

The Born Rule as a Closed Record Pair: Measurement, Objectivity, and the Arrow of Time in a Quantum-Error-Correcting Substrate

David Elliman
Neuro-symbolic Ltd
dave@neusym.ai
<https://neusym.ai>

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Abstract

This note updates the Born-rule paper to the current canon of the It-from-Bit/QEC substrate programme. The earlier version correctly identified measurement with syndrome recording, but left the Born measure itself in the same open bucket as the deeper “why quantum?” problem. The current canon separates those issues. Inside the accepted complex quantum-error-correcting record substrate, the Born rule is conditionally closed in two complementary ways. First, repeatable syndrome readout gives orthogonal projectors; self-dual stabiliser structure gives compatible, non-contextual tests; and Gleason-type uniqueness fixes $p_i = \text{Tr}(\rho P_i)$. Second, the Naimark/Stinespring record map

$$V = \sum_s \Pi_s \otimes |s\rangle_R, \quad V^\dagger V = I$$

makes measurement an isometric copying of syndrome facts into orthogonal record sectors. In a closed forward/backward record history, unmatched sectors vanish and the surviving diagonal is $AA^* = |A|^2$. This is the precise sense in which “Born is the closed record-action pair.” The paper also records the current reconstruction frontier. The minimal balanced record cell theorem forces the byte: in the binary balanced Type-II CSS class, the unique minimal distance-4 record cell is the self-dual doubly-even $[8, 4, 4]$ code. The bare service rate $\alpha_0 = 1/137$ is now grounded as the uniform Born weight of one label in the monitored symmetric record-pair alphabet $\text{Sym}^2(16) + 1$. What remains open is no longer the Born rule inside the substrate, but the philosophical-operational reconstruction floor: why nature is a complex, locally tomographic, record-writing QEC Hilbert system with the monitored service alphabet.

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1 Status and Scope

The Born rule is often stated as an axiom:

$$p_i = |\langle i|\psi\rangle|^2.$$

The question here is not whether ordinary quantum mechanics uses that rule; it does. The question is how far the substrate programme can explain the rule from its own record mechanics.

The answer is sharper than in the previous draft. The Born rule is not an open dynamical coefficient in the current canon. It is closed at foundationally grounded grade *inside* the complex-QEC record substrate. The remaining residual is one level deeper: the reconstruction floor that asks why the world is such a complex, locally tomographic, record-writing QEC system at all. That floor is the analogue of the primitive axiom set in Hardy/CDP/Masanes–Muller style reconstructions of quantum theory [1, 4].

Claim	Current status	Meaning
Pointer basis	Canon-derived	The monitored observables are syndrome projectors; the pointer basis is the joint syndrome eigenbasis.
Orthogonal records	Standard theorem / canon-verified	Repeatable readout gives projectors, and Stinespring/Naimark dilation writes the record isometrically.
Born weights	Conditionally closed	Gleason/non-contextuality and the closed-record-pair calculation both give $ A ^2$ inside the complex-QEC substrate.
Minimal record byte	Theorem in the stated class	Binary balanced Type-II CSS plus distance-4 protection has unique minimal cell [8, 4, 4].
Bare α_0 service rate	Foundationally grounded	Uniform monitored service over the symmetric record-pair alphabet gives 1/137; sector billing maps are separate audits.
Why this substrate?	Open reconstruction floor	The remaining question is why nature obeys the complex, local-tomographic, record-writing QEC axioms.

2 Measurement is Syndrome Recording

In ordinary decoherence theory, a measuring apparatus entangles a system with environmental degrees of freedom, suppressing interference in a preferred pointer basis [7]. In the present framework

the environment is not generic. It is the syndrome ledger of the substrate. A measurement is the substrate noticing that a logical degree of freedom has entered a syndrome sector and writing a record of that fact.

This makes the usual measurement ingredients less mysterious:

- the record environment is the syndrome register;
- the pointer basis is the stabiliser/syndrome eigenbasis;
- repeatability is QND syndrome readout;
- irreversibility is reusable record reset and Landauer export [5].

The point is not that the substrate adds a collapse mechanism. Before reset, record writing is an isometry. Collapse is the effective description seen by an observer who conditions on one recorded branch or discards the record environment. The substrate supplies the *record structure*; it does not replace unitary quantum mechanics by a hidden classical jump.

3 Projectors, Compatibility, and Non-Contextuality

A repeatable yes/no record test must be idempotent on its stable record states. In Hilbert-space language, such tests are projectors. The substrate implements them as syndrome projectors Π_s associated with commuting stabilisers. For the cube record cell, the stabilisers are those of the self-dual [8, 4, 4] code. The CSS commutation condition is

$$X(r)Z(s) = (-1)^{r \cdot s} Z(s)X(r),$$

and self-duality gives $r \cdot s = 0 \pmod{2}$ for the relevant row pairs. Thus the syndrome tests are compatible.

