

A question-led introduction to relativity and gravity

Space, time and gravity

Clocks, causal paths and black holes

David Elliman

Neuro-Symbolic

Modern physics through questions

Volume 3

Preface

Modern physics is fascinating, often surprising, and sometimes hidden behind unfamiliar mathematics. Popular accounts can awaken curiosity without showing enough of the reasoning to satisfy it. University textbooks supply that reasoning, but often assume mathematical fluency that a reader has not yet acquired, or has not used for some time.

At Neuro-Symbolic, we needed material to help able graduates understand the physics relevant to their work. These four books are our attempt to meet that need. They may also interest undergraduates, researchers entering a new field, and readers who want to understand more than a memorable analogy.

The books use questions and answers. We found this approach useful in workshops: a concrete puzzle gives people a reason to examine an idea and discuss their predictions. Each group of questions begins with the physical setting. Explanations use experiments, diagrams and simple arithmetic before introducing symbols. Optional mathematical sections connect that reasoning to the notation used in more advanced texts.

Our aim is to explain what the theories say, why physicists take them seriously, and where their explanations remain incomplete. Established results, assumptions of an example and unsettled proposals are identified separately. A useful calculation need not explain every ingredient that went into it.

We have taken care to check the physics and the examples, but that is no guarantee against error. Mistakes are our responsibility, and we welcome corrections from researchers and readers. The most useful feedback often begins with the exact sentence where an explanation stopped making sense.

Acknowledgement

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How to use this volume

Start with the measurements. A clock records elapsed time along its own path. Light connects an emitter and a receiver. A galaxy image tells us about light that has travelled through an evolving universe. These are concrete observations before they become equations about spacetime.

The first part asks how clocks and light lead to relativity. The next examines photon time, acceleration, gravitons and the limits of a classical gravitational source. We then treat black holes as causal structures and ask about quantum radiation and accelerated detectors. Cosmology has its own fourth volume, *The universe and its history*. Each opening asks for a prediction; worked answers are at the back.

No tensor calculus is needed. Optional pages introduce a few formulas after their arithmetic and meaning have been established. Diagrams are labelled when they are schematic. A picture can clarify a causal relation without constituting a derivation of Einstein's equation.

The companion books, *Quantum before symbols* and *Matter and its interactions*, supply the quantum and particle background. This volume distinguishes tested gravitational physics, controlled quantum approximations and open questions about quantum gravity.

Each part begins by describing the physical setting and the problem that motivates its questions. Read that introduction first. Short transitions then prepare the next question; make the prediction after that context. The optional mathematical pages explain each new operation before using its compact notation.

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Part one

What clocks and light measure

A physical event has a place and a time: a detector clicks, a light pulse reaches a mirror, or two objects meet. To compare events we use clocks, distance measurements and exchanged signals. An observer in relativity means such a measurement arrangement, not merely a person watching from a seat.

A frame is a system of spatial coordinates and synchronised clocks. An inertial frame is one in which a freely moving body has constant velocity when gravity can be neglected. Different inertial frames may move relative to one another. Relativity asks how their measurements of the same events fit together.

Light gives a particularly revealing comparison. Its vacuum speed is the same in local inertial measurements, although the observers need not agree on distances or elapsed coordinate times. Gravity then leads us to compare measurements across regions where no single inertial frame suffices. These first questions introduce that geometry, ask what makes gravity important, and distinguish a useful calculation grid from a claim that space itself is discrete.

1 Why do inertial observers agree on the speed of light?

Before reading on. A light clock moves past you. You see its pulse follow a longer diagonal path than someone riding with the clock. If both measure the same light speed, can both assign the same elapsed time to that leg of the pulse's trip?

A light clock can be imagined as a pulse bouncing between two mirrors. In the clock's rest frame the pulse goes straight between them. An

observer who sees the clock move sideways sees a diagonal path: the receiving mirror moves while the light is in transit.

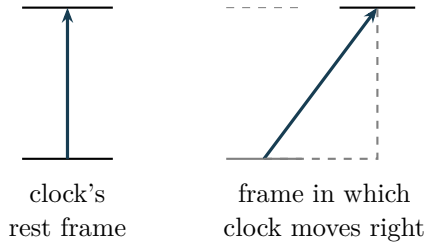


Figure 1: Two descriptions of a light-clock leg. The mirror separation is perpendicular to the clock's motion. In the passing observer's frame the receiving mirror moves sideways, so the pulse follows a diagonal. The diagram compares descriptions of the same apparatus.

Ordinary intuition would add the source's speed to the light's speed and keep a universal time. Special relativity uses a different rule, supported by experiment: inertial observers measuring vacuum light locally with their own rods and clocks obtain the same invariant speed. Their space and time measurements are related accordingly.

The triangle shows one consequence. If the clock moves at 0.6 of the light speed in your frame, the pulse geometry gives a moving-clock elapsed time of 0.8 seconds during 1 second of your coordinate time. Equivalently, one second on the moving clock corresponds to 1.25 seconds in that frame. The optional pages show the arithmetic.

There is no contradiction when each inertial observer describes the other's clock as slow. Comparing separated clocks also requires a definition of simultaneity, and the two frames disagree on which distant events are simultaneous. A reunion comparison must follow the actual paths, including any changes of inertial frame.

The invariant speed belongs to spacetime's causal structure. Maxwell's theory places vacuum electromagnetic waves on that light cone: the boundary between events a light signal can connect and events too far apart for such a signal in the available time. Light in material can have slower propagation speeds; that does not replace the vacuum invariant.

