Native Three-Dimensional Gauge Structure from Orthogonal Octagon Interlocking on a 6-Node Honeycomb Lattice

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Abstract

We construct a three-dimensional lattice by interlocking three mutually perpendicular families of regular octagons via shared axis-aligned edges. The resulting honeycomb has a primitive unit cell of exactly 6 nodes, uniform vertex degree 5, and regular octahedral voids whose 8 triangular faces naturally host a binary code. The Bloch Hamiltonian at the Γ point reduces to the complete graph K_6, with eigenvalues $+5$ ($\times 1$) and -1 ($\times 5$). Under the full octahedral point group O_h, these decompose as A_{1g} (scalar, massive) $\oplus$ T_{1u} (3-component vector, massless) $\oplus$ E_g (symmetric tensor, massless). The characteristic polynomial along Γ–X factors exactly as $(\lambda+1)^3[\lambda^3-3\lambda^2-9\lambda+3-8\cos k_x] = 0$, yielding an analytical bare group velocity $v = \sqrt{2/3}$ for the T_{1u} gauge branch and effective mass $m^* = 9/2$ for the A_{1g} matter branch. The mass gap $\Delta \geq 2$ is maintained across the entire Brillouin zone. The octahedral voids sit on a simple cubic sublattice and are strictly disjoint, connected by single gauge-bridge edges. The two-void scattering geometry involves exactly $8 + 8 = 16$ independent faces, preserving the microstate combinatorics of the companion 2D framework [1, 2]. The face-adjacency graph of each octahedron is Q_3, the 3-dimensional Boolean hypercube—the native topological home for an 8-bit information code. This extension eliminates holographic projection, upgrades the gauge boson from 2 to 3 spatial components, strengthens the mass gap from touching to permanent separation, and predicts sterile right-handed neutrinos as dynamically decoupled dark matter candidates.

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