Compatibility matters because Gleason-type derivations require probabilities that do not depend on which other compatible measurements are performed in the same context [3]. Here that non-contextuality is not an independent physical taste. It is the record-system requirement that a stable syndrome fact remains the same when read together with any other compatible syndrome fact. The stabiliser algebra supplies the compatible-test structure; the record interpretation supplies the operational reason that its marginals cannot depend on a later choice of commuting refinement. This is the substrate version of measurement non-contextuality [6].

4 Gleason Route to the Born Rule

Given non-contextual probabilities over projectors in Hilbert dimension ≥ 3 , Gleason's theorem gives

$$p(P) = \text{Tr}(\rho P).$$

For a pure state and a rank-one projector $P_i = |i\rangle\langle i|$, this is

$$p_i = \text{Tr}(|\psi\rangle\langle\psi| |i\rangle\langle i|) = |\langle i|\psi\rangle|^2.$$

The substrate contribution is that the hard premise is no longer a floating axiom. The tests are syndrome projectors, their compatibility follows from the stabiliser algebra, and their non-contextuality is the statement that a recorded syndrome fact is reusable and stable across compatible readouts. This does not derive Hilbert space from nothing. It does show that, once the complex-QEC record substrate is accepted, the Born weights are the unique consistent probabilities for its projective records.

5 Closed Record Pair Route

The second route is more specific to the framework. Let the record-writing isometry be

$$V = \sum_s \Pi_s \otimes |s\rangle_R, \quad V^\dagger V = \sum_s \Pi_s = I,$$

where $\{|s\rangle_R\}$ are orthogonal record states. If a system amplitude A_s enters sector s , the closed record history contains a forward amplitude and its backward/dual partner. Orthogonality gives

$$\langle r|s\rangle_R = \delta_{rs},$$

so unmatched forward/backward sectors vanish:

$$\sum_{r,s} A_s A_r^* \langle r|s\rangle_R = \sum_s |A_s|^2.$$

The square is therefore not an arbitrary decoration on amplitudes. It is the diagonal left by a closed pair of record histories after orthogonal record sectors remove cross terms. This is the formal content of the canon phrase:

Born weight = closed record-action pair.

This route also explains why a simple equal-counting argument over classical branches is the wrong object. Equal counting over outcome strings counts records after amplitudes have already been discarded. The closed-record-pair calculation keeps the complex amplitude and its conjugate record partner until the orthogonal record trace is taken.

6 Why the Byte Appears

The current reconstruction note strengthens the substrate side of the story. The minimal balanced record cell is not simply assumed to be a byte. In the binary balanced Type-II CSS class, distance-4 erasure protection has a unique minimal cell:

$$[8, 4, 4] = \text{RM}(1, 3).$$

The logic is:

repeatable non-disturbing readout \Rightarrow commuting stabilisers,
 read/write symmetry \Rightarrow CSS/self-dual structure,
 stable complex local phase \Rightarrow doubly-even Type-II code,
 distance-4 finite-noise protection \Rightarrow $[8, 4, 4]$ as the minimal cell.

The remaining caveats are explicit: binary is the minimal record alphabet; CSS is the selected balanced read/write subclass of stabiliser records; and Type-II closure uses the local complex phase operation. Within that stated class, however, the byte is forced. The programme therefore starts less from “assume eight bits” than from stable finite records and arrives at the $[8, 4, 4]$ byte.

7 The Service Rate $\alpha_0 = 1/137$

The earlier Born-rule draft did not cover the later R14 result. It matters because the Born measure and service rate now meet in one place.

The monitored service alphabet is a one-hot record observable. Its labels are not fermionic matter pairs but clonable record pairs. A 16-state orthogonal pointer basis has symmetric record-pair count

$$\text{Sym}^2(16) = \frac{16 \cdot 17}{2} = 136.$$

Including the idle label gives 137 monitored labels. In the connected unital monitor, the stationary state is uniform. One firing label therefore has Born weight

$$p_{\text{fire}} = \frac{1}{137} \equiv \alpha_0.$$

This is the bare service rate, not the electromagnetic dressed $\alpha^{-1} = 137.035999\dots$. The current canon also separates the common rate from sector billing maps: each physical sector must still show that it bills this monitored record observable with the stated multiplicity. The mass sector, CMB source, HBC scalar printer, and gravity erasure route have been audited separately. That is a billing-map question, not a Born-rule question.