A microscopic theory must recover this operational symmetry, not merely make speeds equal along a few chosen directions. Matching one leading

dispersion coefficient is useful evidence within a model, but interactions, higher-order terms and the measurement rules also need to be consistent.

Try 1. If one second on the moving clock corresponds to 1.25 seconds in the stated frame, how many moving-clock seconds pass during 5 seconds in that frame?

Special relativity described measurements in an inertial setting. Around a gravitating body, light from a distant source can arrive from a shifted direction. This is gravitational lensing. The fact that a photon has no rest mass makes the effect especially useful for testing what gravity couples to.

2 If light has no rest mass, why does gravity bend it?

Before reading on. Is rest mass the only property that can enter a gravitational description of a physical system?

A photon has zero rest mass and nonzero energy and momentum. General relativity couples geometry to energy, momentum and stress, not just to a collection of rest masses. A photon therefore does not need an invented tiny rest mass to take part in gravitational physics.

In the ray approximation for vacuum light, when its wavelength is small compared with the scale on which the geometry varies, free light follows null geodesics. Null identifies a lightlike spacetime interval, and geodesic names the appropriate free path in the geometry. Locally, in a freely falling frame small enough to neglect tidal variation, light behaves as it does in special relativity.

Across an extended region, local directions cannot always be combined into the straight lines of one global Euclidean picture. Curvature changes the relation between the ray's initial and final directions. A distant observer can measure a deflection.

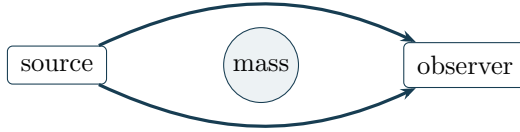


Figure 2: Schematic gravitational lensing. The mass changes light paths between source and observer. The separation of the drawn paths is exaggerated; no numerical deflection is implied.

Light bending, gravitational time delay and gravitational frequency shifts probe related features of the metric. A metric is the rule that connects coordinate differences with physical intervals. The coordinates are labels; the measured clock readings and light signals carry the physical content.

An accelerating laboratory supplies a useful local clue. While a light pulse crosses the room, the accelerating apparatus changes motion, so its recorded path can appear curved. The equivalence principle connects such local descriptions with gravity. It does not by itself calculate every global lensing effect or replace the field equations.

To calculate a lens, we need the gravitational field sourced by the matter and then the light propagation through it. One consistent theory must govern both. Assigning a separate ad hoc bending rule to photons would miss the common geometrical structure that relativity tests.

Try 2. Which photon properties remain nonzero despite its zero rest mass?

Light bending and planetary motion show that gravity has measurable effects. Yet between elementary particles its force can be tiny compared with an electric force. A meaningful comparison must state which particles and conditions are being compared, rather than merely comparing the printed values of constants.

3 Why is gravity so weak?

Before reading on. Does comparing two constants with different units give a meaningful ranking of force strengths?

A number changes when its unit changes. The same distance can be one metre or one hundred centimetres. Comparing the numerical values of dimensional constants without their units cannot tell us which interaction is physically stronger.

Instead, compare forces or dimensionless couplings in a specified situation. Consider two protons sufficiently far apart that their strong interaction is negligible, moving slowly enough for a static force comparison. They attract gravitationally and repel electrically. In the simple inverse-square comparison, both forces have the same distance dependence, so their ratio does not depend on that separation.

The gravitational attraction is extraordinarily small in that comparison. The proton mass, its electric charge and the gravitational and electromagnetic constants enter the ratio. Changing to different particles changes the comparison. The word weak needs those inputs.

Why then does gravity govern planets and galaxies? Most macroscopic matter is nearly electrically neutral: positive and negative charges largely cancel in the long-range electric field. Ordinary mass-energy does not cancel in the same way. Large amounts accumulate, and gravity's long range becomes decisive.

As a counting analogy, imagine a thousand positive contributions and a thousand negative contributions that nearly cancel in one account. In another account, two thousand small positive contributions add. The second total can matter even when each contribution is tiny. This illustrates cancellation; it is not a numerical model of a galaxy.

General relativity takes the gravitational coupling as an input. It relates that coupling to many measurable effects but does not explain its small dimensionless value relative to a chosen particle mass scale. That hierarchy is a deeper physics question.

The hierarchy is also a reminder that successful laws can leave numerical questions unanswered. Measuring the gravitational constant and calculating an orbit are different achievements from explaining its observed relation to particle mass scales.

Try 3. Why can gravity dominate the large-scale motion of nearly neutral matter despite its weakness between two individual protons?

General relativity normally describes smoothly varying spacetime. Computers often approximate a smooth system using a grid of points. That practical step raises a separate physical question: could a discrete structure be fundamental, and what observation would distinguish it?

4 Is space continuous or made of pieces?

Before reading on. If a lattice calculation reproduces a continuum equation at long wavelengths, does that prove space is a lattice?

A lattice can be a tool for calculation or a claim about physical reality. The two uses require different evidence. In numerical field theory, one often discretises a continuum model and studies how results approach a continuum limit as the spacing is reduced.

A fundamental lattice proposal instead assigns physical meaning to some microscopic discreteness. It must explain why familiar continuum behaviour emerges in the observed regime, and identify any departures that remain measurable. Success as a regulator does not establish that stronger claim.

