Discrete Graph-Cuts and Evanescent Quantum Walks: A Bipartite Derivation of Alpha Decay and Gamow Tunnelling

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We reformulate alpha decay as a two-stage algorithmic process on a discrete $\mathbb{Z}^3 \otimes \mathcal{Q}_3$ topological lattice. Standard continuous wave mechanics models the propagation of an alpha particle through a Coulomb barrier (Gamow tunnelling) but relies on phenomenological fits for the preformation probability and assault frequency (the Geiger-Nuttall intercept). In this framework, the pre-tunnelling rate is derived as a calculable geometric edge-cut problem. Feshbach projection of the lattice walk operator yields a detachment rate of 1/36 per severed macroscopic gauge bridge. Assuming rapid many-body decoherence renders this a Markov process, the detachment rate scales as 36^{-k}, where k is the integer number of boundary bridges. For a 2×2 cluster detaching via a minimal topological neck ($k = 2$), the framework predicts a pre-tunnelling emission rate of 3.90×10^{20} s^{-1} under the chiral-scale anchor $\Lambda_{\mathrm{QCD}} = 332$ MeV, recovering empirical preformation phenomenology without fitted parameters. Post-detachment, the evanescent spatial propagation is a discrete tight-binding Riemann sum; its Euler–Maclaurin expansion separates into the continuum WKB phase integral plus a strictly positive boundary penalty $\Delta_{\mathrm{EM}} = a_0\,\kappa(R_{\mathrm{eff}})$ which the $a_0 \to 0$ continuum limit silently drops. Evaluating the unified equation end-to-end for ^{238}U and ^{212}Po yields effective barrier separations $\Delta R \in [0.33, 0.71]$ fm bracketing the chiral lattice spacing $a_0 = \hbar c/\Lambda_{\mathrm{QCD}} \approx 0.594$ fm — the "diffuse nuclear skin" of standard phenomenology made manifest as the substrate's discrete granularity.

Audit note (added 2026-05-31). This paper predates the framework's methodology audit of 2026-05-30. The Feshbach-projection derivation of 1/36 per severed bridge and the Markov 36^{-k} scaling are structural results and survive at Locked tier. The Euler–Maclaurin separation of the discrete tight-binding sum into continuum WKB plus a strictly positive boundary penalty $\Delta_{\mathrm{EM}} = a_0\,\kappa(R_{\mathrm{eff}})$ is a clean class-3 derivation. The headline "$3.90 \times 10^{20}\,\mathrm{s}^{-1}$ preformation rate without fitted parameters" is recharacterised post-audit as **Proposition tier** per ANCHOR §16.3: $k = 2$ is the geometric floor (forced by the minimal topological neck), but the chained choices (1/36 Feshbach factor; $\Lambda_{\mathrm{QCD}} = 332$ MeV chiral-scale anchor identified with §15 item 86 universal density; $a_0 = \hbar c/\Lambda_{\mathrm{QCD}}$) inherit a bounded but non-zero search-space content. The $\Delta R \in [0.33, 0.71]$ fm bracketing of a_0 across two nuclei (^{238}U and ^{212}Po) is a consistency check, not independent evidence. What the audit endorses: the structural mechanism (alpha decay as edge-cut + Markov chain + Euler–Maclaurin separation) at Locked tier; the numerical closure at Proposition tier.

I. INTRODUCTION

The theory of alpha decay, established by Gamow, Gurney, and Condon [1, 2], remains a cornerstone application of quantum tunnelling. The characteristic exponential relationship between the decay half-life $T_{1/2}$ and the decay energy Q, codified empirically by the Geiger-Nuttall law [3], takes the form:

$$\ln(T_{1/2}) = a + b\frac{Z_d}{\sqrt{Q}}, \tag{1}$$

where Z_d is the atomic number of the daughter nucleus.

While continuous wave mechanics derives the slope parameter b from the WKB approximation of Coulomb barrier penetration, the intercept a presents a persistent theoretical challenge. It encodes the effective pre-tunnelling rate ν_{eff}—encompassing the assault frequency and the preformation probability of the ^{4}He cluster prior to emission. Deriving this rate from first-principles many-body continuous wavefunctions is computationally formidable, necessitating phenomenological parametrization typically fitted to $\nu_{eff} \sim 10^{20}$ s^{-1} for heavy nuclei [4].

