

Native Three-Dimensional Gauge Structure from Orthogonal Octagon Interlocking on a 6-Node Honeycomb Lattice

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Abstract

We construct a three-dimensional lattice by interlocking three mutually perpendicular families of regular octagons via shared axis-aligned edges. The resulting honeycomb has a primitive unit cell of exactly 6 nodes, uniform vertex degree 5, and regular octahedral voids whose 8 triangular faces naturally host a binary code. The Bloch Hamiltonian at the Γ point reduces to the complete graph K_6 , with eigenvalues $+5$ ($\times 1$) and -1 ($\times 5$). Under the full octahedral point group O_h , these decompose as A_{1g} (scalar, massive) $\oplus T_{1u}$ (3-component vector, massless) $\oplus E_g$ (symmetric tensor, massless). The characteristic polynomial along Γ - X factors exactly as $(\lambda+1)^3[\lambda^3-3\lambda^2-9\lambda+3-8\cos k_x]=0$, yielding an analytical bare group velocity $v = \sqrt{2/3}$ for the T_{1u} gauge branch and effective mass $m^* = 9/2$ for the A_{1g} matter branch. The mass gap $\Delta \geq 2$ is maintained across the entire Brillouin zone. The octahedral voids sit on a simple cubic sublattice and are strictly disjoint, connected by single gauge-bridge edges. The two-void scattering geometry involves exactly $8 + 8 = 16$ independent faces, preserving the microstate combinatorics of the companion 2D framework [1, 2]. The face-adjacency graph of each octahedron is Q_3 , the 3-dimensional Boolean hypercube—the native topological home for an 8-bit information code. This extension eliminates holographic projection, upgrades the gauge boson from 2 to 3 spatial components, strengthens the mass gap from touching to permanent separation, and predicts sterile right-handed neutrinos as dynamically decoupled dark matter candidates.

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- [2] D. Elliman, “Emergent Minimal Gauge Coupling from C_{4v} Symmetry Reduction on the 4.8.8 Lattice,” Neuro-Symbolic Ltd, Preprint (2026).
- [3] K. G. Wilson, “Confinement of quarks,” *Phys. Rev. D* 10, 2445 (1974).
- [4] J. B. Kogut, “An introduction to lattice gauge theory and spin systems,” *Rev. Mod. Phys.* 51, 659 (1979).
- [5] J. M. Luttinger and W. Kohn, “Motion of Electrons and Holes in Perturbed Periodic Fields,” *Phys. Rev.* 97, 869 (1955).
- [6] B. Grünbaum and G. C. Shephard, *Tilings and Patterns*, W. H. Freeman (1987).
- [7] E. H. Lieb and D. W. Robinson, “The finite group velocity of quantum spin systems,” *Commun. Math. Phys.* 28, 251–257 (1972).
- [8] X. Mi *et al.*, “Information Scrambling in Quantum Circuits,” *Science* 374, 1479–1483 (2021).
- [9] S. Bravyi and A. Kitaev, “Universal quantum computation with ideal Clifford gates and noisy ancillas,” *Phys. Rev. A* 71, 022316 (2005).
- [10] R. L. Workman *et al.* (Particle Data Group), “Review of Particle Physics,” *Phys. Rev. D* 110, 030001 (2024).