A regular spatial grid has preferred directions at its microscopic scale. Symmetry can make the leading long-wavelength propagation isotropic, meaning the same in every spatial direction, but higher-order terms can retain directional or energy dependence. Equality of a few axis speeds does not automatically establish full Lorentz symmetry, the relation

among measurements made by rotated or relatively moving inertial observers.

Consider an illustrative correction proportional to the square of the ratio of spacing to wavelength: first divide the spacing by the wavelength, then square the result. A wavelength ten times the spacing gives a squared ratio of 0.01. A wavelength one hundred times the spacing gives 0.0001. This explains how an effect can be small at long wavelengths. The actual coefficient and form must come from the specified theory; the example does not predict a physical spacing.

Conversely, the success of continuum equations at accessible scales does not prove that arbitrarily small distances are continuous. It constrains alternatives through the accuracy and range of the observations. A discrete theory may also use structures more general than a fixed crystal grid.

The useful question is therefore not which picture looks more natural. It is which physical observable distinguishes the proposals: dispersion, rotational symmetry, causal response or another controlled effect. Without that bridge, a set of cells remains a possible description rather than evidence for the substance of space.

Try 4. In the stated squared-ratio example, increasing wavelength by a factor of ten reduces the correction by what factor?

Part two

Clocks, signals and quantum limits

Relativity describes measurements made with clocks, freely falling objects and light signals. Everyday phrases can hide distinctions between those measurements. A photon has no time sounds like a statement from the photon's point of view; gravity travels at light speed can sound like the discovery of a particle. Neither conclusion follows so directly.

The next questions return to the physical arrangements. We compare different paths through spacetime, a supported laboratory with a falling one, and a change in a gravitational field with a static attraction. We then ask which aspects of spacetime have been measured and which still need a deeper explanation.

Quantum theory enters when matter and fields can occupy coherent combinations of states. That creates a serious question for gravity, but not simply because one experiment cannot measure position and momentum exactly at once.

5 Does a photon experience no time?

Before reading on. If no rest frame exists for a photon, can we put an ordinary clock beside it and ask what that clock reads in the photon's rest frame?

We cannot. Every inertial observer measures a vacuum light ray moving at the same invariant speed. A massive observer can approach that speed relative to another observer, but cannot become an inertial observer travelling alongside the ray.

There is a precise statement behind the phrase. Proper time is the elapsed time read by a clock carried along a path. A path that a material clock can follow is called timelike. A light ray's path is lightlike, or null: the interval rule gives zero after its time and space contributions are

combined. Substituting a null path into the interval relation gives zero proper-time interval.

That mathematical property does not provide a photon's personal frame or experience. It tells us how the ray's path differs from a clock's path. A lightlike path can be parametrised for calculation, but that parameter is not the reading of a clock carried at rest with the photon.

For example, a pulse crosses a laboratory one metre long. The laboratory clocks assign a nonzero travel time, about 3.3 nanoseconds in vacuum. A nanosecond is one billionth of a second. The source can emit before the receiver detects. Neither the null interval nor a change of coordinates makes those laboratory events the same event.

The light-clock example in Question 1 compared two possible clock measurements. Extrapolating its time-dilation formula all the way to a photon does not supply an allowed clock at the limiting speed. The limiting mathematical relation and the physical meaning of a rest frame must be kept separate.

This also prevents a common mistake in cosmology. Light from an early epoch can have a long travel time in the cosmological description even though its ray is null. The age inferred by cosmologists is a time along the appropriate matter clocks, not an age reported by photons.

Try 5. Can emission and detection have a nonzero time separation in the laboratory while the connecting ideal light ray has zero spacetime interval?

A photon has no rest-frame clock, but a small laboratory can carry clocks and compare falling objects. That gives a concrete way to understand Einstein's connection between gravity and acceleration without identifying every gravitational field with a coordinate trick.

6 Is gravity acceleration, or is it curvature?

Before reading on. If one falling observer removes the apparent weight of nearby objects, must the separation of all freely falling objects remain fixed?

Imagine a small freely falling laboratory. Objects released inside it can float approximately alongside the observer. The usual feeling of weight has disappeared because the laboratory and its contents are falling together.

Now imagine a rocket accelerating through a region with negligible gravity. Its floor pushes against the passenger. Released objects fall towards that floor in the rocket's description. In a sufficiently small region and over a sufficiently short time, the comparison captures the equivalence between gravitational effects and acceleration.

The qualifications matter. Two objects falling towards different parts of the Earth need not keep exactly the same separation. Their relative acceleration is a tidal effect. A single freely falling frame cannot remove such relative accelerations throughout an extended region.

Curvature describes this remaining geometrical content. It can be measured through the relative behaviour of nearby free paths. Coordinates can remove certain apparent forces locally; they cannot turn a genuinely curved spacetime into flat spacetime over a finite region. An accelerating rocket in flat spacetime provides the first kind of effect without producing gravitational curvature merely by changing the passenger's coordinates.

Thus the two descriptions answer different questions. The equivalence principle guides the local comparison of gravity and acceleration. Curvature describes how these local comparisons fit together across spacetime.

Einstein's field equation then relates a particular curvature combination to energy, momentum and stress.

Mass is not a substance pressed into a rubber sheet. The metric is a physical field specifying intervals, and the coupled equations determine its response. The source includes radiation and pressure as well as rest mass. A stretched-sheet drawing leaves out time and needs external gravity to make its balls roll.

A microscopic lattice might in principle reproduce such a response. To demonstrate it, one would need an action giving a common response of matter and light and the appropriate gravitational source. Calling a lattice distortion curvature is not yet that demonstration.