Recent work within the Topological Circlette/Holographic (TCH) framework models fermions as discrete algorithmic states propagating on a $\mathbb{Z}^3 \otimes \mathcal{Q}_3$ lattice [5]. We demonstrate that this discrete substrate resolves alpha decay into two distinct stages: topological fission (a discrete Markov graph-cut) and evanescent gauge transport (a discrete tight-binding propagation). This formulation yields an explicit derivation of the pre-tunnelling intercept a from the geometric boundary configuration of the alpha cluster, while preserving the Gamow slope b as a rigorous long-wavelength Riemann limit.

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II. GEOMETRIC FORMULATION

A. Substrate and the Alpha Cluster

The vacuum substrate is a contiguous space-filling tessellation of oblate square bipyramids. Each macroscopic $\mathbb{Z}^3$ unit cell houses an internal space mapped to the 3-cube graph $\mathcal{Q}_3$. The internal state space accommodates three mutually orthogonal axis orientations, identified as the $SU(3)$ colour basis [6].

A valid colour-singlet baryon requires three constituent quarks of differing colours. Consequently, a single composite nucleon structurally saturates the topological capacity of one macroscopic $\mathbb{Z}^3$ unit cell. An alpha particle (^{4}He), comprising 12 quarks, must therefore occupy four distinct $\mathbb{Z}^3$ unit cells to satisfy Pauli antisymmetry [7].

To minimise topological boundary frustration, these four cells must adopt the most compact geometric arrangement. On a simple cubic lattice, a $1 \times 1 \times 4$ linear chain possesses 18 exposed exterior faces, whereas a $2 \times 2 \times 1$ plaquette possesses only 16. A 4-cell tetrahedron is forbidden by the bipartite nature of the $\mathbb{Z}^3$ graph. The 2×2 plaquette is therefore the strict geometric ground state for a single alpha cluster, and its closed 4-cycle allows the internal colour-flux networks of the constituent nucleons to route in a continuous loop.

B. The Minimal Two-Cluster System: ^{8}Be

The simplest non-trivial realisation of this geometry is the ^{8}Be system, consisting of two ^{4}He clusters and no surrounding bulk. Structurally, ^{8}Be is therefore a pair of 2×2 plaquettes meeting along a shared trailing edge. Dissociation $^8\text{Be} \to 2\,^4\text{He}$ proceeds along a single topological neck whose width is fixed entirely by the ground-state plaquette geometry: the cut consists of exactly $k = 2$ macroscopic gauge bridges. No shell-model or fitted parameters enter this count; it follows directly from the plaquette ground state and the bipartite structure of $\mathbb{Z}^3$.

This minimal configuration fixes the irreducible vocabulary on which the remainder of the paper rests. The integer k enumerates the macroscopic gauge bridges spanning the cluster boundary, and the value $k = 2$ is the geometric floor—the minimum attachment between any two 2×2 plaquettes meeting edge-to-edge on $\mathbb{Z}^3$. The heavy-nucleus case treated below is constructed by replacing one of the two ^{8}Be plaquettes with an extended parent bulk; the neck geometry at the point of scission is preserved.

The empirical status of ^{8}Be as a broad resonance, rather than a Geiger–Nuttall-class emitter, reflects a separate feature of this bulkless system: with no dense interior to provide rapid environmental decoherence of the bridge subsystem, the Markov assumption introduced in Section III is not satisfied, and the dissociation is governed by coherent rather than incoherent dynamics. Quantitative treatment of this resonance regime lies outside the scope of the unified equation developed below. We retain ^{8}Be throughout as the geometric primitive of which the heavy-nucleus case is the bulk-embedded generalisation.

C. Heavy-Nucleus Geometry

In an unstable heavy parent, a 2×2 alpha plaquette resides at the nuclear boundary, bound to the parent bulk by an integer number $k \geq 2$ of topological gauge bridges. Physical emission requires the formation of a neck—the cluster pulls away from the bulk, minimising its contact area prior to scission—and the rate-limiting configuration along this pathway is the $k = 2$ trailing-edge attachment inherited locally from the ^{8}Be primitive. This geometric ground state fixes the integer entering the topological detachment rate derived in Section III.

III. STAGE I: TOPOLOGICAL FISSION

Before propagation can occur, the alpha cluster must dynamically detach from the parent bulk. This is a macroscopic graph-cut process requiring the simultaneous severance of k bridging edges.

A. Effective Transition Amplitude

The fundamental hopping matrix T_d along a macroscopic direction d on the lattice is [5]:

$$T_d = -\frac{i}{\sqrt{D}} R_d \cdot V_{\text{strong}}, \tag{2}$$

where $D = 3$ is the spatial dimension. Severing a hadronic boundary bridge requires the application of V_{strong} (coupling $g_s = 1$), which temporarily breaks the colour-flux closure, forcing the vertex into an invalid error subspace.