8 Measurement, Arrow, and Objectivity

The measurement arrow follows from reusable records. A non-selective syndrome measurement dephases:

$$\rho \mapsto \sum_s \Pi_s \rho \Pi_s.$$

If the apparatus is to be used again, its record register must be reset. The many-to-one reset from record states to a blank state exports at least the Shannon entropy of the recorded distribution as heat. That is the local thermodynamic arrow attached to measurement.

Objectivity follows by the same record logic. Orthogonal record states are copyable pointer facts; they can be redundantly broadcast without violating no-cloning, because no-cloning forbids copying unknown superpositions, not copying already-orthogonal classical record labels. Thus quantum Darwinism's redundant environment records [7] are naturally identified with the substrate's broadcast syndrome facts.

9 What Remains Open

It is important not to overstate the closure. The paper's conclusion is a tier split, not a claim to derive physics from no premises.

Closed inside the substrate. Given a complex, locally tomographic QEC Hilbert substrate with repeatable orthogonal records, the Born rule is closed by the Gleason/non-contextuality route and by the closed-record-pair route. Measurement basis, decoherence, record objectivity, and reset entropy are all substrate-level structures.

Foundational reconstruction floor. The deeper question remains:

Why nature is a complex, locally tomographic, record-writing QEC Hilbert system?

The current canon reduces this to a labelled reconstruction ladder, with the minimal byte and bare service rate now substantially grounded. But the philosophical-operational floor itself remains: stable local records, compatible tests, repeatability, local tomography, and finite noisy substrate are the starting axioms of the reconstruction.

Traffic clock and holonomy. The R13 service-history measure is mostly a skeleton with known pieces: record unit, Crooks/KMS split, Born square, and bare α_0 . Its remaining live frontiers are the traffic clock beyond the native QEC/cosmology sector, and the recovery-holonomy/CP sector. A later promotion-gate audit sharpens this statement: the finite service instrument does support one record per event, but the stronger one-clock lift is rejected by the present ledger, while the holonomy phase/orientation sector remains conditional on a new sign-pointer bridge. Those are not defects in the Born rule; they are the remaining dynamics of the record-action measure.

10 Summary

The old wording “the Born measure remains an axiom” is now too pessimistic. The correct current statement is:

Born weights are conditionally closed inside the complex-QEC record substrate. The residual has moved upward to the reconstruction floor that asks why nature is this kind of record-writing Hilbert system.

That is a genuine improvement. The framework no longer treats measurement, probability, objectivity, and entropy as separate mysteries. It reads them as different faces of one process: stable syndrome facts are written, copied, paired with their dual histories, and eventually reset.

A Reproducibility

Each claim in the text is tied to a short self-asserting script in the repository [2]. The scripts exit 0 only when the stated checks pass.

Program	Role
<code>python_code/substrate_born_rule.py</code>	Decoherence, pointer basis, and uniqueness of the non-contextual Born frame function.
<code>python_code/substrate_noncontextuality.py</code>	Non-contextuality from the self-dual $[8, 4, 4]$ stabiliser algebra.
<code>python_code/substrate_pointer_basis.py</code>	The Q -leakage/syndrome channel selects the syndrome eigenbasis.
<code>python_code/born_closed_record_pair.py</code>	Stinespring record map; orthogonal closed-record sectors leave $AA^* = A ^2$.
<code>python_code/measurement_service_h_theorem.py</code>	Non-selective syndrome measurement dephases; reusable reset exports record entropy.
<code>python_code/minimal_balanced_record_cell_theorem.py</code>	Uniqueness of the minimal balanced distance-4 $[8, 4, 4]$ record cell in the stated class.
<code>python_code/r5_record_class_bridge.py</code>	Bridge from stable-record axioms to the balanced CSS/Type-II record class.
<code>python_code/alpha0_record_pair_symmetry_theorem.py</code>	Record-pair count $\text{Sym}^2(16) + 1 = 137$ and record-vs-matter billing distinction.
<code>python_code/r12_r13_record_action_closure_audit.py</code>	Theorem-boundary audit: R12/R13 have derived subclauses but remain a conditional skeleton, not a locked least-action theorem.
<code>python_code/r12_r13_r15_promotion_gate_attempt.py</code>	Promotion-gate audit: one-record-per-event closes at finite-instrument grade; the one-clock universalisation is rejected; recovery holonomy remains conditional on a sign-pointer bridge.
<code>python_code/r13_record_action_measure_status.py</code>	Current status of the record-action measure: native clock closed, universal clock rejected, holonomy sign-pointer frontier still open.

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