Try 6. Which feature can distinguish an extended gravitational field from a mere acceleration of coordinates in flat spacetime: apparent weight at one point, or relative tidal acceleration?

Curvature need not be static. A changing system can produce disturbances that carry energy through space. We need to separate the propagation of such a disturbance from its possible quantum description.

7 Does light-speed gravity prove that gravitons exist?

Before reading on. Would measuring the speed of a wave alone establish that its energy had been detected in individual quanta?

No. A wave's propagation speed and the detection of its individual quanta are different observations. In general relativity, weak gravitational disturbances propagate locally at the invariant light speed on the background geometry, in the vacuum approximation.

Gravitational waves have been detected through their effect on the relative separations of freely responding test masses. Their waveforms test the dynamical theory. The nearly coincident gravitational and electromagnetic signals from the binary-neutron-star event GW170817 also placed strong constraints on differences in propagation speed, with source-emission assumptions included.[6]

This does not mean that every static gravitational attraction should be pictured as a delayed arrow pointing towards where the source used to be. A dynamical field theory contains constraints and velocity-dependent effects as well as propagating radiation. Simply retarding Newton's force direction is not general relativity.

A graviton is the quantum associated with a weak gravitational wave in the usual quantised, linearised description. Under the relevant relativistic assumptions it is massless and has two helicities, plus two and minus two in the usual quantum units. Helicity is the spin component along the direction of motion. The signs distinguish its two circular polarisation possibilities. That description is useful within a low-energy quantum theory; it is not supplied by the speed measurement alone.

The gravitational-wave observations cited here measure classical wave behaviour involving enormous occupation numbers in that quantum interpretation. They are not reports of counting individual gravitons. The practical difficulty of such counting must not be turned into a logical proof that no quantum description can be tested.

The distinction mirrors the earlier discussion of light. Interference measurements reveal wave behaviour; other experiments establish features of its quantised exchanges. A viable quantum theory of gravity must recover the successful classical wave predictions and specify what additional observations would distinguish it.

Try 7. Name one observation supporting propagating gravity, and one stronger claim that observation does not establish.

So far we have used three spatial directions and one time coordinate. Those numbers describe the world in the regime we test. Asking why they occur is different from discovering a numerical resemblance in an internal particle algebra.

8 Why are there three dimensions of space and one of time?

Before reading on. Would a particle with three internal colour labels require the laboratory to have three spatial dimensions?

An internal label and a spatial direction do different jobs. We can change a quark's colour basis without turning the laboratory. Conversely, rotating the laboratory does not merely exchange its colour labels. The appearance of the number three in both places does not identify the two structures.

Operationally, three independent spatial coordinates locate an ordinary small object in our surroundings. A separate time coordinate orders readings of clocks. Relativity combines them into spacetime while retaining a crucial sign difference in its interval rule. That difference distinguishes timelike, lightlike and spacelike separations and defines causal possibilities. A timelike separation allows a slower-than-light clock to connect the events; a lightlike separation allows a vacuum light ray; a spacelike separation allows neither.

General relativity describes this structure; its usual four-dimensional formulation does not derive the number of large dimensions from a register count. Other mathematical theories can be written in other dimensions. Whether any describes nature is an experimental and dynamical question.

Additional dimensions appear in some proposals. They may be compact, meaning that a direction closes on itself over a very short distance, or otherwise difficult to access, but then the proposal must explain why present experiments see the effective dimensionality they do. Naming a hidden direction is not enough: one needs a consistent model and consequences that can be tested.

Arguments about stable planetary orbits, atoms or complex structures can illustrate how physics changes when dimension changes. Such arguments depend on which laws are kept fixed. They do not constitute a unique derivation of three spatial dimensions, nor do they show that no other mathematical world could have interesting structures.

For an information-based proposal, the useful task would be to obtain the observed relations among localisation, propagation and causal order from specified dynamics. Three colour directions or three drawn lattice axes cannot supply that result by themselves. We should retain the question without claiming that a suggestive matching of numbers answers it.

Try 8. What distinguishes an internal colour basis from a set of laboratory spatial axes?

The number and causal roles of spacetime dimensions are part of the gravitational description. A further question arises when the source itself is quantum: which gravitational field should accompany a coherent combination of matter states?

9 Does uncertainty make Einstein's equation impossible?

Before reading on. Must a classical field equation assign exact measured positions and momenta to every microscopic particle before it can predict an average gravitational field?

No. Classical matter is often described by smooth densities, pressures and flows rather than a list of exact microscopic measurements. General relativity can use such effective descriptions where they are adequate. Quantum uncertainty does not invalidate every successful classical prediction.

In semiclassical gravity, one uses a quantum expectation value of stress-energy as the source for a classical metric. An expectation value is an average specified by a quantum state, not a claim that all non-commuting observables have simultaneously definite measured values. Stress-energy collects energy density, momentum flow and pressure. Its

quantum calculation needs a consistent renormalisation prescription: a rule for combining field contributions and model parameters so that the calculated source has a well-defined physical meaning.

That approach has limits. Imagine a mass prepared in a coherent combination of two well-separated locations. An average density has support in both places. It does not by itself specify gravitational correlations with a detector that becomes correlated with the different alternatives, or settle when a single classical metric is an adequate description.

Large fluctuations, entanglement involving gravity, and regimes near a classical singularity raise further questions. They motivate a quantum account of gravitational degrees of freedom. The problem is therefore richer than we cannot measure energy and position exactly, so the equation cannot work.