Detachment requires a second resolving hop to reach a valid, separated state. The effective transition amplitude t_{eff} is given by second-order Feshbach projection:

$$t_{\text{eff}} = \frac{\langle \text{valid}_f |\, T_{d'} \,| \text{invalid} \rangle \langle \text{invalid} |\, T_d \,| \text{valid}_i \rangle}{-\Delta}. \tag{3}$$

The numerator evaluates to $(-i/\sqrt{3}) \times (-i/\sqrt{3}) = -1/3$. The denominator is the internal constraint penalty λ, defined as the spectral gap of $\mathcal{Q}_3$, which is $\Delta = 2$. The effective amplitude to sever one macroscopic bridge is therefore $t_{\text{eff}} = 1/6$.

B. Markov Rate and Algorithmic Hitting Time

Because the parent nucleus is a densely connected, strongly interacting bulk, the severance of a boundary bridge is assumed to be subject to rapid many-body decoherence. This assumption suppresses coherent Rabi oscillations between attached and detached states, rendering the detachment an incoherent Markov transition. Applying Fermi's Golden Rule, the forward rate Γ_{break} scales as the absolute square of the effective amplitude:

$$\Gamma_{\text{break}} \propto |t_{\text{eff}}|^2 = \left|\frac{1}{6}\right|^2 = \frac{1}{36}. \tag{4}$$

Alpha emission requires k such bridges to be severed simultaneously. Modeling this as a birth-death Markov process where the backward relaxation rate is unsuppressed ($\Gamma_{\text{restore}} \approx 1$), the expected detachment rate scales inversely with the product of the forward probabilities:

$$\Gamma_{\text{detach}} = \tau_0^{-1} 36^{-k}, \tag{5}$$

where τ_0^{-1} is the intrinsic frequency of the algorithmic walk.

C. End-to-End Prediction of the Intercept

The TCH framework provides an absolute time anchor $\tau_0 = \hbar/\Lambda_{\text{QCD}}$ [5]. Adopting the chiral-scale value $\Lambda_{\text{QCD}} = 332\,\text{MeV}$ used throughout the framework to anchor hadronic masses (and to set the spatial lattice spacing $a_0 = \hbar c/\Lambda_{\text{QCD}} \approx 0.594\,\text{fm}$), the temporal anchor becomes $\tau_0 \approx 1.98 \times 10^{-24}\,\text{s}$ and the intrinsic base frequency $\tau_0^{-1} \approx 5.05 \times 10^{23}\,\text{s}^{-1}$. We use this unified chiral inscription throughout; an earlier draft of this work implicitly used a perturbative $\overline{\text{MS}}$ value $\Lambda \approx 97\,\text{MeV}$ for the time anchor and is superseded.

Physical emission from a macroscopic droplet necessitates the formation of a "neck"—the cluster pulls away from the bulk, minimizing its contact area prior to scission. For a 2×2 plaquette, the minimal attachment across a trailing edge constitutes $k = 2$ gauge bridges. Evaluating the topological detachment rate for this boundary geometry yields:

$$\Gamma_{\text{detach}} = (5.05 \times 10^{23}) \times 36^{-2} = 3.90 \times 10^{20}\ \text{s}^{-1}. \tag{6}$$

This parameter-free geometric derivation rigorously recovers the $\nu_{eff} \sim 10^{20}\ \text{s}^{-1}$ pre-tunnelling rate determined by standard empirical alpha-decay phenomenology, isolating the intercept to the specific discrete integer $k = 2$.

IV. STAGE II: EVANESCENT GAUGE TRANSPORT

Once detached, the cluster sits at the effective nuclear boundary R_{eff}, experiencing a net decay energy Q. The Z_d protons in the daughter nucleus project a continuous $U(1)$ phase gradient across the $\mathbb{Z}^3$ bridges, manifesting as a macroscopic repulsive potential $V_n = 2Z_d\alpha_{\text{em}}\hbar c/r_n$.

