded_sweep.py`	K04 debris	Runs embedded degree-three $\mathbb{Z}^3$ bond-subset crystallisation jobs.
`python_code/k04_embedded_analysis.py`	K04 debris	Locates ordering windows and shows equilibrium debris is sparse/healable.
`python_code/k04_kz_analysis.py`	K04 debris	Analyzes Kibble–Zurek ramp trapping, mass, and durability readouts.
`python_code/k04_defect_network_abundance.py`	K04 debris	Shows the KZ quench is homology-trivial, the strict topological relic is negligible, and $n_{\text{wall}} \sim 2.63/\xi$.
`python_code/depinning_mobility_gate.py`	K04 debris	Demonstrates the finite transport barrier and substrate pinning of K04 relic walls.
`python_code/advection_nonlocal_ruleout.py`	K04 debris	Rules out printing advection and canonical nonlocal transport escape routes.
`python_code/k04_cooling_driver_energy_bookkeeping.py`	K04 debris	Audits $\gamma = 0.995$, ramp $R \simeq 495.7$, and energy-fraction bookkeeping.
`python_code/k04_entropy_spike_resolution.py`	K04 debris	Tests and rejects the entropy-spike derivation of the cooling driver.
`python_code/k04_absolute_scale_cooling_law_audit.py`	K04 scale	Splits w_4/w_6, w_6/Λ_{QCD}, and physical boundary-printer cooling status.
`python_code/k04_w6_anchor_epoch_selection.py`	K04 scale	Compares ramp-start and transition anchors for $w_6 \leftrightarrow \Lambda_{\text{QCD}}$.
`python_code/k04_w6_line_tension_anchor_theorem.py`	K04 scale	Selects the ramp-start anchor from $\mu = w_4 + 4w_6 = 6w_6$ and rejects transition anchoring as a new response-temperature postulate.
`python_code/k04_kz_fss_analysis.py`	K04 debris	Finite-size scaling of the KZ ramp sweeps; pins $\beta = 0.16 \pm 0.02$ and the glassy $d \simeq 0.77$ plateau.
`python_code/k04_cooling_normalization.py`	K04 scale	Derives H_c, $\xi(R)$, and the $\Omega^{\text{bare}}_{\text{K04}} \sim 10^{31}$ overclosure / shadow bound.
`python_code/k04_orphan_policy_audit.py`	wall shadow	Excludes strict-pair and fully adaptive policies; records boundary-local rescue as the archived partial branch.
`python_code/k04_island_floor_surface.py`	wall shadow	Measures the archived zero-inflated boundary-local-rescue island-floor diagnostic.
`python_code/item132_chi_unit_poisson.py`	MOND	Proves the conditional matched-rate Poisson theorem and $\chi_{R4} = 1$.
`python_code/item132_r4_kraus_poisson_lift_audit.py`	MOND	Audits the finite R4 Kraus map and sets up the later Stinespring-history lift.
`python_code/item132_scheduler_form_closure.py`	MOND	Retires the KMS zero-bias obstruction by scheduler-clocked record counting.
`python_code/item132_r4_stinespring_fock_lift.py`	MOND	Builds the actual R4 Kraus/Stinespring repair channel and closes the nonexclusive count ledger under the service-history reading.
`python_code/item132_p1_service_class_reduction.py`	MOND	Splits P1: same monitored service class closes the Poisson scheduler leg, while the a_0 scale remains separate.
`python_code/item132_a0_phase_latch_minimality.py`	MOND	Shows the primitive KMS phase-return latch would force $a_0 = cH_0/(2\pi)$ and exclude hidden integer cadences, but is not derived from finite R4 labels alone.
`python_code/item132_r4_local_action_lift.py`	MOND	Derives the local cubic action from $d = 1$ support and edge stiffness.
`python_code/item132_halo_closure.py`	MOND	Audits BTFR, cored profile, and Jeans failure modes.
`python_code/item132_jeans_consistency.py`	MOND	Reconciles r_c $(1 - \pi/4$ vs $1/3)$ against the MOND phantom core; closes Jeans support as variable log-tension.
`python_code/item132_residuals_status_audit.py`	MOND	Consolidates remaining support, stiffness, and core-profile residuals.
`python_code/gravity_mond_closure_gate.py`	MOND/gravity	Checks that MOND closure remains distinct from the horizon-class a_0 and G questions.
`python_code/item123_cmb_zero_mode_theorem.py`	CMB/halo	Builds the conditional pressureless $R4$ zero-mode reservoir/effective-dust gate.
`python_code/item123_nuR_absolute_density_boot_qec.py`	CMB source	Computes the $\alpha_0/208$ sterile source law and total dark density candidate.
`python_code/item123_omega_dark_absolute_chain.py`	CMB source	Consolidates the one-port sterile source and 4:1 zero-mode incidence into the current conditional $\Omega_{\text{dark}}h^2$ chain.
`python_code/item123_sterile_generation_s`	CMB source	Proves generation-blind one-port release for the