General relativity is also more constrained than an arbitrary curve fitted independently to each experiment. The same geometrical structure relates clocks, free fall, light propagation and gravitational waves. Its field equations have consistency relations. Those achievements do not establish that smooth classical spacetime is fundamental at every scale.

A lattice or register model could offer a different underlying description, but must earn that claim. It needs physical states, dynamics and observables that recover the tested limits and make sense of the quantum source. A suggestive cell deformation or a list of matter labels is not a substitute for those steps.

Try 9. Why can an expectation-value source be useful without being a complete description of gravity entangled with a mass in two locations?

Part three

What a horizon allows

A falling body, a light signal and a rocket have different possible paths. In relativity we can ask, at each event, which later events a signal could reach without exceeding the local light-speed limit. A light cone is a diagram of those possibilities. It is not a physical cone surrounding an object.

A black-hole horizon is a boundary defined by whether signals can escape to the distant exterior. To make the first two questions precise, we use the ideal classical Schwarzschild black hole: uncharged, nonrotating and isolated. More complicated black holes require their own geometry.

This setting separates three issues that are often confused: the locally measured speed of light, the labels used on a spacetime diagram, and the places a future signal can reach. The later questions add quantum fields. Hawking radiation changes the physical account, but it does not require an ordinary photon to travel outwards from within the classical horizon.

10 Why can't light escape a black hole?

Before reading on. Must light slow down locally at a horizon for escape to become impossible?

The defining feature of a black-hole event horizon is causal. It separates events that can send signals to the specified distant future from those that cannot. In the ideal classical black-hole geometry, escape from inside is not among the future-directed lightlike paths.

Light still travels locally at the invariant speed. The obstruction is the arrangement of possible future paths in the geometry, not a local friction that slows the photon.

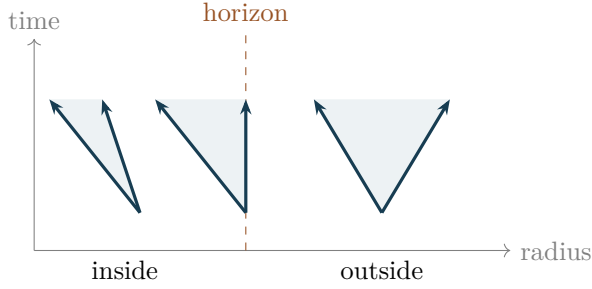


Figure 3: A schematic horizon-crossing view of the ideal Schwarzschild black hole described in this part. Radius increases to the right and a chosen regular time coordinate increases upwards. Inside the horizon, both radial future light directions head towards smaller radius. At the horizon the outward direction lies along it. Cone slopes depend on coordinates; the escape restriction is the causal statement.

An event horizon is not generally a material surface that an infalling observer strikes. For a sufficiently large, quiet black hole, local tidal effects at the horizon can be small enough that crossing has no special local drama. Later conditions can be very different.

The word event also signals a global definition. Locating the event horizon precisely depends on the spacetime's future causal structure. Other horizon notions can be useful in dynamical calculations and need not coincide in every situation.

The Newtonian picture of an escape speed reaching the speed of light can suggest a characteristic radius, but it does not establish this causal account. In general relativity, one must examine the allowed paths and the metric.

Nor does an outside observer's coordinate description replace the infaller's clock. Signals from an infalling object become delayed and redshifted in the familiar ideal exterior account. The object can nevertheless cross the horizon in finite proper time along its own path.

Keeping local measurement, coordinate time and global causal reach separate removes much of the apparent contradiction. No photon has to decide to stop being light at the horizon.

Try 10. Does the inability to escape imply that an infalling observer measures light moving locally below its usual vacuum speed?

The horizon drawing used coordinates that remain regular during crossing. Other familiar coordinates fail at the horizon and can make space and time appear to exchange roles. We must separate a change of labels from the allowed paths of clocks and signals.

11 Do space and time swap inside a black hole?

Before reading on. If a coordinate changes its causal role, does a physical substance called space turn into a different substance called time?

The slogan refers to the Schwarzschild solution, the ideal nonrotating, uncharged black hole. In its usual coordinates, the radial coordinate has a timelike role inside the horizon, while the coordinate named time no longer has the exterior timelike role.

The physical content is more useful than the slogan. In that ideal interior, future-directed causal paths proceed towards smaller areal radius. An observer cannot remain at a fixed radius by firing a sufficiently powerful rocket. The restriction is comparable to the ordinary requirement that a physical trajectory proceeds into its future.

Areal radius is defined through the area of the symmetry spheres, rather than simply being the distance a ruler measures from the centre. The coordinate description must be interpreted through the metric.

The usual Schwarzschild coordinates become singular at the horizon, but the ideal geometry need not be singular there. Regular coordinates can describe a smooth horizon crossing. A failure of one chart is not automatically a physical singularity.

This is a familiar lesson from maps in a more unfamiliar setting. Longitude labels behave badly at the Earth's poles; the label failure does not tear the surface. The black-hole analogy has limits, but it illustrates why coordinate pathology and physical pathology must be distinguished.

The central singularity of the ideal solution raises a different issue: curvature and the classical description break down there. One expects a more complete theory to be needed, but the classical coordinate slogan does not supply that theory.

Rotating, charged and dynamical black-hole models have more complicated interiors. The simple statement about monotonically decreasing radius should not be applied unchanged to all of them. It belongs to the stated ideal Schwarzschild case.

Try 11. In the ideal Schwarzschild interior, is remaining at fixed areal radius a possible future-directed timelike trajectory?