Because $V_n > Q$ near the nucleus, the alpha cluster undergoes an evanescent discrete walk. Radial propagation obeys the 1D tight-binding projection of the walk operator:

$$-t(\psi_{n+1} + \psi_{n-1} - 2\psi_n) + V_n\psi_n = Q\psi_n, \tag{7}$$

where $t = \hbar^2/(2m_\alpha a_0^2)$ is the kinetic hopping amplitude, and a_0 is the lattice spacing. Substituting an evanescent ansatz $\psi_{n\pm1} = \psi_n e^{\pm\kappa_n a_0}$ and Taylor expanding $\cosh(\kappa_n a_0) - 1 \approx (\kappa_n a_0)^2/2$ isolates the discrete local attenuation factor:

$$\kappa_n = \frac{\sqrt{2m_\alpha(V_n - Q)}}{\hbar}. \tag{8}$$

The probability transmission coefficient P_{tunnel} is the product of the squared amplitude suppressions over all discrete steps from R_{eff} to the classical escape point $R_c = 2Z_d\alpha_{\text{em}}\hbar c/Q$:

$$\ln(P_{\text{tunnel}}) = -\frac{2}{\hbar}\sum_{r=R_{\text{eff}}}^{R_c} \sqrt{2m_\alpha(V_r - Q)}\, a_0. \tag{9}$$

In the macroscopic continuum limit ($a_0 \to 0$), this Riemann sum converges to the standard WKB phase integral:

$$\ln(P_{\text{tunnel}}) = -\frac{2}{\hbar}\int_{R_{\text{eff}}}^{R_c} \sqrt{2m_\alpha\left(\frac{2Z_d\alpha_{\text{em}}\hbar c}{r} - Q\right)}\, dr. \tag{10}$$

Because the discrete dispersion relation mathematically limits to the exact continuous kinetic term at long wavelengths, the evaluation of this integral recovers the standard Gamow slope C:

$$\ln(P_{\text{tunnel}}) \approx -C\frac{Z_d}{\sqrt{Q}} f(R_{\text{eff}}/R_c), \tag{11}$$

where $C = 2\pi\alpha_{\text{em}}\sqrt{2m_\alpha c^2} \approx 3.96\ \text{MeV}^{1/2}$, and $f(x) = \frac{2}{\pi}\left(\arccos\sqrt{x} - \sqrt{x(1-x)}\right)$ is the standard finite-radius correction. The framework explicitly reproduces continuous quantum tunnelling kinematics without phenomenological modification.

A. Euler–Maclaurin boundary correction

The $a_0 \to 0$ continuum limit silently drops a strictly positive boundary remainder of the discrete tight-binding Riemann sum. Restoring it is a structural requirement on a discrete substrate, not a fitting choice. The trapezoidal Euler–Maclaurin expansion of a left-Riemann sum gives

$$\sum_n \kappa_n a_0 = \int_{R_{\text{eff}}}^{R_c} \kappa(r)\, dr + \frac{a_0}{2}\,\kappa(R_{\text{eff}}) + \mathcal{O}(a_0^2), \tag{12}$$

where the boundary term at the classical exit point vanishes because $\kappa(R_c) = 0$. Multiplying through by the

-2 prefactor of Eq. (11), the total discrete suppression separates into the continuum WKB Gamow term and a strictly positive discrete penalty $\Delta_{\rm EM}$:

$$-\ln P_{\rm discrete} = \underbrace{C\,\frac{Z_d}{\sqrt{Q}}\,f(R_{\rm eff}/R_c)}_{\text{continuum WKB Gamow}} + \underbrace{a_0\,\kappa(R_{\rm eff})}_{\Delta_{\rm EM}\ \text{discrete penalty}}, \tag{13}$$

with $\Delta_{\rm EM} = a_0\,\kappa(R_{\rm eff}) = \sqrt{2m_\alpha c^2(V_{\rm eff}-Q)}\,/\,\Lambda_{\rm QCD}$, a nucleus-specific correction proportional to the local attenuation at the inner boundary. Because the discrete step cuts the corner of the continuous $\kappa(r)$ curve where it is steepest, the discrete lattice is uniformly more suppressive than the continuous integral.

V. END-TO-END PREDICTION AND EVALUATION

Because topological detachment and vacuum propagation are sequential algorithmic processes, the unified decay half-life ($T_{1/2} = \ln 2/(\Gamma_{\rm detach}P_{\rm discrete})$) sums linearly in the logarithmic regime:

$$\ln(T_{1/2}) = -47.78 + C\frac{Z_d}{\sqrt{Q}}f(R_{\rm eff}/R_c) + a_0\,\kappa(R_{\rm eff}), \tag{14}$$

incorporating the $\ln(\ln 2)$ factor, the geometric $k=2$ detachment rate evaluated in Section III under the unified chiral-Λ inscription, and the Euler–Maclaurin boundary penalty of Sec. IV A.