References

[1] Jacob Bekenstein and Mordehai Milgrom. Does the missing mass problem signal the breakdown of Newtonian gravity? *Astrophys. J.*, 286:7–14, 1984.

[2] G. Bertone, D. Hooper, and J. Silk. Particle dark matter: evidence, candidates and constraints. *Phys. Rep.*, 405:279–390, 2005.

[3] J. David Brown and Karel V. Kuchař. Dust as a standard of space and time in canonical quantum gravity. *Phys. Rev. D*, 51:5600–5629, 1995.

[4] D. Clowe, M. Bradač, A. H. Gonzalez, M. Markevitch, S. W. Randall, C. Jones, and D. Zaritsky. A direct empirical proof of the existence of dark matter. *Astrophys. J. Lett.*, 648:L109–L113, 2006.

[5] W. J. G. de Blok. The core-cusp problem. *Adv. Astron.*, 2010:789293, 2010.

[6] A. del Campo and W. H. Zurek. Universality of phase transition dynamics: topological defects from symmetry breaking. *Int. J. Mod. Phys. A*, 29:1430018, 2014.

[7] S. Dodelson and L. M. Widrow. Sterile neutrinos as dark matter. *Phys. Rev. Lett.*, 72:17–20, 1994.

[8] David Elliman. It-for-bit model: Reproducibility repository. `https://github.com/dgedge/itforbit_model.git`, 2026. Code and reproducibility repository for the canon snapshot series.

[9] Benoit Famaey and Stacy S. McGaugh. Modified Newtonian dynamics (MOND): observational phenomenology and relativistic extensions. *Living Rev. Relativity*, 15:10, 2012.

[10] Tom W. B. Kibble. Topology of cosmic domains and strings. *J. Phys. A*, 9:1387–1398, 1976.

[11] Antony Lewis, Anthony Challinor, and Anthony Lasenby. Efficient computation of cosmic microwave background anisotropies in closed FRW models. *Astrophys. J.*, 538:473–476, 2000.

[12] S. S. McGaugh, J. M. Schombert, G. D. Bothun, and W. J. G. de Blok. The baryonic Tully–Fisher relation. *Astrophys. J. Lett.*, 533:L99–L102, 2000.

[13] S. S. McGaugh, F. Lelli, and J. M. Schombert. Radial acceleration relation in rotationally supported galaxies. *Phys. Rev. Lett.*, 117:201101, 2016.

[14] Mordehai Milgrom. A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis. *Astrophys. J.*, 270:365–370, 1983.

[15] J. F. Navarro, C. S. Frenk, and S. D. M. White. The structure of cold dark matter halos. *Astrophys. J.*, 462:563–575, 1996.

[16] Planck Collaboration. Planck 2018 results. VI. cosmological parameters. *Astron. Astrophys.*, 641:A6, 2020.

[17] S. W. Randall, M. Markevitch, D. Clowe, A. H. Gonzalez, and M. Bradač. Constraints on the self-interaction cross section of dark matter from numerical simulations of the merging galaxy cluster 1E 0657-56. *Astrophys. J.*, 679:1173–1180, 2008.

[18] Constantinos Skordis and Tom Złośnik. New relativistic theory for modified Newtonian dynamics. *Phys. Rev. Lett.*, 127:161302, 2021.

[19] R. B. Tully and J. R. Fisher. A new method of determining distances to galaxies. *Astron. Astrophys.*, 54:661–673, 1977.

[20] Alexander Vilenkin and E. Paul S. Shellard. *Cosmic Strings and Other Topological Defects.* Cambridge University Press, 1994.

[21] Ya. B. Zel'dovich, I. Yu. Kobzarev, and L. B. Okun. Cosmological consequences of a spontaneous breakdown of a discrete symmetry. *Zh. Eksp. Teor. Fiz.*, 67:3–11, 1974. [Sov. Phys. JETP 40, 1 (1975)].

[22] Wojciech H. Zurek. Cosmological experiments in superfluid helium? *Nature*, 317:505–508, 1985.