Classical geometry tells us which light signals can escape. Quantum field theory asks what a detector measures when a field is in a specified state on that geometry. Combining those descriptions leads to the radiation prediction, with a domain of validity that must remain explicit.

12 Why do black holes evaporate?

Before reading on. If classical light cannot escape from inside a horizon, must every quantum prediction of radiation require a photon to travel out from that interior?

Hawking radiation is a prediction of quantum field theory on a black-hole spacetime, with a specified state appropriate to its formation or environment. The relation between field modes in the past and those detected far away leads to an outgoing flux.

For an ideal stationary, uncharged, nonrotating black hole, the temperature is inversely proportional to its mass. Double the mass and the Hawking temperature halves. Radiation reaching infinity also passes through curvature-dependent scattering, so the observed spectrum includes transmission factors as well as the thermal scale.

The popular picture of a pair appearing beside the horizon, with one member escaping, is at best a mnemonic. It does not derive the temperature or spectrum. Taken literally, it invites incorrect stories about borrowing energy and about a particle escaping from inside the horizon.

Energy conservation is expressed through the appropriate gravitational and field account. A black hole emitting more energy than it absorbs loses mass. A black hole in a sufficiently hot environment can instead gain mass. Radiates and shrinks now are therefore not identical statements.

Astrophysical black holes have extremely low predicted Hawking temperatures. Direct observation of their Hawking emission has not been established. Analogue systems can test related wave effects under their own assumptions, but they are not direct detections of radiation from an astrophysical event horizon.

The semiclassical result also raises the information problem. How does the complete evaporation process relate an initial quantum state to the outgoing radiation, and what happens when the classical background approximation becomes inadequate? The simple thermal calculation does not settle every part of that question.

We should preserve both the strength and the limit of the result. Hawking's calculation connects quantum fields, causal geometry and thermodynamics. It is not a complete microscopic theory of the final stages of evaporation.

Try 12. In the ideal temperature–mass relation, what happens to temperature when mass doubles? What additional comparison decides whether the hole loses mass overall?

Black-hole radiation relates a quantum field to the spacetime and the observer's measurements. A related surprise already occurs in flat spacetime when a detector accelerates.

13 Can acceleration make empty space feel warm?

Before reading on. Must an inertial detector and an accelerating detector coupled to the same quantum field have identical responses?

A detector measures through an interaction along its path. Change the path and the field correlations sampled by that interaction change too. The vacuum contains no particles therefore needs a specification of the state and the particle description being used.

Proper acceleration is what an accelerometer carried with the detector reads, as opposed to a change of coordinate velocity seen from far away. The ideal Unruh result considers uniform proper acceleration through the Minkowski vacuum, the vacuum associated with inertial motion in flat spacetime. For a suitable detector and the appropriate long-time limit, the rates of gaining and losing internal energy have the same ratio as they would in a heat bath. This is the thermal relation being predicted; it refers to specified detector transitions. The corresponding temperature is proportional to the proper acceleration. Double that acceleration and the ideal temperature doubles.

This does not mean that acceleration fills the laboratory with an observer-independent gas of ordinary photons. The result applies to quantum fields and detector couplings; the simple calculation is often presented with a scalar field, a field with spin-zero excitations. Calling it a sea of photons can hide those assumptions.

Nor does an accelerated detector obtain unlimited energy for free. The agent maintaining the motion and the detector-field interaction belong in the energy account. An inertial description can explain the same physical detector processes without treating the inertial vacuum as a hot material bath.

A real finite experiment also switches the coupling on and off, changes the acceleration and has background noise. Transient excitations are not automatically evidence for the ideal steady thermal relation. Analogue experiments and related acceleration effects must be distinguished from directly observing that full idealised setup.

The effect is a substantial theoretical prediction of quantum field theory, discussed with these qualifications in the review cited here.[7] It is not established for a proposed register framework merely because that framework uses quantum states. Recovering it would require the appropriate vacuum correlations, detector interaction and accelerated trajectory in a controlled regime.

Try 13. In the ideal Unruh relation, what happens to the temperature when proper acceleration doubles? Why must the mechanism maintaining the acceleration enter the energy account?

Optional

Three bridges to the equations

The main text described what clocks measure and how a gravitational field affects light and matter. The optional equations organise those statements. We will not attempt a full course in relativity. We will learn to recognise which quantity a formula relates to which, and what additional information would be needed to use it.

The light-clock example uses Pythagoras and a square root. Einstein's equation requires a different idea: a mathematical object with components that change together when the coordinate labels change. Such an object is called a tensor. We will explain why it is needed before inspecting the compressed equation.

The final question returns to entropy and information. A black hole's entropy scales with horizon area in the semiclassical theory. Understanding that statement requires distinguishing area from volume and a physical entropy from an ordinary list of detector records. It does not require assuming that every region is literally a hollow storage device.

14 How does the light-clock arithmetic work?

Before reading on. A triangle has diagonal length 1 and horizontal side 0.6 in chosen units. What is its vertical side?

For a right-angled triangle, Pythagoras says that the square of the diagonal equals the sum of the squares of the other two sides. Squaring means multiplying a number by itself. To find a side from its square we take the positive square root. These operations give the answer without any new relativistic notation. The square of the vertical side is $1 - 0.6^2 = 0.64$, so the side is 0.8. That is the triangle behind the numerical light-clock example. The units have been chosen so that

the diagonal represents the light distance in one unit of the passing observer's time.