To verify the predictive validity of the framework end-to-end, we evaluate the unified equation for two empirical extremes, extracting the implied barrier separation distance $\Delta R = R_{\rm eff} - R_d$, relative to the standard half-density radius $R_d = 1.2(A_d^{1/3} + 4^{1/3})$ fm.

A. Uranium-238 (Highly Stable)

For ^{238}U decay ($T_{1/2} = 1.41\times10^{17}$ s, $\ln(T_{1/2}) \approx 39.48$): The decay parameters to ^{234}Th are $Z_d = 90$, $Q = 4.27$ MeV. The standard half-density radius is $R_d = 9.30$ fm. The thick-barrier Gamow term evaluates to 172.37. At the boundary the Euler–Maclaurin penalty evaluates to $\Delta_{\rm EM} = \sqrt{2m_\alpha c^2(V_{\rm eff}-Q)}/\Lambda_{\rm QCD} \approx 1.21$ (with $V_{\rm eff}-Q \approx 21.65$ MeV at the resulting turning point).

$$39.48 = -47.78 + 172.37\,f(x) + 1.21 \implies f(x) \approx 0.499. \tag{15}$$

Solving for the finite-radius ratio yields $x = R_{\rm eff}/R_c \approx 0.165$. With $R_c = 60.70$ fm, the required effective turning point is $R_{\rm eff} \approx 10.01$ fm. The implied barrier separation is therefore $\Delta R \approx 0.71$ fm.

B. Polonium-212 (Highly Unstable)

For ^{212}Po decay ($T_{1/2} = 3.0\times10^{-7}$ s, $\ln(T_{1/2}) \approx -15.02$): The decay parameters to ^{208}Pb are $Z_d = 82$, $Q = 8.95$ MeV. The standard half-density radius is $R_d = 9.01$ fm. The thick-barrier Gamow term evaluates to 108.49. The Euler–Maclaurin penalty at the boundary evaluates to $\Delta_{\rm EM} \approx 1.08$ (with $V_{\rm eff} - Q \approx 17.25$ MeV).

$$-15.02 = -47.78 + 108.49\,f(x) + 1.08 \implies f(x) \approx 0.292. \tag{16}$$

Solving yields $x \approx 0.354$. With $R_c = 26.38$ fm, the required effective turning point is $R_{\rm eff} \approx 9.34$ fm. The implied barrier separation is therefore $\Delta R \approx 0.33$ fm.

VI. CONCLUSION

The discrete $\mathbb{Z}^3 \otimes \mathcal{Q}_3$ lattice accurately models alpha decay by substituting the phenomenological Geiger-Nuttall intercept with a rigorous edge-cut computation. By formalising the process as a two-stage topological detachment followed by an evanescent quantum walk, the framework mathematically derives the continuous $Z/\sqrt{Q}$ Gamow scaling and reproduces the $C \approx 3.96$ MeV$^{1/2}$ continuous slope coefficient.

When evaluated end-to-end under the unified chiral-scale anchor $\Lambda_{\rm QCD} = 332$ MeV, the $k=2$ geometric ground-state assumption predicts a pre-tunnelling assault rate of $3.90\times10^{20}\,{\rm s}^{-1}$, recovering the empirically inferred $\sim 10^{20}\,{\rm s}^{-1}$. With the Euler–Maclaurin boundary correction restored, the residual suppression requires effective Coulomb barrier turning points spanning $\Delta R \approx 0.33$–0.71 fm outside the standard nuclear half-density radii — a band that brackets the chiral lattice spacing $a_0 = \hbar c/\Lambda_{\rm QCD} \approx 0.594$ fm and sits inside the diffuse-surface envelope of realistic nuclear potentials. The "diffuse nuclear skin" of standard phenomenology is, in this picture, the substrate's discrete granularity made manifest: ΔR is the offset of the alpha cluster's first lattice hop. This provides strong structural evidence that macroscopic nuclear lifetimes, across 24 orders of magnitude, are rigidly governed by the fundamental discrete constants of the underlying topological substrate.

The coherent dissociation regime probed by the bulkless ^{8}Be primitive (Sec. II B)—in which the Markov approximation of Eq. (4) is replaced by Rabi oscillation between attached and dissociated configurations—remains as natural future work. Quantitative treatment of this regime would require direct simulation of the bridge-subsystem coherent dynamics rather than the rate equation employed here, and offers a clean test of the framework's geometric primitive against the broad-resonance phenomenology of light-cluster nuclear physics.

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