The horizontal side represents the clock's displacement during that interval. Its speed is 0.6 of the light speed. The vertical side is the mirror separation, which lies perpendicular to the relative motion and is unchanged by that Lorentz boost, the coordinate transformation between the two uniformly moving frames.

In the clock's own rest frame, light crosses that vertical separation directly. The elapsed proper time is therefore 0.8 of the passing frame's coordinate time. Proper time means the time a clock records along its own path.

The example has given us a rule to generalise: multiply the chosen frame's elapsed time by the square root of one minus the squared speed fraction. A speed fraction of zero gives a factor of one; a fraction of 0.6 gives 0.8. A material clock follows a timelike path, so its speed in this inertial frame is less than the vacuum light speed. For uniform motion, the general relation is

$$\Delta\tau = \Delta t \sqrt{1 - v^2/c^2}.$$

The symbol $\Delta\tau$ denotes the moving clock's proper time interval; Δt denotes the coordinate time interval in the specified inertial frame. Delta means the difference between the final and initial readings. The letter tau is a name for the clock's own time, not another mathematical operation. The ratio v/c is its speed as a fraction of the invariant light speed.

Putting the fraction 0.6 into the square root gives the 0.8 already calculated. Reversing the comparison, one unit of proper time corresponds to $1/0.8 = 1.25$ coordinate-time units. Both numbers describe the same relation.

For a changing velocity, a single constant factor cannot describe the whole trip. The proper-time contributions must be accumulated along the actual path. In a gravitational field, the metric supplies the corresponding local interval relation.

This is why moving clocks run slow is an incomplete instruction for every clock comparison. It must identify the frame, the events being

compared and the paths. The formula is compact, but the experimental meaning remains part of it.

Try 14. Using the 0.8 factor, what proper time accumulates during 2.5 seconds of the specified coordinate time?

A clock formula tells us what a known geometry predicts. To calculate the geometry produced by a star or a cloud of gas, we also need a relation between geometry and its sources. Einstein's equation is that relation in general relativity.

15 What does Einstein's equation relate?

Before reading on. Would a rule for curvature alone tell us how much curvature a specified distribution of matter produces?

The metric tells us how to compute intervals and free paths. Curvature is built from particular combinations of changes in the metric. Merely seeing varying metric coefficients is not enough: polar coordinates on a flat plane already produce such coefficients. Curvature identifies what cannot be removed by relabeling the coordinates. A field equation is needed to relate the geometry to the physical source. Einstein's equation supplies that relation in general relativity.

To see why components are needed, imagine describing a flow of energy. Its amount is not enough; we also need the direction. A fluid has pressures and transfers of momentum in different directions too. A table can collect these quantities relative to a chosen set of axes and clocks.

A tensor is more than an arbitrary table: it comes with a rule for changing its components when the coordinate system changes. The transformed entries describe the same physical situation. Einstein's equation relates such objects, so that different observers can use different components without changing the law.

Its conventional compact form is

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}.$$

The indexed object $G_{\mu\nu}$ is a particular combination of curvature, called the Einstein tensor. The metric is $g_{\mu\nu}$. The source object $T_{\mu\nu}$ contains

energy density, energy flow, momentum and stress. The two indices organise the components involving one time and three space coordinates.

The unindexed letter G in the coefficient is Newton's gravitational constant, not the Einstein tensor. The cosmological constant is written Λ . These names are enough to read the structure: a specified curvature expression and cosmological term are related to the source.

The two small Greek indices label a row and a column of components, each associated with one time or one of three space directions. They are labels, not quantities multiplied together. The displayed equation stands for the corresponding component relations. The π in the coefficient is the familiar circle constant; the fourth power of the light speed is part of the conversion between the source and geometrical units.

The source on the right does not determine every aspect of curvature locally. Regions with no local matter can still contain a gravitational field or a passing gravitational wave. The equations must be solved with appropriate initial and boundary data.

The source includes more than rest mass. Radiation contributes, as do pressures and momentum flows. This is why the question about massless light cannot be answered by rest-mass bookkeeping alone.

The equation has strong internal consistency relations tied to local energy-momentum conservation. In a coupled matter-gravity theory, the source should come from the same action that determines the matter dynamics. The action is a mathematical specification of the model from which its equations of motion are obtained. Assigning a convenient source separately can destroy that consistency.

Reading the equation is not the same as solving it. We must specify the matter laws, boundary or initial data and an appropriate approximation. A weak-field solution is valuable within its regime, but does not automatically settle strong fields, nonlinear evolution or quantum gravity.

The central achievement is the common structure: clocks, freely falling matter and light all respond to one metric, while matter's energy and stress help determine that metric. The coupling value itself remains an input to the theory.

Try 15. Name two source contributions other than rest-mass density that appear in the gravitational account.

Thermodynamics connects energy exchanges, temperature and entropy. Black holes have a corresponding set of relations once quantum effects are included. Their entropy depends on horizon area, which raises a question about state counting rather than merely about the size of a storage container.

16 Why do black holes connect information with area?

Before reading on. If black-hole entropy scales with horizon area, does it follow that every ordinary region must store all its information physically on its surface?

Entropy measures the multiplicity or uncertainty of microscopic states compatible with a stated description, using an appropriate physical definition. Boltzmann's constant converts a dimensionless entropy measure into thermodynamic units. For equally probable microstates, entropy grows as the logarithm of their number: each doubling adds the same amount, rather than doubling the entropy. Black-hole thermodynamics assigns an entropy proportional to horizon area. In the usual semi-classical relation, the entropy in units of Boltzmann's constant is one quarter of the area measured in Planck-area units. A Planck area is a very small area unit built from the gravitational constant, the reduced Planck constant and the vacuum light speed. Expressing the horizon area in that unit and dividing by four gives the entropy in units of Boltzmann's constant. The coefficient connects gravity, quantum theory and thermodynamics.

For a spherical horizon, doubling the radius multiplies its area by four. The associated black-hole entropy therefore increases by four under that comparison. It does not grow by the volume factor of eight. That difference is a clue about gravitational state counting.

For illustration, a horizon area of 100 million Planck-area units would give an entropy of 25 million Boltzmann units. Increasing the area

to 400 million would give 100 million. These numbers illustrate the proportionality, not a prediction for an astrophysical hole. Boltzmann units are not a literal count of binary memory cells; a bit-based entropy uses a different logarithm convention.

Try 16. If a spherical horizon's radius doubles, what happens to its area and to the entropy assigned by the semiclassical area law?

The area law is not a literal photograph of microscopic bits painted on a surface. A duality is a correspondence between two descriptions that match their physical predictions. It can encode information in variables organised very differently from the original description. Holographic examples make such relations precise under specific assumptions. They do not license a universal replacement of every bulk quantum state by an arbitrary boundary ledger.

Quantum fields also have correlations across boundaries. Ignoring one part of an entangled state gives a mixed description of the remaining part. Relations between this entanglement and geometric area depend on the state, the theory and the short-distance description.

There are serious routes from thermodynamic or entanglement conditions to gravitational equations. Jacobson's 1995 argument uses local horizons, a universal area-entropy relation, the temperature seen by accelerated observers and a local heat relation to obtain Einstein's equation. It assumes a spacetime setting and specific equilibrium conditions.[2]

His later entanglement-equilibrium argument relates first-order variations of specified local vacuum states to the semiclassical Einstein equation, with additional qualifications for fields without conformal symmetry, the local scale symmetry used in the simplest form of the argument.[3] In a holographic setting, boundary entanglement constraints can give gravitational equations linearised about anti-de Sitter space. Linearised means retaining the first-order response to a small disturbance of the reference geometry. Anti-de Sitter space is a negatively curved reference geometry.[4] These are substantive connections with physical assumptions, rather than consequences of information counting alone.

For checking

Answers to the Try questions

Try the questions before reading these answers. Compare the physical assumptions and the first step of the reasoning, not just the final number. A matching number can hide a different interpretation.

1. Four seconds: $5/1.25 = 4$, or $5 \times 0.8 = 4$, under the stated constant-speed comparison.
2. Energy and momentum. General relativity's source and propagation laws are not restricted to rest mass.
3. Large amounts of mass-energy accumulate, while positive and negative electric charges nearly cancel in ordinary neutral matter.
4. By a factor of one hundred. The wavelength appears in the denominator of the stated squared ratio.
5. Yes. Null interval does not mean identical events or zero laboratory travel time. There is no photon rest-frame clock.
6. Relative tidal acceleration is the relevant curvature test. Apparent weight at one point can arise from acceleration of coordinates in flat spacetime.
7. Detected gravitational waveforms support propagating gravity. They do not constitute detection of individual gravitons.
8. A colour basis organises internal field components; spatial axes organise localisation and laboratory rotations. Matching the number of labels does not identify those roles.
9. An average source can approximate a mean field while omitting fluctuations and correlations with the different matter alternatives.
10. No. Local vacuum light speed remains the invariant speed. The horizon concerns which future causal paths can reach distant observers.
11. No, in that ideal interior. Smaller areal radius lies along every future-directed timelike path. The statement should not be copied unchanged to all black-hole interiors.

12. Temperature halves. Net mass loss requires emitted energy to exceed absorbed energy.
13. The ideal temperature doubles. The external agent and detector interaction must be included to account for the energy of excitation.
14. Two seconds: $2.5 \times 0.8 = 2$.
15. Examples include radiation energy, momentum density, energy flow and pressure or other stresses.
16. Both increase by a factor of four. Volume would instead increase by a factor of eight.

For returning to

Words and further reading

Words to return to

Inertial frame

Coordinates and clocks for which free motion is uniform in the setting where gravity is negligible, or locally in a freely falling laboratory.

Proper time

The time measured by a clock along its own path.

Coordinate time

A time label in a specified spacetime chart. It is not automatically the reading of every clock in that chart.

Metric

The rule converting nearby coordinate differences into spacetime intervals and clock readings.

Curvature

The geometrical content that cannot be removed simply by relabelling coordinates. Varying coefficients alone do not establish it.

Scale factor

A function describing how separations along the idealised large-scale cosmic flow change.

Event horizon

A boundary defined by which events can send signals to the specified distant future; locating it is a global question.

Semiclassical

An approximation combining quantum fields with a classical spacetime geometry. Its use does not establish a complete quantum theory of gravity.

Sources and their scope

Hawking derives semiclassical radiation, not a complete microscopic account of evaporation.[1] Jacobson and Faulkner and colleagues connect information and gravity under specific equilibrium or perturbative

assumptions; these results do not validate every proposed information substrate.[2, 3, 4]

Tong’s notes develop coordinates, metrics and curvature for readers ready for more mathematics.[5] The GW170817 analysis constrains wave propagation under stated emission assumptions; it does not count gravitons.[6] Crispino, Higuchi and Matsas review the Unruh effect and its interpretation.[7]

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